High-precision two-dimensional atom localization via probe absorption in an M-scheme atomic system*

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In the present paper, we investigate the behavior of two-dimensional atom localization in a five-level M-scheme atomic system driven by two orthogonal standing-wave fields. We find that the precision and resolution of the atom localization depends on the probe field detuning significantly. And because of the effect of the microwave field, an atom can be located at a particular position via adjusting the system parameters.

Keywords: two-dimensional atom localization, probe absorption

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1. Introduction

In recent years, the behavior of atom localization in the sub-wavelength domain has attracted much attention and has been extensively researched because of its wide applications such as laser cooling and trapping neutral atoms,[1,2] atomic nanolithography,[3,4] Bose–Einstein condensation,[5,6] etc. It is well known that atomic coherence and quantum interference have brought about many interesting physical phenomena, such as lasing without inversion,[7] electromagnetically induced transparency,[8–10] sub- and super-luminal light propagation,[11,12] etc. Besides, one-dimensional atom localization based on atomic coherence and quantum interference has been extensively discussed in multi-level atomic system. For example, Herkommer *et al*.[13] has realized one-dimensional localization via the Auter–Townes spontaneous emission spectrum. Qumar *et al*.[14] has proposed a simple model to obtain one-dimensional atom localization based on resonance fluorescence. Paspalakis and Knight[15] and Paspalakis *et al*.[16] have proposed a three-level Λ-type atomic system to realize sub-wavelength atom localization via the measurement of the upper-state population. Additionally, one-dimensional atom localization can also be obtained through the appropriate adjusting of the Raman gain process[17] and amplitude and phase control of the absorption spectrum.[18,19]

The behaviors of the two-dimensional atom localization in a multi-level atomic system have been studied in detail because of their practical applications. And efficient two-dimensional atom localization can be obtained via measuring the upper-state and ground-state populations,[20] via detecting the probe absorption spectrum,[21–23] or via measuring the spontaneously emitted photon.[24–26] It is found that only when two orthogonal standing-wave fields are coupled to the same atomic transition in a multi-level system with a cycle-configuration, can one locate the atom at an expected position.[22–25] When they are coupled to different atomic transitions, the precise localization of an atom at a particular position cannot be realized.[20,21,26]

According to the above observation, we try to realize two-dimensional atom localization in an M-scheme atomic system via the probe absorption. First, we investigate the effect of the existence of a cycle-configuration in an M-scheme atomic system on two-dimensional atom localization. We find that in the case of a cycle-configuration, an atom can be located at a particular position via adjusting the system parameters; second, when the atom is localized at a particular position, the corresponding probe detuning is not unique, but the spatial resolution of the atom localization is significantly different for different probe detunings.

2. Model and equations

We consider an M-scheme atom with two upper levels $|4\rangle$, and $|5\rangle$ and three ground levels $|1\rangle$, $|2\rangle$, and $|3\rangle$ as shown in Fig. 1. A weak probe field with Rabi frequency $\Omega_{\rm P}$ is applied to the transition $|4\rangle \leftrightarrow |1\rangle$, while the transition $|5\rangle \leftrightarrow |2\rangle$ is driven by two orthogonal standing-wave fields with Rabi frequency $\Omega_{xy} = \Omega_2[\sin(kx) + \sin(ky)]$. The transitions $|4\rangle \leftrightarrow |2\rangle$ and $|5\rangle \leftrightarrow |3\rangle$ are coupled by two travelling-wave fields with their Rabi frequency Ω_1 and Ω_3. And we add a microwave field with Rabi frequency $\Omega_{\rm m}$ to the forbidden transition $|3\rangle \leftrightarrow |2\rangle$ to form a cyclic configuration in the M-scheme atomic system. In the following equation derivation process, we will calculate the analytical expression of the M-scheme model with a cyclic configuration. However, the

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analytical expression of the M-scheme model without the microwave field can be obtained when we set $\Omega_{\rm m}=0$.

