General hybrid projective complete dislocated synchronization with non-derivative and derivative coupling based on parameter identification in several chaotic and hyperchaotic systems*

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According to the Lyapunov stability theorem, a new general hybrid projective complete dislocated synchronization scheme with non-derivative and derivative coupling based on parameter identification is proposed under the framework of drive-response systems. Every state variable of the response system equals the summation of the hybrid drive systems in the previous hybrid synchronization. However, every state variable of the drive system equals the summation of the hybrid response systems while evolving with time in our method. Complete synchronization, hybrid dislocated synchronization, projective synchronization, non-derivative and derivative coupling, and parameter identification are included as its special item. The Lorenz chaotic system, Rössler chaotic system, memristor chaotic oscillator system, and hyperchaotic Lü system are discussed to show the effectiveness of the proposed methods.

Keywords: complete dislocated synchronization, parameter identification, non-derivative and derivative coupling, memristor chaotic oscillator system

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1. Introduction

Chaos in control systems and controlling chaos in dynamical systems have attracted much interest in recent years. A chaotic system has complex dynamical behaviors that possess special features, such as excessive sensitivity to initial conditions, broad spectrums of Fourier transform, bounded and fractal properties of the motion in the phase space, and so on. A synchronization method for two identical chaotic systems with different initial conditions has been introduced.[1,2] Chaos synchronization has been studied extensively by many researchers during the last few decades. So far, several theoretical methods have been developed to realize chaos synchronization, such as the feedback control method,[3–6] the adaptive control method,[7–9] the active-control control method,[10] the impulsive control method,[11] the passive control method,[12] the Bang–Bang control method,[13] and the robust control method,[14] etc. Up until now, different kinds of synchronization have been intensively studied and many theoretical results have been obtained, including generalized synchronization,[15–17] complete synchronization,[18] partial synchronization,[19] phase synchronization,[20,21] anti-phase synchronization,[22,23] lag synchronization,[24,25] and projective synchronization.[26–29]

Hu *et al*.[30] considered a full state hybrid projective synchronization (FSHPS), which bridges the gap from chaos control to chaos synchronization in some literature. It is obvious that complete synchronization and anti-synchronization are special cases of the above synchronization. A novel adaptive projective synchronization scheme is proposed to ensure that the drive system and the response system with nonlinearity inputs can be rapidly synchronized up to the given scaling factor.[31] As far as we know, a general hybrid projective complete dislocated synchronization with non-derivative and derivative coupling based on parameter identification has not been reported via adaptive method control, which motivates the current study.

Inspired by the above discussions, in this paper we will consider a novel scheme of a general hybrid projective complete dislocated synchronization with non-derivative and derivative coupling based on parameter identification. Compared with the previous methods, the summation of hybrid drive systems is modified for the summation of hybrid response systems. Non-derivative and derivative coupling, and parameter identification, are also considered during general hybrid projective complete dislocated synchronization. This seems to be a well-known idea in the chaos research field. The design is employed to realize the full state hybrid projective synchronization with non-derivative and derivative coupling between two identical chaotic systems and two different chaotic systems.

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This work is arranged as follows. In Section 2, we show general hybrid projective complete dislocated synchronization of two identical chaotic systems. In Section 3, general hybrid projective complete dislocated synchronization of two different chaotic systems is shown. Section 4 gives the conclusion of the paper.

2. General hybrid projective complete dislocated synchronization of two identical chaotic systems

In this section, hybrid projective complete dislocated synchronization with non-derivative and derivative coupling based on the parameter identification synchronization method is designed to achieve synchronization.

2.1. General scheme description for synchronization and parameter identification of two identical chaotic systems

Consider a drive system

$$\dot{x} = f(x) + g(x)\alpha, \tag{1}$$

and the corresponding response system

$$\dot{y} = f(y) + g(y)\tilde{\alpha} + u, \tag{2}$$

where $x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in R^n$ and $y = (y_1, y_2, \ldots, y_n)^{\mathrm{T}} \in R^n$ are state vectors, $f : R^n \to R^n$ and $g : R^n \to R^{n\times m}$ are two continuous functions, the estimated parameter vector of the parameter vector $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_m)^{\mathrm{T}} \in R^m$ is $\tilde{\alpha} = (\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_m)^{\mathrm{T}} \in R^m$, and u is a controller to be designed.

Definition 1 If there exists at least a non-zero constant $d_{ij} \neq 0$ $(i = 1, 2, \ldots, n, j = 1, 2, \ldots, m)$, the following condition

$$\lim_{t\to\infty} || \sum_{j=1, i\neq j}^{n} d_{ij}\dot{y}_j + x_i|| = 0 \tag{3}$$

can hold true, then the drive system (1) and the corresponding response system (2) are hybrid projective complete dislocated synchronization with non-derivative and derivative coupling based on parameter identification.

In other words, the error dynamics system of general hybrid projective complete dislocated synchronization with non-derivative and derivative coupling based on parameter identification is described by the following expressions

$$\begin{aligned} \dot{e} = & \left(\sum_{i=2}^{m} d_{1i}\dot{y}_i + \dot{x}_1, \sum_{i=1, i\neq 2}^{m} d_{2i}\dot{y}_i + \dot{x}_2, \ldots, \right. \\ & \left. \sum_{i=1, i\neq n}^{m} d_{ni}\dot{y}_i + \dot{x}_n \right)^{\mathrm{T}}, \end{aligned} \tag{4}$$

$$\dot{e}_\alpha = \dot{\tilde{\alpha}} - \dot{\alpha} = (\dot{\tilde{\alpha}}_1 - \dot{\alpha}_1, \dot{\tilde{\alpha}}_2 - \dot{\alpha}_2, \ldots, \dot{\tilde{\alpha}}_m - \dot{\alpha}_m)^{\mathrm{T}}. \tag{5}$$

The Lyapunov function is chosen as follows:

$$V = \frac{1}{2}e^{\mathrm{T}}Pe + \frac{1}{2}e_\alpha^{\mathrm{T}}Qe_\alpha, \tag{6}$$

where P and Q are the positive-definite constant matrices. The time derivative of V along the trajectories of Eq. (6) is

$$\dot{V} = \frac{1}{2}(\dot{e}^{\mathrm{T}}Pe + e^{\mathrm{T}}P\dot{e}) + \frac{1}{2}(\dot{e}_\alpha^{\mathrm{T}}Qe_\alpha + e_\alpha^{\mathrm{T}}Q\dot{e}_\alpha). \tag{7}$$

If a suitable feedback control law $u(x, y) \in R^n$ and constants d_{ij} exist to make $\dot{V} \leq 0$ hold true, then according to the Lyapunov stability theorem, the drive system (1) and the corresponding response system (2) are general hybrid projective complete dislocated synchronization with non-derivative and derivative coupling based on parameter identification.

Remark 1 When $d_{ij} = 1$ $(i = 1, 2, \ldots, n, j = 1, 2, \ldots, m, i \neq j)$, and the rest $d_{ij} = 0$, then complete dislocated synchronization will appear. When $d_{ij} = -1$ $(i = 1, 2, \ldots, n, j = 1, 2, \ldots, m, i \neq j)$, and the rest $d_{ij} = 0$, then dislocated anti-synchronization will appear. When $d_{ij} = -\lambda_i$ $(i = 1, 2, \ldots, n, \lambda_i$ is constant, $\lambda_i \neq 0, 1, i \neq j)$, and the rest $d_{ij} = 0$, then projective dislocated synchronization will appear.

2.2. Simulation and results

The memristor chaotic oscillator system has been modified,[32]

$$\begin{cases} \dot{x}_1 = a_1(x_3 - \varphi(x_4)x_1), \\ \dot{x}_2 = a_2x_2 - a_3x_3, \\ \dot{x}_3 = x_2 - x_1 - a_4x_3, \\ \dot{x}_4 = x_1, \end{cases} \tag{8}$$

where x_1, x_2, x_3, x_4 are state variables, and a_1, a_2, a_3, a_4 are the real constants. $q(w)$ is a piece-wise linear function

$$q(w) = bw + 0.5(a - b)(|w + 1| - |w - 1|), \tag{9}$$

where w is the state variable, and a and b are constants. $\varphi(w)$ is provided by the following expression

$$\varphi(w) = \frac{\mathrm{d}q(w)}{\mathrm{d}w} = \begin{cases} a, & |w| < 1, \\ b, & |w| > 1. \end{cases} \tag{10}$$

In fact, the system (8) is chaotic when $a_1 = 0.31$, $a_2 = 0.35$, $a_3 = 0.29$, $a_4 = 0.41$. We assume that we have two memristor chaotic oscillator systems, and the drive system with the variable x drives the response system with identical equations denoted the variable y. The drive system is system (8), and the response system is defined as follows:

$$\begin{cases} \dot{y}_1 = b_1(y_3 - \varphi(y_4)y_1) + u_1, \\ \dot{y}_2 = b_2y_2 - b_3y_3 + u_2, \\ \dot{y}_3 = y_2 - y_1 - b_4y_3 + u_3, \\ \dot{y}_4 = y_1 + u_4, \end{cases} \tag{11}$$

where b_1, b_2, b_3, and b_4 are the parameters of the response system (11) which needs to be estimated, and u_1, u_2, u_3, and u_4 are the nonlinear controllers such that two chaotic systems (8) and (11) can be synchronized.

Let

$$\begin{cases} e_1 = x_1 + d_{12}\dot{y}_2 + d_{13}\dot{y}_3 + d_{14}\dot{y}_4, \\ e_2 = x_2 + d_{21}\dot{y}_1 + d_{23}\dot{y}_3 + d_{24}\dot{y}_4, \\ e_3 = x_3 + d_{31}\dot{y}_1 + d_{32}\dot{y}_2 + d_{34}\dot{y}_4, \\ e_4 = x_4 + d_{41}\dot{y}_1 + d_{42}\dot{y}_2 + d_{43}\dot{y}_3, \end{cases} \tag{12}$$

where d_{ij} $(i = 1,2,3,4, j = 1,2,3,4, i \neq j)$ are constants.

