Rhythm dynamics of complex neuronal networks with mixed bursting neurons*

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The spatiotemporal order and rhythm dynamics of a complex neuronal network with mixed bursting neurons are studied in this paper. A quantitative characteristic, the width factor, is introduced to describe the rhythm dynamics of an individual neuron, and the average width factor is used to characterize the rhythm dynamics of a neuronal network. An r parameter is introduced to denote the ratio of the short bursting neurons in the network. Then we investigate the effect of the ratio on the rhythm dynamics of the neuronal network. The critical value of r is derived, and the neurons in the network always remain short bursting when the r ratio is larger than the critical value.

Keywords: rhythm dynamics, complex network, neuron

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1. Introduction

The human brain is a complex network consisting of approximately 10^{11} neurons, linked together by 10^{14} to 10^{15} connections, amounting to 10^4 synapses per neuron.[1] An emergent property of this dynamical system is that a set of neurons synchronize and fire impulses simultaneously. Neural synchronization plays a key role in the mechanisms of information transformation and processing within different areas of the brain.[2,3] Clinical evidence points out that synchronization is also the origin of neurological diseases such as Parkinson's, essential tremors, and epilepsy.[4]

The basic pattern of neuronal firing is bursting in some areas of the brain cortex, which alternates between a quiescent state and spiking trains.[5] In studies on the synchronization of neural networks, the channel-based neuron models expressed by some ordinary differential equations (ODE) are most commonly used. Recently, map-based neuron models[6] have also been introduced for their simplicity in reproducing the characteristic behaviors of biological neurons. One could gain a great advantage in understanding the collective behavior of real-world neuronal populations by studying the networks of discrete-time systems.

Recently, there have been various approaches to studying the synchronization of chaos systems such as adaptive control, backstepping control, and so on. Due to the plasticity of a synapse between neurons, Yan *et al.*[7] studied the synchronization of the nervous system by synaptic adaptive learning. Chai *et al.*[8] investigated the synchronization of multi-spatiotemporal chaotic systems by adding control to a terminal node. Both burst synchronization and spike synchronization can be found in relating the neural synchronization with the firing rhythm of individual neurons. Burst synchronization has attracted more and more interest in recent years[9–16] since it was found that burst synchronization is also important in enhancing the reliability of communications between neurons by facilitating transmitter release.[17,18] Obviously, the information flow in coupled systems is not instantaneous, as the transmission process gives rise to a time delay. In Ref. [19], the authors investigated the synchronization of the minimal models of bursting neurons coupled by chemical or electrical synapses. Wang *et al.*[20] studied the dependence of the synchronization transition of bursting oscillators on the information transmission delay over scale-free neuronal networks with attractive and repulsive coupling. Spatiotemporal order, that is, temporal coherence and spatial synchronization, is one of the most attractive phenomena of coupled nonlinear elements. Ordering spatiotemporal chaos of small-world neural networks with electrical synaptic coupling has been studied extensively in recent years.[21,22] In Ref. [21], the authors indicated that the time delay in the coupling can enhance the spatiotemporal order. Du *et al.*[22] investigated ordering of spatiotemporal chaos in two-dimensional map neuron networks with small-world connection, and found that the spatiotemporal order is related to the randomness p, and the maximal spatiotemporal order appears at a particular value of p.

The above results, based on bursting neurons (chaotic or nonchaotic) as the basic units of networks, presented some quantitative characteristics to evaluate burst synchronization and the spatiotemporal order degree of neuronal networks. Though there have been many works on studying neuronal

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synchronization in the past few decades, we are still far away from a real understanding of neuronal information processing and the complex dynamic behaviors of brain networks. The rhythm dynamics of complex networks with mixed bursting neurons has been seldom considered. Wang *et al.*[23] investigated the dynamics of phase locking in a minimal neuronal network to explore the nervous dynamical mechanism. The rhythm dynamics of bursting neurons in the complex neuronal networks was studied by using the non-chaotic Rulkov map in Refs. [15] and [16]. However, the results were based on networks with long bursting neurons or short bursting neurons. In fact, because of the complexity of the brain's neural system, the types of bursting pattern of neurons in a neuronal network are not always the same. So it is necessary to study the spatiotemporal order and the rhythm dynamics of neuronal networks consisting of mixed bursting neurons.

In this work, we mainly investigate the effect of the ratio of long bursting and short bursting neurons in a neuronal network on the spatiotemporal order and rhythm dynamics of a neuronal network. An outline of this article is given as follows. A map-based non-chaotic neuron model and a complex neuronal network with electrical coupling are introduced in Section 2. Then, the spatiotemporal order and rhythm dynamics of the neuronal networks are presented in Section 3. Finally, the conclusions are given in Section 4.