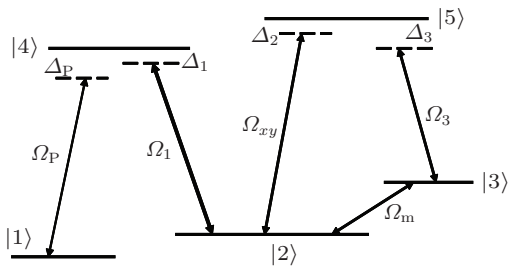


Fig. 1. Schematic diagram of M-type atomic system.

We assume that the center-of-mass position distribution of the atom is approximately in accordance with the direction of the standing wave, and we ignore the kinetic part of the atom in Hamiltonian by applying the Raman–Nath approximation.[27] Under the conditions of the electric dipole approximation and rotating-wave approximation, the Hamiltonian of the system can be written as ($\hbar=1$)

$$\begin{aligned} H \;=\; & \delta_2 |2\rangle\langle 2| + \delta_3 |3\rangle\langle 3| + \Delta_{\rm P} |4\rangle\langle 4| + \delta_5 |5\rangle\langle 5| \\ & + [\Omega_{\rm P} |4\rangle\langle 1| + \Omega_1 |4\rangle\langle 2| + \Omega_{xy} |5\rangle\langle 2| \\ & + \Omega_3 |5\rangle\langle 3| + \Omega_{\rm m} |3\rangle\langle 2| + {\rm h.c.}], \end{aligned} \tag{1}$$

where $\delta_2=\Delta_{\rm P}-\Delta_1$, $\delta_5=\delta_2+\Delta_2$, $\delta_3=\delta_5-\Delta_3$, with $\Delta_{\rm P}$, Δ_2, Δ_1, and Δ_3 being the detuning of the probe, standing-wave, and two traveling-wave fields, respectively; $\Omega_{\rm i}=\mu_{\rm i}E_{\rm i}/\hbar$ is the Rabi frequency of the corresponding optical field, with $\mu_{\rm i}$ and $E_{\rm i}$ being the electric dipole matrix element and the slowly varying amplitude of the corresponding optical field, respectively.

The dynamics of the system can be demonstrated by using the density matrix

$$\dot{\rho} = -\frac{\rm i}{\hbar}[H,\rho] - \frac{1}{2}\{\Gamma,\rho\}. \tag{2}$$

Substituting Eq. (1) into Eq. (2), the relevant off-diagonal density-matrix element equations can be obtained as follows:

$$\begin{aligned} {\rm i}\dot{\rho}_{21} &= \delta_2\rho_{21} + \Omega_{\rm m}\rho_{31} + \Omega_1\rho_{41} - \Omega_{\rm P}\rho_{24} + \Omega_{xy}\rho_{51}, \\ {\rm i}\dot{\rho}_{31} &= \delta_3\rho_{31} + \Omega_{\rm m}\rho_{21} - \Omega_{\rm P}\rho_{34} + \Omega_3\rho_{51}, \\ {\rm i}\dot{\rho}_{41} &= (\Delta_{\rm P} - {\rm i}\Gamma_4)\rho_{41} - \Omega_{\rm P}(\rho_{44}-\rho_{11}) + \Omega_1\rho_{21}, \\ {\rm i}\dot{\rho}_{51} &= (\delta_5 - {\rm i}\Gamma_5)\rho_{51} + \Omega_{xy}\rho_{21} + \Omega_3\rho_{31} - \Omega_{\rm P}\rho_{54}, \end{aligned} \tag{3}$$

where Γ_4 and Γ_5 represent the total decay rates from the upper levels $|4\rangle$ and $|5\rangle$ to the ground levels. Now, we can solve the long-time solution of the equation set (3). When the probe field is sufficiently weak, the perturbation approach can be used to solve the density-matrix equation set, and the population will be in the ground level $|1\rangle$, so we have