Substituting Eq. (11) into Eq. (12), we have

$$\begin{cases} e_1 = x_1 + d_{12}(b_2y_2 - b_3y_3 + u_2) \\ \quad + d_{13}(y_2 - y_1 - b_4y_3 + u_3) + d_{14}(y_1 + u_4), \\ e_2 = x_2 + d_{21}[b_1(y_3 - \varphi(y_4)y_1) + u_1] \\ \quad + d_{23}(y_2 - y_1 - b_4y_3 + u_3) + d_{24}(y_1 + u_4), \\ e_3 = x_3 + d_{31}[b_1(y_3 - \varphi(y_4)y_1) + u_1] \\ \quad + d_{32}(b_2y_2 - b_3y_3 + u_2) + d_{34}(y_1 + u_4), \\ e_4 = x_4 + d_{41}[b_1(y_3 - \varphi(y_4)y_1) + u_1] \\ \quad + d_{42}(b_2y_2 - b_3y_3 + u_2) + d_{43}(y_2 - y_1 - b_4y_3 + u_3). \end{cases} \tag{13}$$

Let

$$\begin{aligned} E \;=\; & d_{13}d_{24}d_{31}d_{42} - d_{12}d_{24}d_{31}d_{43} - d_{14}d_{23}d_{31}d_{42} \\ & - d_{13}d_{21}d_{34}d_{42} - d_{14}d_{21}d_{32}d_{43} \\ & + d_{12}d_{21}d_{34}d_{43} - d_{13}d_{24}d_{32}d_{41} \\ & + d_{14}d_{23}d_{32}d_{41} - d_{12}d_{23}d_{34}d_{41}, \end{aligned} \tag{14}$$

$$\begin{cases} A_1 = A(d_{23}d_{34}d_{42} + d_{32}d_{24}d_{43}) \\ \quad + B(d_{12}d_{34}d_{43} - d_{14}d_{32}d_{43} - d_{13}d_{34}d_{42}) \\ \quad + C(d_{13}d_{24}d_{42} - d_{12}d_{24}d_{43} - d_{14}d_{23}d_{42}) \\ \quad + D(d_{14}d_{23}d_{32} - d_{12}d_{23}d_{34} - d_{13}d_{24}d_{32}), \\ A_2 = A(d_{21}d_{34}d_{43} - d_{31}d_{24}d_{43} - d_{23}d_{34}d_{41}) \\ \quad + B(d_{13}d_{34}d_{41} + d_{14}d_{31}d_{43}) \\ \quad + C(d_{14}d_{41}d_{23} - d_{14}d_{21}d_{43} - d_{13}d_{24}d_{41}) \\ \quad + D(d_{13}d_{24}d_{31} - d_{13}d_{21}d_{34} - d_{14}d_{23}d_{31}), \\ A_3 = -A(d_{24}d_{32}d_{41} + d_{21}d_{34}d_{42} - d_{24}d_{31}d_{42}) \\ \quad - B(d_{14}d_{31}d_{42} + d_{12}d_{34}d_{41} - d_{14}d_{32}d_{41}) \\ \quad + C(d_{12}d_{24}d_{41} + d_{14}d_{21}d_{42}) \\ \quad - D(d_{12}d_{24}d_{31} + d_{14}d_{21}d_{32} - d_{12}d_{21}d_{34}), \\ A_4 = A(-d_{21}d_{32}d_{43} - d_{23}d_{31}d_{42} + d_{23}d_{32}d_{41}) \\ \quad + B(-d_{32}d_{13}d_{41} - d_{12}d_{31}d_{43} + d_{13}d_{31}d_{42}) \\ \quad + C(-d_{13}d_{21}d_{42} - d_{12}d_{23}d_{41} + d_{12}d_{21}d_{43}) \\ \quad + D(d_{13}d_{21}d_{32} + d_{12}d_{23}d_{31}), \end{cases} \tag{15}$$

and

$$\begin{cases} A = (b_1 - a_1) - (x_1 + d_{12}\dot{y}_2 + d_{13}\dot{y}_3 + d_{14}\dot{y}_4) \\ \quad - a_1[x_3 - \varphi(x_4)x_1] - d_{12}(b_2\dot{y}_2 - b_3\dot{y}_3) \\ \quad - d_{13}(\dot{y}_2 - \dot{y}_1 - b_4\dot{y}_3) - d_{14}\dot{y}_1, \\ B = (b_2 - a_2) - (x_2 + d_{21}\dot{y}_1 + d_{23}\dot{y}_3 + d_{24}\dot{y}_4) \\ \quad - (a_2x_2 - a_3x_3) - d_{21}b_1[\dot{y}_3 - \varphi(y_4)\dot{y}_1] \\ \quad - d_{23}(\dot{y}_2 - \dot{y}_1 - b_4\dot{y}_3) - d_{24}\dot{y}_1, \\ C = (b_3 - a_3) - (x_3 + d_{31}\dot{y}_1 + d_{32}\dot{y}_2 + d_{34}\dot{y}_4) \\ \quad - (x_2 - x_1 - a_4x_3) - d_{31}b_1[\dot{y}_3 - \varphi(y_4)\dot{y}_1] \\ \quad - d_{32}(b_2\dot{y}_2 - b_3\dot{y}_3) - d_{34}\dot{y}_1, \\ D = (b_4 - a_4) - (x_4 + d_{41}\dot{y}_1 + d_{42}\dot{y}_2 + d_{43}\dot{y}_3) \\ \quad - x_1 - d_{41}b_1[\dot{y}_3 - \varphi(y_4)\dot{y}_1] - d_{42}(b_2\dot{y}_2 - b_3\dot{y}_3) \\ \quad - d_{43}(\dot{y}_2 - \dot{y}_1 - b_4\dot{y}_3). \end{cases} \tag{16}$$

Substituting Eq. (11) into Eq. (16), we obtain

$$\begin{cases} A = (b_1 - a_1) - [x_1 + d_{12}(b_2y_2 - b_3y_3 + u_2) + d_{13}(y_2 - y_1 - b_4y_3 + u_3) \\ \quad + d_{14}(y_1 + u_4)] - a_1(x_3 - \varphi(x_4)x_1) - d_{12}[b_2(b_2y_2 - b_3y_3 + u_2) - b_3(y_2 \\ \quad - y_1 - b_4y_3 + u_3)] - d_{13}\{(b_2y_2 - b_3y_3 + u_2) - [b_1(y_3 - \varphi(y_4)y_1) + u_1] \\ \quad - b_4(y_2 - y_1 - b_4y_3 + u_3)\} - d_{14}[b_1(y_3 - \varphi(y_4)y_1) + u_1], \\ B = (b_2 - a_2) - \{x_2 + d_{21}[b_1(y_3 - \varphi(y_4)y_1) + u_1] + d_{23}(y_2 - y_1 - b_4y_3 + u_3) \\ \quad + d_{24}(y_1 + u_4)\} - (a_2x_2 - a_3x_3) - d_{21}b_1\{(y_2 - y_1 - b_4y_3 + u_3) \\ \quad - \varphi(y_4)(b_1(y_3 - \varphi(y_4)y_1) + u_1)\} - d_{23}\{(b_2y_2 - b_3y_3 + u_2) - [b_1(y_3 \\ \quad - \varphi(y_4)y_1) + u_1] - b_4(y_2 - y_1 - b_4y_3 + u_3)\} - d_{24}[b_1(y_3 - \varphi(y_4)y_1) + u_1], \\ C = (b_3 - a_3) - [x_3 + d_{31}(b_1(y_3 - \varphi(y_4)y_1) + u_1) + d_{32}(b_2y_2 - b_3y_3 + u_2) + d_{34}(y_1 \\ \quad + u_4)] - (x_2 - x_1 - a_4x_3) - d_{31}b_1\{(y_2 - y_1 - b_4y_3 + u_3) - \varphi(y_4)[b_1(y_3 \\ \quad - \varphi(y_4)y_1) + u_1]\} - d_{32}[b_2(b_2y_2 - b_3y_3 + u_2) - b_3(y_2 - y_1 - b_4y_3 + u_3)] \\ \quad - d_{34}[b_1(y_3 - \varphi(y_4)y_1) + u_1], \\ D = (b_4 - a_4) - \{x_4 + d_{41}[b_1(y_3 - \varphi(y_4)y_1) + u_1] + d_{42}(b_2y_2 - b_3y_3 + u_2) \\ \quad + d_{43}(y_2 - y_1 - b_4y_3 + u_3)\} - x_1 - d_{41}b_1\{(y_2 - y_1 - b_4y_3 + u_3) - \varphi(y_4)[b_1(y_3 \\ \quad - \varphi(y_4)y_1) + u_1]\} - d_{42}[b_2(b_2y_2 - b_3y_3 + u_2) - b_3(y_2 - y_1 - b_4y_3 + u_3)] \\ \quad - d_{43}\{(b_2y_2 - b_3y_3 + u_2) - [b_1(y_3 - \varphi(y_4)y_1) + u_1] - b_4(y_2 - y_1 - b_4y_3 + u_3)\}. \end{cases} \tag{17}$$

Let the control law be as follows:

$$\begin{cases} \dot{u}_1 = A_1/E, \\ \dot{u}_2 = A_2/E, \\ \dot{u}_3 = A_3/E, \\ \dot{u}_4 = A_4/E. \end{cases} \tag{18}$$

Therefore, the implemented control input can be obtained as

$$\begin{cases} u_1 = \int_0^t A_1/E\,\mathrm{d}t, \\ u_2 = \int_0^t A_2/E\,\mathrm{d}t, \\ u_3 = \int_0^t A_3/E\,\mathrm{d}t, \\ u_4 = \int_0^t A_4/E\,\mathrm{d}t, \end{cases} \tag{19}$$

where $u_1(0) = u_2(0) = u_3(0) = u_4(0) = 0$.

The updated rules for unknown parameters a_1, a_2, a_3, and a_4 are described as follows:

$$\begin{cases} \dot{a}_1 = -e_1, \\ \dot{a}_2 = -e_2, \\ \dot{a}_3 = -e_3, \\ \dot{a}_4 = -e_4. \end{cases} \tag{20}$$

Theorem 1 The response system (11) synchronizes the drive system (8) for any initial conditions $(x_1(0), x_2(0), x_3(0), x_4(0))$, and $(y_1(0), y_2(0), y_3(0), y_4(0))$ by the control law (19) and the update law (20).

Proof It is easy to see that the error dynamics can be obtained as follows:

$$\begin{cases} \dot{e}_1 = \dot{x}_1 + d_{12}(b_2\dot{y}_2 - b_3\dot{y}_3 + \dot{u}_2) \\ \quad + d_{13}(\dot{y}_2 - \dot{y}_1 - b_4\dot{y}_3 + \dot{u}_3) + d_{14}(\dot{y}_1 + \dot{u}_4), \\ \dot{e}_2 = \dot{x}_2 + d_{21}[b_1(\dot{y}_3 - \varphi(y_4)\dot{y}_1) + \dot{u}_1] \\ \quad + d_{23}(\dot{y}_2 - \dot{y}_1 - b_4\dot{y}_3 + \dot{u}_3) + d_{24}(\dot{y}_1 + \dot{u}_4), \\ \dot{e}_3 = \dot{x}_3 + d_{31}[b_1(\dot{y}_3 - \varphi(y_4)\dot{y}_1) + \dot{u}_1] \\ \quad + d_{32}(b_2\dot{y}_2 - b_3\dot{y}_3 + \dot{u}_2) + d_{34}(\dot{y}_1 + \dot{u}_4), \\ \dot{e}_4 = \dot{x}_4 + d_{41}[b_1(\dot{y}_3 - \varphi(y_4)\dot{y}_1) + \dot{u}_1] \\ \quad + d_{42}(b_2\dot{y}_2 - b_3\dot{y}_3 + \dot{u}_2) + d_{43}(\dot{y}_2 - \dot{y}_1 - b_4\dot{y}_3 + \dot{u}_3). \end{cases} \tag{21}$$

Substituting Eqs. (14), (15), and (18) into Eq. (21), we have

$$\begin{cases} \dot{e}_1 = \dot{x}_1 + d_{12}(b_2\dot{y}_2 - b_3\dot{y}_3) \\ \quad + d_{13}(\dot{y}_2 - \dot{y}_1 - b_4\dot{y}_3) + d_{14}\dot{y}_1 + A, \\ \dot{e}_2 = \dot{x}_2 + d_{21}[b_1(\dot{y}_3 - \varphi(y_4)\dot{y}_1)] \\ \quad + d_{23}(\dot{y}_2 - \dot{y}_1 - b_4\dot{y}_3) + d_{24}\dot{y}_1 + B, \\ \dot{e}_3 = \dot{x}_3 + d_{31}[b_1(\dot{y}_3 - \varphi(y_4)\dot{y}_1)] \\ \quad + d_{32}(b_2\dot{y}_2 - b_3\dot{y}_3) + d_{34}\dot{y}_1 + C, \\ \dot{e}_4 = \dot{x}_4 + d_{41}[b_1(\dot{y}_3 - \varphi(y_4)\dot{y}_1)] \\ \quad + d_{42}(b_2\dot{y}_2 - b_3\dot{y}_3) + d_{43}(\dot{y}_2 - \dot{y}_1 - b_4\dot{y}_3) + D. \end{cases} \tag{22}$$

Substituting Eqs. (8), (11), and (17) into Eq. (22), we obtain

$$\begin{cases} \dot{e}_1 = (b_1 - a_1) - [x_1 + d_{12}(b_2y_2 - b_3y_3 + u_2) \\ \quad + d_{13}(y_2 - y_1 - b_4y_3 + u_3) + d_{14}(y_1 + u_4)], \\ \dot{e}_2 = (b_2 - a_2) - \{x_2 + d_{21}[b_1(y_3 - \varphi(y_4)y_1) + u_1] \\ \quad + d_{23}(y_2 - y_1 - b_4y_3 + u_3) + d_{24}(y_1 + u_4)\}, \\ \dot{e}_3 = (b_3 - a_3) - \{x_3 + d_{31}[b_1(y_3 - \varphi(y_4)y_1) + u_1] \\ \quad + d_{32}(b_2y_2 - b_3y_3 + u_2) + d_{34}(y_1 + u_4)\}, \\ \dot{e}_4 = (b_4 - a_4) - \{x_4 + d_{41}[b_1(y_3 - \varphi(y_4)y_1) + u_1] \\ \quad + d_{42}(b_2y_2 - b_3y_3 + u_2) \\ \quad + d_{43}(y_2 - y_1 - b_4y_3 + u_3)\}. \end{cases} \tag{23}$$

Choose the following Lyapunov candidate

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_{a_1}^2 + e_{a_2}^2 + e_{a_3}^2 + e_{a_4}^2), \tag{24}$$

where

$$e_{a_1} = b_1 - a_1, \quad e_{a_2} = b_2 - a_2, \quad e_{a_3} = b_3 - a_3, \quad e_{a_4} = b_4 - a_4.$$

Therefore, the differential of the Lyapunov function along the trajectory of Eq. (24) is

$$\begin{aligned} &\dot{V}(e_1, e_2, e_3, e_4, e_{a_1}, e_{a_2}, e_{a_3}, e_{a_4}) \\ &= \dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 e_3 + \dot{e}_4 e_4 + \dot{e}_{a_1} e_{a_1} \\ &\quad + \dot{e}_{a_2} e_{a_2} + \dot{e}_{a_3} e_{a_3} + \dot{e}_{a_4} e_{a_4}. \end{aligned} \tag{25}$$

Substituting Eq. (23) into Eq. (24), we gain

$$\begin{aligned} &\dot{V}(e_1, e_2, e_3, e_4, e_{a_1}, e_{a_2}, e_{a_3}, e_{a_4}) \\ &= \{(b_1 - a_1) - [x_1 + d_{12}(b_2y_2 - b_3y_3 + u_2) \\ &\quad + d_{13}(y_2 - y_1 - b_4y_3 + u_3) + d_{14}(y_1 + u_4)]\}e_1 \\ &\quad + \{(b_2 - a_2) - \{x_2 + d_{21}[b_1(y_3 - \varphi(y_4)y_1) + u_1] \\ &\quad + d_{23}(y_2 - y_1 - b_4y_3 + u_3) + d_{24}(y_1 \\ &\quad + u_4)\}\}e_2 + \{(b_3 - a_3) - \{x_3 + d_{31}[b_1(y_3 - \varphi(y_4)y_1) + u_1] \\ &\quad + d_{32}(b_2y_2 - b_3y_3 + u_2) + d_{34}(y_1 + u_4)\}\}e_3 \\ &\quad + \{(b_4 - a_4) - \{x_4 + d_{41}[b_1(y_3 - \varphi(y_4)y_1) + u_1] \\ &\quad + d_{42}(b_2y_2 - b_3y_3 + u_2) + d_{43}(y_2 - y_1 - b_4y_3 + u_3)\}\}e_4 \\ &\quad - (b_1 - a_1)e_1 - (b_2 - a_2)e_2 \\ &\quad - (b_3 - a_3)e_3 - (b_4 - a_4)e_4. \end{aligned} \tag{26}$$

From Eq. (19), we have

$$\begin{aligned} &\dot{V}(e_1, e_2, e_3, e_4, e_{a_1}, e_{a_2}, e_{a_3}, e_{a_4}) \\ &= (b_1 - a_1 - e_1)e_1 + (b_2 - a_2 - e_2)e_2 \\ &\quad + (b_3 - a_3 - e_3)e_3 + (b_4 - a_4 - e_4)e_4 - (b_1 - a_1)e_1 \\ &\quad - (b_2 - a_2)e_2 - (b_3 - a_3)e_3 - (b_4 - a_4)e_4 \\ &= -e_1^2 - e_2^2 - e_3^2 - e_4^2 \\ &= -e^{\mathrm{T}}e, \end{aligned} \tag{27}$$

where $e = [e_1, e_2, e_3, e_4]^{\mathrm{T}}$. Since $\dot{V}$ is negative-semidefinite, we cannot immediately find that the origin of Eq. (23) is asymptotically stable. In fact, as $\dot{V} \leq 0$, then $e_1, e_2, e_3, e_4 \in \ell_\infty$ and $e_{a_1}, e_{a_2}, e_{a_3}, e_{a_4} \in \ell_\infty$. From the error system (11), we have $\dot{e}_1$, $\dot{e}_2$, $\dot{e}_3$, $\dot{e}_4 \in \ell_\infty$, $\dot{e}_{a_1}$, $\dot{e}_{a_2}$, $\dot{e}_{a_3}$, $\dot{e}_{a_4} \in \ell_\infty$. Since $\dot{V} = -e^{\mathrm{T}}e$, then we have

$$\begin{aligned} &\int_0^t ||e||^2 \mathrm{d}t \leq \int_0^t e^{\mathrm{T}}e \mathrm{d}t \leq \int_0^t -\dot{V} \mathrm{d}t \\ &= V(0) - V(t) \leq V(0) < \infty. \end{aligned} \tag{28}$$

Thus e_1, e_2, e_3, $e_4 \in \ell_2$, according to Barbalat's lemma, we have e_1, e_2, e_3, $e_4 \to 0$ $(t \to \infty)$. Therefore, the response system (11) synchronizes the drive system (8) by the control law (19) and the update law (20). This completes the proof.

Remark 2 In this paper, we only consider the case where a chaotic system with unknown parameters cannot be known in advance, but it has a certain topology structure.

Remark 3 The control inputs u_1, u_2, u_3, and u_4 are too complex to achieve the synchronization problem. u_1, u_2, u_3, and u_4 are the key to our method.

In the simulation, the fourth-order Runge–Kutta method is used to solve the systems with the time step size 0.001. We assume that the initial states, $(x_1(0), x_2(0), x_3(0), x_4(0)) = (10^{-4}, 2 \times 10^{-4}, 3 \times 10^{-4}, 4 \times 10^{-5})$ and $(y_1(0), y_2(0),$

$y_3(0)$,$y_4(0)) = (2\times10^{-4}, 3\times10^{-4}, 4\times10^{-4}, 5\times10^{-5})$ are employed, respectively. The parameters are chosen as $d_{12} = -0.002$, $d_{13} = 0.003$, $d_{14} = 0.001$, $d_{21} = -0.002$, $d_{23} = 0.003$, $d_{24} = 0.001$, $d_{31} = -0.002$, $d_{32} = 0.003$, $d_{34} = 0.001$, $d_{41} = -0.002$, $d_{42} = 0.003$, $d_{43} = 0.001$, respectively. Synchronization of the systems (8) and (11) via hybrid dislocated adaptive control law (19) and the update law (20) with the initial estimated parameters $a_1 = 0.5$, $a_2 = 0.5$, $a_3 = 0.5$, and $a_4 = 0.5$ are shown in Figs. 1 and 2. Figure 1 displays the synchronization errors of systems (8) and (11). Figure 2 shows that the estimated values of the unknown parameters a_1, a_2, a_3, a_4 converge to $a_1 = 0.31$, $a_2 = 0.35$, $a_3 = 0.29$, and $a_4 = 0.41$.