$$\rho_{11}=1, \quad \rho_{ij}=0 \quad (i\neq 1,\ j\neq 1). \tag{4}$$

To derive the off-diagonal matrix element ρ_{41} relating to the polarization of the probe field, we just need to solve the following equation set:

$$\begin{aligned} &\delta_2\rho_{21} + \Omega_{\rm m}\rho_{31} + \Omega_1\rho_{41} + \Omega_{xy}\rho_{51} = 0, \\ &\delta_3\rho_{31} + \Omega_{\rm m}\rho_{21} + \Omega_3\rho_{51} = 0, \\ &(\Delta_{\rm P} - {\rm i}\Gamma_4)\rho_{41} + \Omega_{\rm P}\rho_{11} + \Omega_1\rho_{21} = 0, \\ &(\delta_5 - {\rm i}\Gamma_5)\rho_{51} + \Omega_{\rm xy}\rho_{21} + \Omega_3\rho_{31} = 0. \end{aligned} \tag{5}$$

Through some simple algebraic operations, we can obtain

$$\rho_{41} = \frac{\Omega_{\rm P}[(\delta_2\delta_3-\Omega_{\rm m}^2)(\delta_3\delta_5-{\rm i}\delta_3\Gamma_5-\Omega_3^2)-(\Omega_{\rm m}\Omega_3-\delta_3\Omega_{xy})^2]}{(\Delta_{\rm P}-{\rm i}\Gamma_4)[(\Omega_{\rm m}\Omega_3-\delta_3\Omega_{xy})^2-(\delta_2\delta_3-\Omega_{\rm m}^2)(\delta_3\delta_5-{\rm i}\delta_3\Gamma_5-\Omega_3^2)]+\delta_3\Omega_1^2(\delta_3\delta_5-{\rm i}\delta_3\Gamma_5-\Omega_3^2)}. \tag{6}$$

The polarization relating to the probe field is[10]

$$\chi = \frac{N\mu_{41}^2\rho_{41}}{\varepsilon_0\hbar\Omega_{\rm P}} = \frac{K}{\Omega_{\rm P}}\rho_{41}. \tag{7}$$

Here, N represents the atomic density of the medium. From Eq. (7), the normalized absorption of the probe field can be obtained directly as

$$\chi' = \frac{1}{K}\,{\rm Im}\left[\frac{\rho_{41}}{\Omega_{\rm P}}\right]. \tag{8}$$

The result of Eq. (8) reflects the atomic position probability distribution under some conditions, which is the main task of our present work.

3. Results and discussion

In this section, we will analyze the results about two-dimensional atom localization according to Eq. (8) and observe the effect caused by the cyclic configuration in the M-scheme atomic system on the precise atomic localization. For simplicity, we take $\Delta_1=\Delta_2=\Delta_3=0$, $\Omega_1=20$, $\Omega_2=\Omega_3=10$, $\Gamma_4=\Gamma_5=1$.

First, we study the case without microwave field (no cyclic configuration in M-type atomic system). In Fig. 2, we depict the normalized probe absorption χ' versus the positions (kx,ky) for the different probe detunings. It can be seen from the figure that the conditional position probability distribution strongly depends on the probe detuning. When $\Delta_{\rm P}=6.8$, the probe absorption exhibits a spike-like pattern and the atom

is mainly localized in the first and the third quadrant [see Fig. 2(a)]. When we set $\Delta_P = 20$, it is shown that the atom mainly distributes in the second and the fourth quadrants [see Fig. 2(b)]. To increase the detuning $\Delta_P = 29.2$, the atomic distribution comes back to the first and third quadrants of the x–y plane, and at this time the peak values of the probe absorption are sharper than those in Fig. 2(a).