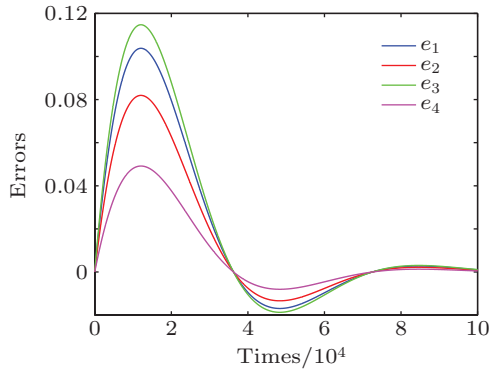


Fig. 1. (color online) Synchronization errors e_1, e_2, e_3, e_4 of the coupled memristor chaotic oscillator system.

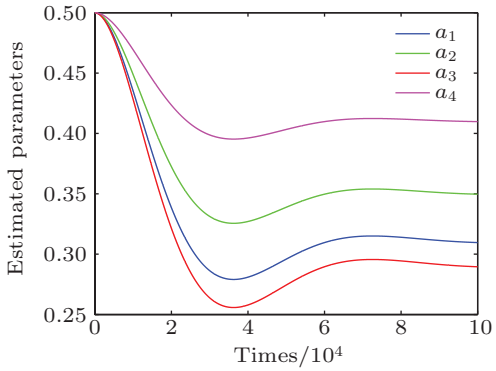


Fig. 2. (color online) Estimated parameters of the controlled memristor chaotic oscillator system.

Remark 4 In the simulation, d_{ij} $(i = 1,2,3,4, j = 1,2,3,4, i \neq j)$ are chosen to make

$$d_{13}d_{24}d_{31}d_{42} - d_{12}d_{24}d_{31}d_{43} - d_{14}d_{23}d_{31}d_{42} - d_{13}d_{21}d_{34}d_{42} - d_{14}d_{21}d_{32}d_{43} + d_{12}d_{21}d_{34}d_{43} - d_{13}d_{24}d_{32}d_{41} + d_{14}d_{23}d_{32}d_{41} - d_{12}d_{23}d_{34}d_{41} \neq 0$$

hold true.

3. General hybrid projective complete dislocated synchronization of two different chaotic systems

In this section, a general description scheme for general hybrid projective complete dislocated synchronization with non-derivative and derivative coupling based on parameter identification is designed to achieve synchronization for two different chaotic systems. Two kinds of cases are considered.

3.1. General description scheme for synchronization and parameter identification of two different chaotic systems (I)

Consider a drive system

$$\dot{x} = f_1(x) + g_1(x)\alpha, \tag{29}$$

and the corresponding response system

$$\dot{y} = f_2(y) + g_2(y)\beta + u, \tag{30}$$

where $x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in R^n$ and $y = (y_1, y_2, \ldots, y_n)^{\mathrm{T}} \in R^n$ are state vectors, $f_1, f_2 : R^n \to R^n$ are two continuous functions, $g_1, g_2 : R^n \to R^{n\times m}$ are two continuous matrix functions, the estimated parameter vectors of the parameter vectors $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_m)^{\mathrm{T}} \in R^m$ and $\beta = (\beta_1, \beta_2, \ldots, \beta_m)^{\mathrm{T}} \in R^m$ are $\tilde{\alpha} = (\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_m)^{\mathrm{T}} \in R^m$ and $\tilde{\beta} = (\tilde{\beta}_1, \tilde{\beta}_2, \ldots, \tilde{\beta}_m)^{\mathrm{T}} \in R^m$, respectively, and u is a controller to be designed.

Definition 2 If a non-zero constant $d_{ij} \neq 0$ $(i = 1,2,\ldots,n, j = 1,2,\ldots,m)$ exists, then the following condition

$$\lim_{t\to\infty} || \sum_{j=1, i\neq j}^{n} d_{ij}\dot{y}_j + x_i || = 0 \tag{31}$$

can hold true, and then the drive system (29) and the corresponding response system (30) are hybrid projective complete dislocated synchronization with non-derivative and derivative coupling based on parameter identification.

In other words, the error dynamics system of general hybrid projective complete dislocated synchronization with non-derivative and derivative coupling based on parameter identification can be obtained as follows:

$$\dot{e} = \left(\sum_{i=2}^{n} d_{1i}\ddot{y}_i + \dot{x}_1, \sum_{i=1, i\neq 2}^{n} d_{2i}\ddot{y}_i + \dot{x}_2, \cdots, \sum_{i=1, i\neq m}^{n} d_{mi}\ddot{y}_i + \dot{x}_m \right)^{\mathrm{T}}, \tag{32}$$

$$\dot{e}_\alpha = \dot{\tilde{\alpha}} - \dot{\alpha} = (\dot{\tilde{\alpha}}_1 - \dot{\alpha}_1, \dot{\tilde{\alpha}}_2 - \dot{\alpha}_2, \ldots, \dot{\tilde{\alpha}}_m - \dot{\alpha}_m)^{\mathrm{T}}, \tag{33}$$

$$\dot{e}_\beta = \dot{\tilde{\beta}} - \dot{\beta} = (\dot{\tilde{\beta}}_1 - \dot{\beta}_1, \dot{\tilde{\beta}}_2 - \dot{\beta}_2, \ldots, \dot{\tilde{\beta}}_m - \dot{\beta}_m)^{\mathrm{T}}. \tag{34}$$

The Lyapunov function is chosen as follows:

$$V = \frac{1}{2}e^{\mathrm{T}}Pe + \frac{1}{2}e_\alpha^{\mathrm{T}}Qe_\alpha + \frac{1}{2}e_\beta^{\mathrm{T}}Re_\beta, \tag{35}$$

where P, Q, and R are the positive-definite constant matrices. The time derivative of V along the trajectories of Eq. (35) is

$$\dot{V} = \frac{1}{2}(\dot{e}^{\mathrm{T}}Pe + e^{\mathrm{T}}P\dot{e}) + \frac{1}{2}(\dot{e}_\alpha^{\mathrm{T}}Qe_\alpha + e_\alpha^{\mathrm{T}}Q\dot{e}_\alpha) + \frac{1}{2}(\dot{e}_\beta^{\mathrm{T}}Re_\beta + e_\beta^{\mathrm{T}}R\dot{e}_\beta). \tag{36}$$

If there exists a suitable feedback control law $u(x,y) \in R^m$ and constants d_{ij} to make $\dot{V} \leq 0$ hold true, then according to the Lyapunov stability theorem, the drive system (29) and the corresponding response system (30) are general hybrid projective complete dislocated synchronization with non-derivative and derivative coupling based on parameter identification.

3.2. Simulation and results

The Lorenz system is described by

$$\begin{cases} \dot{x}_1 = a_1(x_2 - x_1), \\ \dot{x}_2 = a_2 x_1 - x_1 x_3 - x_2, \\ \dot{x}_3 = x_1 x_2 - a_3 x_3, \end{cases} \tag{37}$$

where x_1, x_2, x_3 are state variables, and a_1, a_2, a_3 are parameters. The response Rössler system is defined as follows:

$$\begin{cases} \dot{y}_1 = -(y_2 + y_1) + u_1, \\ \dot{y}_2 = y_1 - b_1 y_2 + u_2, \\ \dot{y}_3 = y_3(y_1 - b_2) + b_3 + u_3, \end{cases} \tag{38}$$

where y_1, y_2, y_3 are state variables, b_1, b_2, b_3 are parameters of the response system (38) which need to be estimated, and u_1, u_2, and u_3 are the nonlinear control law such that two chaotic systems, (37) and (38), can be synchronized.

Let

$$\begin{cases} e_1 = x_1 + d_{12}\dot{y}_2 + d_{13}\dot{y}_3, \\ e_2 = x_2 + d_{21}\dot{y}_1 + d_{23}\dot{y}_3, \\ e_3 = x_3 + d_{31}\dot{y}_1 + d_{32}\dot{y}_2, \end{cases} \tag{39}$$

where d_{ij} $(i = 1,2,3, j = 1,2,3, i \neq j)$ are constants.

Substituting Eq. (38) into Eq. (39), we have

$$\begin{cases} e_1 = x_1 + d_{12}(y_1 - b_1 y_2 + u_2) \\ \quad + d_{13}[y_3(y_1 - b_2) + b_3 + u_3], \\ e_2 = x_2 + d_{21}[-(y_2 + y_1) + u_1] \\ \quad + d_{23}[y_3(y_1 - b_2) + b_3 + u_3], \\ e_3 = x_3 + d_{31}[-(y_2 + y_1) + u_1] \\ \quad + d_{32}(y_1 - b_1 y_2 + u_2). \end{cases} \tag{40}$$

Let the control law be as follows:

$$\begin{cases} \dot{u}_1 = (-d_{23}d_{32}A + d_{13}d_{32}B + d_{12}d_{23}C)/(d_{12}d_{23}d_{31} + d_{13}d_{21}d_{32}), \\ \dot{u}_2 = (d_{23}d_{31}A - d_{13}d_{31}B + d_{13}d_{21}C)/(d_{12}d_{23}d_{31} + d_{13}d_{21}d_{32}), \\ \dot{u}_3 = (d_{21}d_{32}A + d_{12}d_{31}B - d_{12}d_{21}C)/(d_{12}d_{23}d_{31} + d_{13}d_{21}d_{32}). \end{cases} \tag{41}$$

Therefore, the implemented control input can be obtained as

$$\begin{cases} u_1 = \int_0^t (-d_{23}d_{32}A + d_{13}d_{32}B + d_{12}d_{23}C)/(d_{12}d_{23}d_{31} + d_{13}d_{21}d_{32})\,\mathrm{d}t, \\ u_2 = \int_0^t (d_{23}d_{31}A - d_{13}d_{31}B + d_{13}d_{21}C)/(d_{12}d_{23}d_{31} + d_{13}d_{21}d_{32})\,\mathrm{d}t, \\ u_3 = \int_0^t (d_{21}d_{32}A + d_{12}d_{31}B - d_{12}d_{21}C)/(d_{12}d_{23}d_{31} + d_{13}d_{21}d_{32})\,\mathrm{d}t, \end{cases} \tag{42}$$

where $u_1(0) = u_2(0) = u_3(0) = 0$ with

$$\begin{cases} A = (a_1^* - a_1 + b_1^* - b_1) - (x_1 + d_{12}\dot{y}_2 + d_{13}\dot{y}_3) - a_1(x_2 - x_1) \\ \quad - d_{12}(\dot{y}_1 - b_1\dot{y}_2) - d_{13}(\dot{y}_3 y_1 + y_3\dot{y}_1 - b_2), \\ B = (a_2^* - a_2 + b_2^* - b_2) - (x_2 + d_{21}\dot{y}_1 + d_{23}\dot{y}_3) - (a_2 x_1 - x_1 x_3 \\ \quad - x_2) + d_{21}(\dot{y}_2 + \dot{y}_1) - d_{23}(\dot{y}_3 y_1 + y_3\dot{y}_1 - b_2), \\ C = (a_3^* - a_3 + b_3^* - b_3) - (x_3 + d_{31}\dot{y}_1 + d_{32}\dot{y}_2) - (x_1 x_2 - a_3 x_3) \\ \quad + d_{31}(\dot{y}_2 + \dot{y}_1) - d_{32}(\dot{y}_1 - b_1\dot{y}_2). \end{cases} \tag{43}$$

Substituting Eq. (38) into Eq. (43), we get

$$\begin{cases} A = (a_1^* - a_1 + b_1^* - b_1) - \{x_1 + d_{12}(y_1 - b_1 y_2 + u_2) + d_{13}[y_3(y_1 - b_2) + b_3 + u_3]\} \\ \quad - a_1(x_2 - x_1) - d_{12}[-(y_2 + y_1) + u_1 - b_1(y_1 - b_1 y_2 + u_2)] \\ \quad - d_{13}\{[y_3(y_1 - b_2) + b_3 + u_3]y_1 + y_3[-(y_2 + y_1) + u_1] - b_2\}, \\ B = (a_2^* - a_2 + b_2^* - b_2) - \{x_2 + d_{21}[-(y_2 + y_1) + u_1] + d_{23}[y_3(y_1 - b_2) + b_3 + u_3]\} \\ \quad - (a_2 x_1 - x_1 x_3 - x_2) + d_{21}[(y_1 - b_1 y_2 + u_2) - (y_2 + y_1) + u_1] \\ \quad - d_{23}\{[y_3(y_1 - b_2) + b_3 + u_3]y_1 + y_3[-(y_2 + y_1) + u_1] - b_2\}, \\ C = (a_3^* - a_3 + b_3^* - b_3) - \{x_3 + d_{31}[-(y_2 + y_1) + u_1] + d_{32}(y_1 - b_1 y_2 + u_2)\} \\ \quad - (x_1 x_2 - a_3 x_3) + d_{31}[(y_1 - b_1 y_2 + u_2) - (y_2 + y_1) + u_1] - d_{32}[-(y_2 + y_1) \\ \quad + u_1 - b_1(y_1 - b_1 y_2 + u_2)]. \end{cases} \tag{44}$$

The update rules for unknown parameters a_1^*, a_2^*, a_3^*, b_1^*, b_2^*, and b_3^* are described as follows:

$$\begin{cases} \dot{a}_1^* = -e_1, \\ \dot{a}_2^* = -e_2, \\ \dot{a}_3^* = -e_3, \end{cases} \tag{45}$$

$$\begin{cases} \dot{b}_1^* = -e_1, \\ \dot{b}_2^* = -e_2, \\ \dot{b}_3^* = -e_3. \end{cases} \tag{46}$$

Theorem 2 The response system (38) synchronizes the drive system (37) for any initial conditions $(x_1(0), x_2(0), x_3(0))$ and $(y_1(0), y_2(0), y_3(0))$ by the control law (41) and the update laws (45) and (46).

Proof It is easy to see that the error dynamics can be obtained as follows:

$$\begin{cases} \dot{e}_1 = \dot{x}_1 + d_{12}(\dot{y}_1 - b_1\dot{y}_2 + \dot{u}_2) \\ \qquad + d_{13}(\dot{y}_3 y_1 + \dot{y}_1 y_3 - b_2 + \dot{u}_3), \\ \dot{e}_2 = \dot{x}_2 + d_{21}[-(\dot{y}_2 + \dot{y}_1) + \dot{u}_1] \\ \qquad + d_{23}(\dot{y}_3 y_1 + \dot{y}_1 y_3 - b_2 + \dot{u}_3), \\ \dot{e}_3 = \dot{x}_3 + d_{31}[-(\dot{y}_2 + \dot{y}_1) + \dot{u}_1] \\ \qquad + d_{32}(\dot{y}_1 - b_1\dot{y}_2 + \dot{u}_2). \end{cases} \tag{47}$$

Substituting Eq. (41) into Eq. (47), we have

$$\begin{cases} \dot{e}_1 = \dot{x}_1 + d_{12}(\dot{y}_1 - b_1\dot{y}_2) \\ \qquad + d_{13}(\dot{y}_3 y_1 + \dot{y}_1 y_3 - b_2) + A, \\ \dot{e}_2 = \dot{x}_2 + d_{21}[-(\dot{y}_2 + \dot{y}_1)] \\ \qquad + d_{23}(\dot{y}_3 y_1 + \dot{y}_1 y_3 - b_2) + B, \\ \dot{e}_3 = \dot{x}_3 + d_{31}[-(\dot{y}_2 + \dot{y}_1)] \\ \qquad + d_{32}(\dot{y}_1 - b_1\dot{y}_2) + C. \end{cases} \tag{48}$$

Substituting Eqs. (37), (38), and (44) into Eq. (48), we gain

$$\begin{cases} \dot{e}_1 = (a_1^* - a_1 + b_1^* - b_1) - \{x_1 + d_{12}(y_1 - b_1 y_2 + u_2) \\ \qquad + d_{13}[y_3(y_1 - b_2) + b_3 + u_3]\}, \\ \dot{e}_2 = (a_2^* - a_2 + b_2^* - b_2) - [x_2 + d_{21}(-(y_2 + y_1) + u_1) \\ \qquad + d_{23}(y_3(y_1 - b_2) + b_3 + u_3)], \\ \dot{e}_3 = (a_3^* - a_3 + b_3^* - b_3) - [x_3 + d_{31}(-(y_2 + y_1) + u_1) \\ \qquad + d_{32}(y_1 - b_1 y_2 + u_2)]. \end{cases} \tag{49}$$

Choose the following Lyapunov candidate

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_{a_1}^2 + e_{a_2}^2 + e_{a_3}^2 + e_{b_1}^2 + e_{b_2}^2 + e_{b_3}^2), \tag{50}$$

where

$$e_{a_1} = a_1^* - a_1, \quad e_{a_2} = a_2^* - a_2, \quad e_{a_3} = a_3^* - a_3,$$
$$e_{b_1} = b_1^* - b_1, \quad e_{b_2} = b_2^* - b_2, \quad e_{b_3} = b_3^* - b_3.$$

Therefore, the differential of the Lyapunov function along the trajectory of Eq. (50) is

$$\begin{aligned} &\dot{V}(e_1, e_2, e_3, e_{a_1}, e_{a_2}, e_{a_3}, e_{b_1}, e_{b_2}, e_{b_3}) \\ &= \dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 e_3 + \dot{e}_{a_1} e_{a_1} + \dot{e}_{a_2} e_{a_2} \\ &\quad + \dot{e}_{a_3} e_{a_3} + \dot{e}_{b_1} e_{b_1} + \dot{e}_{b_2} e_{b_2} + \dot{e}_{b_3} e_{b_3}. \end{aligned} \tag{51}$$

Substituting Eq. (49) into Eq. (51), we get

$$\begin{aligned} &\dot{V}(e_1, e_2, e_3, e_{a_1}, e_{a_2}, e_{a_3}, e_{b_1}, e_{b_2}, e_{b_3}) \\ &= \{(a_1^* - a_1 + b_1^* - b_1) - [x_1 + d_{12}(y_1 - b_1 y_2 + u_2) \\ &\quad + d_{13}(y_3(y_1 - b_2) + b_3 + u_3)]\}e_1 \\ &\quad + \{(a_2^* - a_2 + b_2^* - b_2) - [x_2 + d_{21}(-(y_2 + y_1) + u_1) \\ &\quad + d_{23}(y_3(y_1 - b_2) + b_3 + u_3)]\}e_2 \\ &\quad + \{(a_3^* - a_3 + b_3^* - b_3) - [x_3 + d_{31}(-(y_2 + y_1) + u_1) \\ &\quad + d_{32}(y_1 - b_1 y_2 + u_2)]\}e_3 - (a_1^* - a_1)e_1 \\ &\quad - (a_2^* - a_2)e_2 - (a_3^* - a_3)e_3 \\ &\quad - (b_1^* - b_1)e_1 - (b_2^* - b_2)e_2 - (b_3^* - b_3)e_3. \end{aligned} \tag{52}$$

By using Eq. (41), we have

$$\begin{aligned} &\dot{V}(e_1, e_2, e_3, e_{a_1}, e_{a_2}, e_{a_3}, e_{b_1}, e_{b_2}, e_{b_3}) \\ &= (a_1^* - a_1 + b_1^* - b_1 - e_1)e_1 + (a_2^* - a_2 + b_2^* - b_2 - e_2)e_2 \\ &\quad + (a_3^* - a_3 + b_3^* - b_3 - e_3)e_3 \\ &\quad - (a_1^* - a_1)e_1 - (a_2^* - a_2)e_2 - (a_3^* - a_3)e_3 \\ &\quad - (b_1^* - b_1)e_1 - (b_2^* - b_2)e_2 - (b_3^* - b_3)e_3 \\ &= -e_1^2 - e_2^2 - e_3^2 \\ &= -e^{\mathrm{T}}e, \end{aligned} \tag{53}$$

where $e = [e_1,\ e_2,\ e_3]^{\mathrm{T}}$. Since $\dot{V}$ is negative-semidefinite, we cannot immediately find that the origin of Eq. (40) is asymptotically stable. In fact, as $\dot{V} \le 0$, then $e_1, e_2, e_3 \in \ell_\infty$, $e_{a_1}, e_{a_2}, e_{a_3} \in \ell_\infty$, and $e_{b_1}, e_{b_2}, e_{b_3} \in \ell_\infty$. From the error system (40), we have $\dot{e}_1, \dot{e}_2, \dot{e}_3 \in \ell_\infty$, $\dot{e}_{a_1}, \dot{e}_{a_2}, \dot{e}_{a_3} \in \ell_\infty$, and $\dot{e}_{b_1}, \dot{e}_{b_2}, \dot{e}_{b_3} \in \ell_\infty$. Since $\dot{V} = -e^{\mathrm{T}}e$, then we have

$$\begin{aligned} &\int_0^t ||e||^2 \mathrm{d}t \le \int_0^t e^{\mathrm{T}} e \mathrm{d}t \le \int_0^t -\dot{V}\,\mathrm{d}t \\ &= V(0) - V(t) \le V(0) < \infty. \end{aligned} \tag{54}$$

Thus e_1, e_2, $e_3 \in \ell_2$, according to Barbalat's lemma, we have e_1, e_2, $e_3 \to 0$ $(t \to \infty)$. The response system (38) therefore synchronizes the drive system (37) by the control law (41) and the update laws (45) and (46). This completes the proof.