From Fig. 2, we can find that the probe absorption distributions are symmetrical along two diagonals of the square in the x–y plane. In dynamic numerical simulation process, when $0 < \Delta_P < 40$, the position distributions of the probe absorption along two diagonals are symmetrical. It means that we can locate the atom in two of the four quadrants by choosing appropriate parameters.

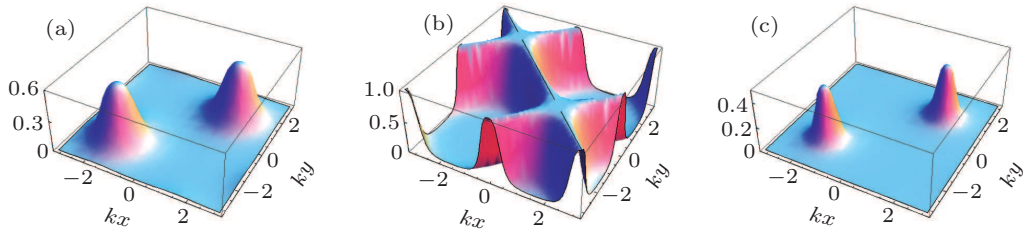


Fig. 2. (color online) Curves of probe absorption χ' versus positions (kx, ky) for different frequency detunings of probe field $\Delta_P = 6.8$ (a), 20 (b), and 29.2 (c).

In the following, we will study the case with the microwave field existing, and we set $\Omega_m = 10$. Figure 3 shows the normalized probe absorption χ' versus the positions (kx, ky) for different probe detunings. When $\Delta_P = 4.5$, the probe absorption is mainly distributed in the third quadrant [see Fig. 3(a)]. When $\Delta_P = 22.3$, the absorption peaks of the probe field are distributed in four different quadrants, but the amount in the third quadrant is least [see Fig. 3(b)]. With the further increase of Δ_P [$\Delta_P = 33$, see Fig. 3(c)], the absorption spectrum is located only in the third quadrant of the x–y plane, and absorption peak turns sharper than the absorption peak in Fig. 3(a). Based on the above analysis, the difference between Figs. 3 and 2 is obvious. In Fig. 3, the figures are symmetrical along one of the two diagonals of the square in the x–y plane.

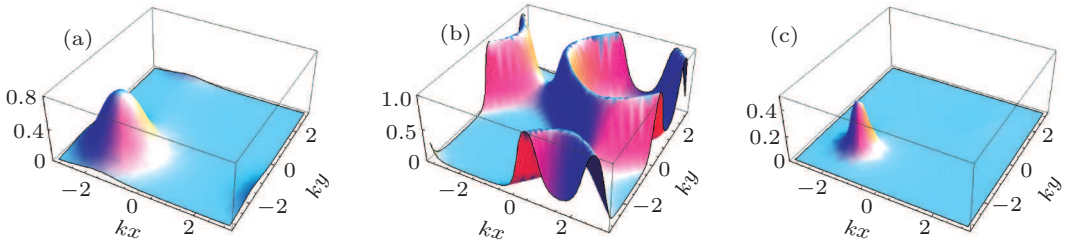


Fig. 3. (color online) Curves of probe absorption χ' versus positions (kx, ky) for different frequency detunings of probe field $\Delta_P = 4.5$ (a), 22.3 (b), and 33 (c).

From the above analysis, we can localize an atom at a particular position in an M-scheme atomic system with a cyclic-configuration. Comparing Fig. 3 with Fig. 2, the probability of finding the atom at a particular position increases from 1/2 to 1.

4. Conclusions

Two-dimensional atom localization behavior in an M-scheme atomic system is investigated via measuring the probe absorption spectrum, and the effect of the microwave field on precise atom localization is discussed. It is found that the atom can be located in two of the four quadrants in the model without cyclic-configuration via adjusting the system parameters; and the atom can be located in one of the four quadrants in the model with a cyclic-configuration with appropriate parameters. Additionally, we can improve the two-dimensional atom localization resolution via choosing appropriate probe detuning.

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