In the simulation, the fourth-order Runge–Kutta method is used to solve the systems with step size 0.01. We assume that the initial states, $(x_1(0), x_2(0), x_3(0)) = (1.5, 0.2, 0.3)$ and $(y_1(0), y_2(0),\ y_3(0)) = (4, 5, 6)$ are employed, respectively. The parameters are chosen as $d_{12} = -1, d_{13} = 2, d_{21} = 1, d_{23} = -1, d_{31} = 1, d_{32} = -2$. Synchronization of systems (37) and (38) are completed via the control law (41) and the update laws (45) and (46) in Fig. 3. The initial states for the drive system (37) and response system (38) are taken as $a_1 = 10$, $a_2 = 8/3$, $a_3 = 28$ and $b_1 = 0.5$, $b_2 = 0.5$, $b_3 = 0.5$, respectively. Figures 4 and 5 demonstrate the estimated values of the unknown parameters for the uncertain drive system, which imply that the uncertain parameters can be identified via control law (41) and the update laws (45) and (46).

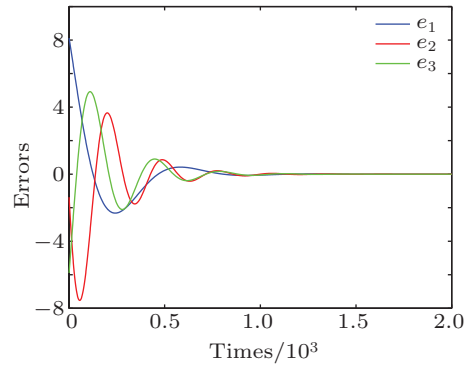


Fig. 3. (color online) Synchronization errors e_1, e_2, e_3 between the Lorenz system and the Rössler system.

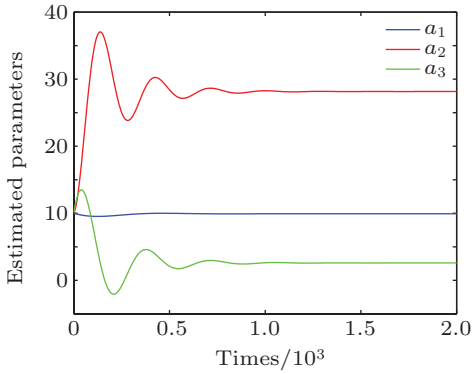


Fig. 4. (color online) The estimated parameters of the Lorenz system.

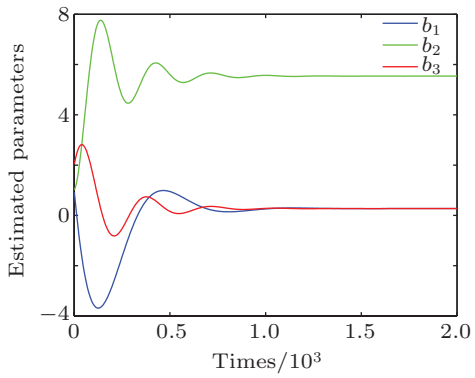


Fig. 5. (color online) The estimated parameters of the Rössler system.

Remark 5 In the simulation, d_{ij} $(i = 1,2,3,\ j = 1,2,3,\ i \neq j)$ are chosen to make $d_{12}d_{23}d_{31} + d_{13}d_{21}d_{32} \neq 0$ hold true.

3.3. General description scheme for synchronization and parameter identification of two different chaotic systems (II)

Definition 3 For the drive system (29) and the response system (30), let the vector error state be

$$e = A\dot{Y} + BX, \tag{55}$$

if there exists a suitable feedback controller $u(x,y) \in R^m$ to make

$$\lim_{t\to\infty} ||e(t)|| = 0,$$

hold true, then the general hybrid projective complete dislocated synchronization between chaotic systems (29) and (30) is achieved, where $e = [e_1, e_2, \ldots, e_m]^{\mathrm{T}}$, $Y = [y_1, y_2, \ldots, y_m]^{\mathrm{T}}$, $X = [x_1, x_2, \ldots, x_n]^{\mathrm{T}}$, $A = (d_{ij})_{m\times m}$, $B = (d_{ij})_{m\times n}$.

In the following, we will give a principle to find suitable feedback control law $u(x,y)$ such that the two chaotic systems are hybrid projective complete dislocated synchronization with non-derivative and derivative coupling based on parameter identification. Construct a dynamical Lyapunov function

$$V = \frac{1}{2}e^{\mathrm{T}}Pe + \frac{1}{2}e_\alpha^{\mathrm{T}}Qe_\alpha + \frac{1}{2}e_\beta^{\mathrm{T}}Re_\beta, \tag{56}$$

where P, Q, and R are the positive-definite constant matrices. The time derivative of V along the trajectories of Eq. (56) is

$$\begin{aligned}\dot{V} = {} & \frac{1}{2}(\dot{e}^{\mathrm{T}}Pe + e^{\mathrm{T}}P\dot{e}) + \frac{1}{2}(\dot{e}_\alpha^{\mathrm{T}}Qe_\alpha + e_\alpha^{\mathrm{T}}Q\dot{e}_\alpha) \\ & + \frac{1}{2}(\dot{e}_\beta^{\mathrm{T}}Re_\beta + e_\beta^{\mathrm{T}}R\dot{e}_\beta).\end{aligned} \tag{57}$$

Suppose we can select an appropriate control law $u(x,y)$ such that $\dot{V}$ is negative-semidefinite. Then, based on Lyapunov's direct method, the general hybrid projective complete dislocated synchronization with non-derivative and derivative coupling based on parameter identification of chaotic systems (29) and (30) is achieved under the certain feedback control law $u(x,y)$.

3.4. Simulation and results

In this subsection, a simulation example is given to show the effectiveness of the above method. We assume that the memristor chaotic oscillator system (8) drives the following hyperchaotic Lü system

$$\begin{cases} \dot{y}_1 = -b_1(y_2 - y_1) + u_1, \\ \dot{y}_2 = b_2y_2 - y_1y_3 + u_2, \\ \dot{y}_3 = y_1y_2 - b_3y_3 + u_3, \\ \dot{y}_4 = y_1y_3 - b_4y_4 + u_4. \end{cases} \tag{58}$$

Let

$$\begin{cases} e_1 = d_{11}\dot{y}_1 + d_{12}\dot{y}_2 + d_{13}x_3 + d_{14}x_4, \\ e_2 = d_{21}\dot{y}_2 + d_{22}\dot{y}_3 + d_{23}x_1 + d_{24}x_4, \\ e_3 = d_{31}\dot{y}_3 + d_{32}\dot{y}_4 + d_{33}x_1 + d_{34}x_2, \\ e_4 = d_{41}\dot{y}_4 + d_{42}\dot{y}_1 + d_{43}x_2 + d_{44}x_3, \end{cases} \tag{59}$$

where d_{ij} $(i = 1,2,3,4, j = 1,2,3,4)$ are constants.

Substituting Eq. (58) into Eq. (59), we have

$$\begin{cases} e_1 = d_{11}[-b_1(y_2 - y_1) + u_1] \\ \quad + d_{12}(b_2y_2 - y_1y_3 + u_2) + d_{13}x_3 + d_{14}x_4, \\ e_2 = d_{21}(b_2y_2 - y_1y_3 + u_2) \\ \quad + d_{22}(y_1y_2 - b_3y_3 + u_3) + d_{23}x_1 + d_{24}x_4, \\ e_3 = d_{31}(y_1y_2 - b_3y_3 + u_3) \\ \quad + d_{32}(y_1y_3 - b_4y_4 + u_4) + d_{33}x_1 + d_{34}x_2, \\ e_4 = d_{41}(y_1y_3 - b_4y_4 + u_4) \\ \quad + d_{42}[-b_1(y_2 - y_1) + u_1] + d_{43}x_2 + d_{44}x_3. \end{cases} \tag{60}$$

Let

$$E = d_{11}d_{21}d_{31}d_{41} - d_{12}d_{22}d_{32}d_{42}, \tag{61}$$

$$\begin{cases} A_1 = d_{21}d_{31}d_{41}A - d_{12}d_{31}d_{41}B + d_{12}d_{22}d_{41}C - d_{12}d_{22}d_{32}D, \\ A_2 = -d_{22}d_{32}d_{42}A + d_{11}d_{31}d_{41}B - d_{11}d_{22}d_{41}C + d_{11}d_{22}d_{32}D, \\ A_3 = d_{21}d_{32}d_{21}A - d_{12}d_{32}d_{42}B + d_{11}d_{21}d_{41}C - d_{11}d_{21}d_{32}D, \\ A_4 = -d_{21}d_{31}d_{42}A + d_{12}d_{31}d_{41}B - d_{12}d_{22}d_{42}C + d_{11}d_{21}d_{31}D, \end{cases} \tag{62}$$

$$\begin{cases} A = (a_1 - a_1^* + b_1 - b_1^*) - (d_{11}\dot{y}_1 + d_{12}\dot{y}_2 + d_{13}x_3 + d_{14}x_4) + d_{11}b_1(\dot{y}_2 \\ \quad - \dot{y}_1) - d_{12}(b_2\dot{y}_2 - y_1\dot{y}_3 - \dot{y}_1y_3) - d_{13}(x_2 - x_1 - a_4x_3) - d_{14}x_1, \\ B = (a_2 - a_2^* + b_2 - b_2^*) - (d_{21}\dot{y}_2 + d_{22}\dot{y}_3 + d_{23}x_1 + d_{24}x_4) - d_{21}(b_2\dot{y}_2 \\ \quad - y_1\dot{y}_3 - \dot{y}_1y_3) - d_{22}(y_1\dot{y}_2 + \dot{y}_1y_2 - b_3\dot{y}_3) - d_{23}(a_1(x_3 - \varphi(x_4)x_1)) - d_{24}x_1, \\ C = (a_3 - a_3^* + b_3 - b_3^*) - (d_{31}\dot{y}_3 + d_{32}\dot{y}_4 + d_{33}x_1 + d_{34}x_2) - d_{31}(y_1\dot{y}_2 + \dot{y}_1y_2 \\ \quad - b_3\dot{y}_3) - d_{32}(y_1\dot{y}_3 + \dot{y}_1y_3 - b_4\dot{y}_4) - d_{33}a_1(x_3 - \varphi(x_4)x_1) - d_{34}(a_2x_2 - a_3x_3), \\ D = (a_4 - a_4^* + b_4 - b_4^*) - (d_{41}\dot{y}_4 + d_{42}\dot{y}_1 + d_{43}x_2 + d_{44}x_3) - d_{41}(y_1\dot{y}_3 + \dot{y}_1y_3 - b_4\dot{y}_4) \\ \quad + d_{42}b_1(\dot{y}_2 - \dot{y}_1) - d_{43}(a_2x_2 - a_3x_3) - d_{44}(x_2 - x_1 - a_4x_3). \end{cases} \tag{63}$$

Substituting Eq. (58) into Eq. (63), we obtain

$$\begin{cases} A = (a_1 - a_1^* + b_1 - b_1^*) - \{d_{11}[-b_1(y_2 - y_1) + u_1] + d_{12}(b_2y_2 - y_1y_3 + u_2) + d_{13}x_3 + d_{14}x_4\} \\ \quad + d_{11}\{b_1[b_2y_2 - y_1y_3 + u_2 + b_1(y_2 - y_1) - u_1]\} - d_{12}\{b_2(b_2y_2 - y_1y_3 + u_2) - y_1(y_1y_2 - b_3y_3 + u_3) \\ \quad - [-b_1(y_2 - y_1) + u_1]y_3\} - d_{13}(x_2 - x_1 - a_4x_3) - d_{14}x_1, \\ B = (a_2 - a_2^* + b_2 - b_2^*) - [d_{21}(b_2y_2 - y_1y_3 + u_2) + d_{22}(y_1y_2 - b_3y_3 + u_3) + d_{23}x_1 + d_{24}x_4] \\ \quad - d_{21}\{b_2(b_2y_2 - y_1y_3 + u_2) - y_1(y_1y_2 - b_3y_3 + u_3) - [-b_1(y_2 - y_1) + u_1]y_3\} \\ \quad - d_{22}\{y_1(b_2y_2 - y_1y_3 + u_2) + [-b_1(y_2 - y_1) + u_1]y_2 - b_3(y_1y_2 - b_3y_3 + u_3)\} - d_{23}[a_1(x_3 - \varphi(x_4)x_1)] - d_{24}x_1, \\ C = (a_3 - a_3^* + b_3 - b_3^*) - [d_{31}(y_1y_2 - b_3y_3 + u_3) + d_{32}(y_1y_3 - b_4y_4 + u_4) + d_{33}x_1 + d_{34}x_2] - d_{31}\{y_1(b_2y_2 - y_1y_3 + u_2) \\ \quad + [-b_1(y_2 - y_1) + u_1]y_2 - b_3(y_1y_2 - b_3y_3 + u_3)\} - d_{32}\{y_1(y_1y_2 - b_3y_3 + u_3) \\ \quad + [-b_1(y_2 - y_1) + u_1]y_3 - b_4(y_1y_3 - b_4y_4 + u_4)\} - d_{33}a_1(x_3 - \varphi(x_4)x_1) - d_{34}(a_2x_2 - a_3x_3), \\ D = (a_4 - a_4^* + b_4 - b_4^*) - \{d_{41}(y_1y_3 - b_4y_4 + u_4) + d_{42}[-b_1(y_2 - y_1) + u_1] + d_{43}x_2 + d_{44}x_3\} \\ \quad - d_{41}\{y_1(y_1y_2 - b_3y_3 + u_3) + [-b_1(y_2 - y_1) + u_1]y_3 - b_4(y_1y_3 - b_4y_4 + u_4)\} \\ \quad - d_{42}\{-b_1[b_2y_2 - y_1y_3 + u_2 + b_1(y_2 - y_1) - u_1]\} - d_{43}(a_2x_2 - a_3x_3) - d_{44}(x_2 - x_1 - a_4x_3). \end{cases} \tag{64}$$

Let the control law be as follows:

$$\begin{cases} \dot{u}_1 = A_1/E, \\ \dot{u}_2 = A_2/E, \\ \dot{u}_3 = A_3/E, \\ \dot{u}_4 = A_4/E. \end{cases} \tag{65}$$

Therefore, the implemented control input can be obtained as

$$\begin{cases} u_1 = \int_0^t A_1/E\,\mathrm{d}t, \\ u_2 = \int_0^t A_2/E\,\mathrm{d}t, \\ u_3 = \int_0^t A_3/E\,\mathrm{d}t, \\ u_4 = \int_0^t A_4/E\,\mathrm{d}t, \end{cases} \tag{66}$$

where $u_1(0) = u_2(0) = u_3(0) = u_4(0) = 0$, and the update rules for unknown parameters a_1^*, a_2^*, a_3^*, a_4^*, b_1^*, b_2^*, b_3^*, and b_4^* are described as follows:

$$\begin{cases} \dot{a}_1^* = -e_1, \\ \dot{a}_2^* = -e_2, \\ \dot{a}_3^* = -e_3, \\ \dot{a}_4^* = -e_4, \end{cases} \tag{67}$$

$$\begin{cases} \dot{b}_1^* = -e_1, \\ \dot{b}_2^* = -e_2, \\ \dot{b}_3^* = -e_3, \\ \dot{b}_4^* = -e_4. \end{cases} \tag{68}$$

Theorem 3 The response system (58) synchronizes the drive system (8) for any initial conditions $(x_1(0), x_2(0), x_3(0), x_4(0))$, and $(y_1(0), y_2(0), y_3(0), y_4(0))$ by the control law (66) and the update laws (67) and (68).

Proof It is easy to see that the error dynamics can be obtained as follows:

$$\begin{cases} \dot{e}_1 = d_{11}[-b_1(\dot{y}_2 - \dot{y}_1) + \dot{u}_1] \\ \quad + d_{12}(b_2\dot{y}_2 - y_1\dot{y}_3 - \dot{y}_1y_3 + \dot{u}_2) + d_{13}\dot{x}_3 + d_{14}\dot{x}_4, \\ \dot{e}_2 = d_{21}(b_2\dot{y}_2 - y_1\dot{y}_3 - \dot{y}_1y_3 + \dot{u}_2) \\ \quad + d_{22}(y_1\dot{y}_2 + \dot{y}_1y_2 - b_3\dot{y}_3 + \dot{u}_3) + d_{23}\dot{x}_1 + d_{24}\dot{x}_4, \\ \dot{e}_3 = d_{31}(y_1\dot{y}_2 + \dot{y}_1y_2 - b_3\dot{y}_3 + \dot{u}_3) \\ \quad + d_{32}(y_1\dot{y}_3 + \dot{y}_1y_3 - b_4\dot{y}_4 + \dot{u}_4) + d_{33}\dot{x}_1 + d_{34}\dot{x}_2, \\ \dot{e}_4 = d_{41}(y_1\dot{y}_3 + \dot{y}_1y_3 - b_4\dot{y}_4 + \dot{u}_4) \\ \quad + d_{42}[-b_1(\dot{y}_2 - \dot{y}_1) + \dot{u}_1] + d_{43}\dot{x}_2 + d_{44}\dot{x}_3. \end{cases} \tag{69}$$

Substituting Eqs. (61), (62), and (65) into Eq. (69), we have

$$\begin{cases} \dot{e}_1 = d_{11}[-b_1(\dot{y}_2 - \dot{y}_1)] + d_{12}(b_2\dot{y}_2 - y_1\dot{y}_3 - \dot{y}_1y_3) \\ \quad + d_{13}\dot{x}_3 + d_{14}\dot{x}_4 + A, \\ \dot{e}_2 = d_{21}(b_2\dot{y}_2 - y_1\dot{y}_3 - \dot{y}_1y_3) \\ \quad + d_{22}(y_1\dot{y}_2 + \dot{y}_1y_2 - b_3\dot{y}_3) + d_{23}\dot{x}_1 + d_{24}\dot{x}_4 + B, \\ \dot{e}_3 = d_{31}(y_1\dot{y}_2 + \dot{y}_1y_2 - b_3\dot{y}_3) \\ \quad + d_{32}(y_1\dot{y}_3 + \dot{y}_1y_3 - b_4\dot{y}_4) + d_{33}\dot{x}_1 + d_{34}\dot{x}_2 + C, \\ \dot{e}_4 = d_{41}(y_1\dot{y}_3 + \dot{y}_1y_3 - b_4\dot{y}_4) \\ \quad + d_{42}[-b_1(\dot{y}_2 - \dot{y}_1)] + d_{43}\dot{x}_2 + d_{44}\dot{x}_3 + D. \end{cases} \tag{70}$$

Substituting Eqs. (8), (58), and (64) into Eq. (70), we have

$$\begin{cases} \dot{e}_1 = (a_1 - a_1^* + b_1 - b_1^*) - \{d_{11}[-b_1(y_2 - y_1) + u_1] \\ \quad + d_{12}(b_2 y_2 - y_1 y_3 + u_2) + d_{13}x_3 + d_{14}x_4\}, \\ \dot{e}_2 = (a_2 - a_2^* + b_2 - b_2^*) - [d_{21}(b_2 y_2 - y_1 y_3 + u_2) \\ \quad + d_{22}(y_1 y_2 - b_3 y_3 + u_3) + d_{23}x_1 + d_{24}x_4], \\ \dot{e}_3 = (a_3 - a_3^* + b_3 - b_3^*) - [d_{31}(y_1 y_2 - b_3 y_3 + u_3) \\ \quad + d_{32}(y_1 y_3 - b_4 y_4 + u_4) + d_{33}x_1 + d_{34}x_2], \\ \dot{e}_4 = (a_4 - a_4^* + b_4 - b_4^*) - \{d_{41}(y_1 y_3 - b_4 y_4 + u_4) \\ \quad + d_{42}[-b_1(y_2 - y_1) + u_1] + d_{43}x_2 + d_{44}x_3\}. \end{cases} \tag{71}$$

Choose the following Lyapunov candidate

$$\begin{aligned} V = & \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_{a_1}^2 + e_{a_2}^2 + e_{a_3}^2 \\ & + e_{a_4}^2 + e_{b_1}^2 + e_{b_2}^2 + e_{b_3}^2 + e_{b_4}^2), \end{aligned} \tag{72}$$

where

$$\begin{aligned} & e_{a_1} = a_1^* - a_1, \quad e_{a_2} = a_2^* - a_2, \quad e_{a_3} = a_3^* - a_3, \\ & e_{a_4} = a_4^* - a_4, \quad e_{b_1} = b_1^* - b_1, \quad e_{b_2} = b_2^* - b_2, \\ & e_{b_3} = b_3^* - b_3, \quad e_{b_4} = b_4^* - b_4. \end{aligned}$$

Therefore, the differential of the Lyapunov function along the trajectory of Eq. (60) is

$$\begin{aligned} & \dot{V}(e_1, e_2, e_3, e_{a_1}, e_{a_2}, e_{a_3}, e_{a_4}, e_{b_1}, e_{b_2}, e_{b_3}, e_{b_4}) \\ = & \dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 e_3 + \dot{e}_4 e_4 + \dot{e}_{a_1} e_{a_1} + \dot{e}_{a_2} e_{a_2} \\ & + \dot{e}_{a_3} e_{a_3} + \dot{e}_{a_4} e_{a_4} + \dot{e}_{b_1} e_{b_1} + \dot{e}_{b_2} e_{b_2} + \dot{e}_{b_3} e_{b_3} + \dot{e}_{b_4} e_{b_4}. \end{aligned} \tag{73}$$

Substituting Eq. (71) into Eq. (73), we obtain

$$\begin{aligned} & \dot{V}(e_1, e_2, e_3, e_{a_1}, e_{a_2}, e_{a_3}, e_{a_4}, e_{b_1}, e_{b_2}, e_{b_3}, e_{b_4}) \\ = & \{(a_1 - a_1^* + b_1 - b_1^*) - \{d_{11}[-b_1(y_2 - y_1) + u_1] \\ & + d_{12}(b_2 y_2 - y_1 y_3 + u_2) + d_{13}x_3 + d_{14}x_4\}\}e_1 \\ & + \{(a_2 - a_2^* + b_2 - b_2^*) - [d_{21}(b_2 y_2 - y_1 y_3 + u_2) \\ & + d_{22}(y_1 y_2 - b_3 y_3 + u_3) + d_{23}x_1 + d_{24}x_4]\}e_2 \\ & + \{(a_3 - a_3^* + b_3 - b_3^*) - [d_{31}(y_1 y_2 - b_3 y_3 + u_3) \\ & + d_{32}(y_1 y_3 - b_4 y_4 + u_4) + d_{33}x_1 + d_{34}x_2]\}e_3 \\ & + \{(a_4 - a_4^* + b_4 - b_4^*) - \{d_{41}(y_1 y_3 - b_4 y_4 + u_4) \\ & + d_{42}[-b_1(y_2 - y_1) + u_1] + d_{43}x_2 + d_{44}x_3\}\}e_4 \\ & - (a_1^* - a_1)e_1 - (a_2^* - a_2)e_2 - (a_3^* - a_3)e_3 - (a_4^* - a_4)e_4 \\ & - (b_1^* - b_1)e_1 - (b_2^* - b_2)e_2 \\ & - (b_3^* - b_3)e_3 - (b_4^* - b_4)e_4. \end{aligned} \tag{74}$$

By using Eq. (66), we have

$$\begin{aligned} & \dot{V}(e_1, e_2, e_3, e_{a_1}, e_{a_2}, e_{a_3}, e_{a_4}, e_{b_1}, e_{b_2}, e_{b_3}, e_{b_4}) \\ = & (a_1^* - a_1 + b_1^* - b_1 - e_1)e_1 + (a_2^* - a_2 + b_2^* - b_2 - e_2)e_2 \\ & + (a_3^* - a_3 + b_3^* - b_3 - e_3)e_3 \\ & + (a_4^* - a_4 + b_4^* - b_4 - e_4)e_4 - (a_1^* - a_1)e_1 \\ & - (a_2^* - a_2)e_2 - (a_3^* - a_3)e_3 - (a_4^* - a_4)e_4 \\ & - (b_1^* - b_1)e_1 - (b_2^* - b_2)e_2 - (b_3^* - b_3)e_3 - (b_4^* - b_4)e_4 \\ = & -e_1^2 - e_2^2 - e_3^2 - e_4^2 \\ = & -e^{\mathrm{T}}e, \end{aligned} \tag{75}$$

where $e = [e_1, e_2, e_3, e_4]^{\mathrm{T}}$. Since $\dot{V}$ is negative-semidefinite, we cannot immediately find that the origin of Eq. (60) is asymptotically stable. In fact, as $\dot{V} \leq 0$, then $e_1, e_2, e_3, e_4 \in \ell_\infty$, e_{a_1}, e_{a_2}, e_{a_3}, $e_{a_4} \in \ell_\infty$, and e_{b_1}, e_{b_2}, e_{b_3}, $e_{b_4} \in \ell_\infty$. From the error system (60), we have $\dot{e}_1$, $\dot{e}_2$, $\dot{e}_3$, $\dot{e}_4 \in \ell_\infty$, $\dot{e}_{a_1}$, $\dot{e}_{a_2}$, $\dot{e}_{a_3}$, $\dot{e}_{a_4} \in \ell_\infty$, and $\dot{e}_{b_1}$, $\dot{e}_{b_2}$, $\dot{e}_{b_3}$, $\dot{e}_{b_4} \in \ell_\infty$. Since $\dot{V} = -e^{\mathrm{T}}e$, then we have

$$\begin{aligned} & \int_0^t ||e||^2 \mathrm{d}t \leq \int_0^t e^{\mathrm{T}} e \mathrm{d}t \leq \int_0^t -\dot{V} \mathrm{d}t \\ = & V(0) - V(t) \leq V(0) < \infty. \end{aligned} \tag{76}$$

Thus e_1, e_2, e_3, $e_4 \in \ell_2$, according to Barbalat's lemma, we have e_1, e_2, e_3, $e_4 \to 0$ ($t \to \infty$), and therefore the response system (58) synchronizes the drive system (8) by control law (66) and the update laws (67) and (68). This completes the proof.

In simulations, the fourth-order Runge–Kutta method is also used to solve the systems with step size 0.001. We assume that the initial states, $(x_1(0), x_2(0), x_3(0), x_4(0)) = (10^{-5}, 2\times10^{-4}, 3\times10^{-4}, 8\times10^{-5})$ and $(y_1(0), y_2(0), y_3(0), y_4(0)) = (10^{-5}, 8\times10^{-4}, 7\times10^{-4}, 6\times10^{-5})$ are employed, respectively. The parameters are chosen as $d_{11} = -0.002$, $d_{12} = 0.003$, $d_{13} = 0.002$, $d_{14} = 0.002$, $d_{21} = -0.002$, $d_{22} = 0.003$, $d_{23} = 0.002$, $d_{24} = 0.002$, $d_{31} = -0.002$, $d_{32} = 0.003$, $d_{33} = 0.002$, $d_{34} = 0.002$, $d_{41} = -0.002$, $d_{42} = 0.003$, $d_{43} = 0.002$, $d_{44} = 0.002$, respectively. Synchronization of systems (8) and (58), the control law (66) and the update laws (67) and (68) with the initial estimated parameters $a_1 = 0.5$, $a_2 = 0.5$, $a_3 = 0.5$, $a_4 = 0.5$, and $b_1 = 38, b_2 = 13, b_3 = 5, b_4 = 10$, are shown in Figs. 6–8. Figure 6 displays the synchronization errors of systems (8) and (58). Figures 7 and 8 show that the estimated values of the unknown parameters a_1, a_2, a_3, a_4 and b_1, b_2, b_3, b_4 converge to $a_1 = 0.31$, $a_2 = 0.35$, $a_3 = 0.29$, $a_4 = 0.41$ and $b_1 = 35$, $b_2 = 12$, $b_3 = 3$, $b_4 = 7$, respectively, which implies that the uncertain parameters can be identified via the control law (66) and the update laws (67) and (68).

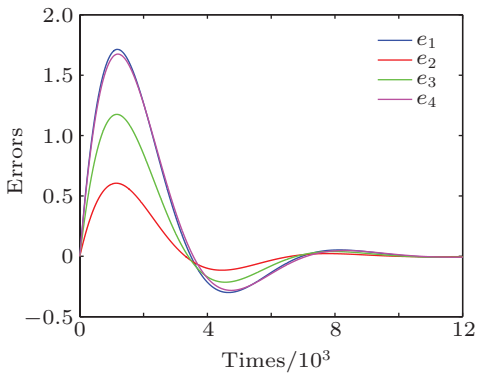


Fig. 6. (color online) Synchronization errors e_1, e_2, e_3, e_4 between the memristor chaotic oscillator system and the hyperchaotic Lü system.

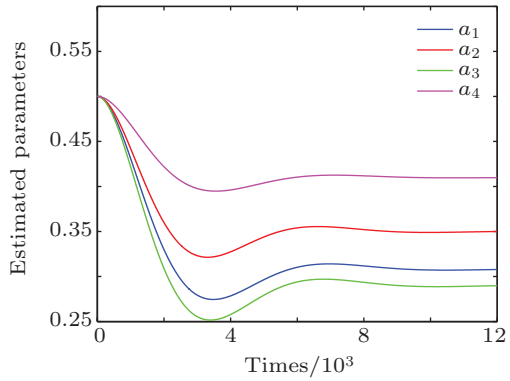


Fig. 7. (color online) The estimated parameters of the memristor chaotic oscillator system.

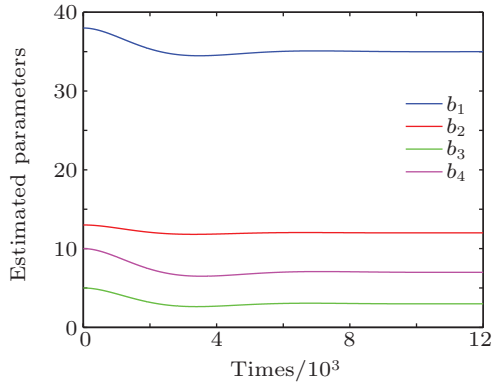


Fig. 8. (color online) The estimated parameters of the hyperchaotic Lü system.

Remark 6 In the simulation, d_{ij} $(i = 1,2,3,4, j = 1,2)$ are chosen to make $d_{11}d_{21}d_{31}d_{41} - d_{12}d_{22}d_{32}d_{42} \neq 0$ hold true.

4. Conclusion

This paper proposes a new general hybrid projective complete dislocated synchronization with non-derivative and derivative coupling based on the parameter identification method according to the Lyapunov stability theorem. To get better chaotic system synchronization results, we consider complete synchronization, hybrid dislocated synchronization, projective synchronization, non-derivative and derivative coupling, and parameter identification as its special item. By choosing a proper control input and the update rules, we realize the general hybrid projective complete dislocated synchronization with non-derivative and derivative coupling based on parameter identification between two different chaotic systems and two identical chaotic systems. The feasibility and effectiveness of the presented synchronization scheme has been validated by computer simulations.

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