2. Model description

2.1. Single neuronal model

Bursting neurons present multiple time scales. That is, fast variables relating to the action-potential firing, and the slow process modulating the fast ones to generate bursting. The discrete time system proposed by Rulkov[6] is

$$\begin{aligned} x_{n+1} &= F_\alpha(x_n, y_n), \\ y_{n+1} &= y_n - \mu(x_n + 1) + \mu\sigma, \end{aligned} \tag{1}$$

where x represents the transmembrane voltage of the neuron and y the slow gating process. The difference in the time scales between the two subsystems is determined by a sufficiently small value of the parameter μ with $0 < \mu \ll 1$. The parameter σ describes the dc current injected into the cell. The α parameter directly affects the spiking time-scale of the spikes, and the nonlinear function $F_\alpha(x,y)$ is responsible for the generation and reset of spikes, which is given by

$$F_\alpha(x,y) = \begin{cases} \dfrac{\alpha}{1-x} + y, & \text{if} \quad x < 0, \\ \alpha + y, & \text{if} \quad 0 \le x < \alpha + y, \\ -1, & \text{if} \quad x \ge \alpha + y. \end{cases}$$

Depending on the values of the parameters α and σ, the map reproduces a variety of neuronal spiking-bursting activities. In this study, we fix $\alpha = 5.0$, $\mu = 0.001$, and control the value of σ to generate various regular neuronal firing patterns.

2.2. Rhythm dynamics of a single neuron

In order to study the bursting rhythm dynamics of the single neuron, a characteristic quantity, called the width factor,[12] is introduced. It is defined as follows:

$$f = \left\langle \frac{T_\mathrm{b}}{T_\mathrm{r}} \right\rangle, \tag{2}$$

where T_b is the duration of a burst and T_r is the subsequent period of refractory time, and $\langle\cdot\rangle$ represents the average value of $T_\mathrm{b}/T_\mathrm{r}$ over the total number of bursts during the observation time. If the width factor $f > 1$, we define these types of bursts as long bursts. Alternately, the bursts with width factor $f < 1$ are called short ones. Based on the above definition, we find that the width factor is larger than 1 after $\sigma = 0.15$ by numerical simulation. The results are shown in Fig. 1(a). Therefore, $\sigma = 0.15$ is the critical value that is used to divide the bursting patterns of neurons into two types . The illustration of these two types of bursting is presented in Fig. 1(b).

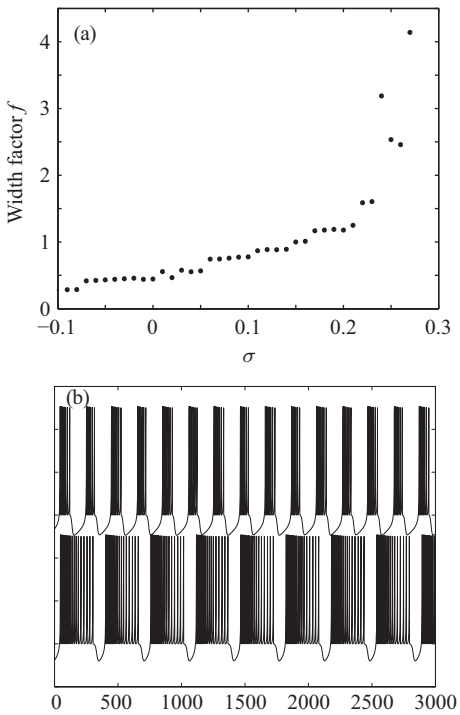


Fig. 1. (a) The width factor of a single neuron. (b) Examples of the firing patterns of a single neuron for higher external input $\sigma = 0.2$ (lower panel) and lower external input $\sigma = 0.02$ (upper panel).

2.3. Network model

The complex neuronal network is described by the following maps

$$\begin{aligned} x_{i,n+1} &= F_\alpha(x_{i,n}, y_{i,n} + \beta_{i,n}), \\ y_{i,n+1} &= y_{i,n} - \mu(x_{i,n} + 1) + \mu\sigma_i, \end{aligned} \tag{3}$$

where indexes $i = 1, 2, \ldots, M$ and $n = 1, 2, \ldots, N$ represent the neuron index and the discrete time series, respectively. The

coupling among neurons is provided by the current from one to the other with $\beta_{i,n}$ as the coupling term to the i-th neuron. In what follows, the coupling is modeled by

$$\beta_{i,n} = g_c b_{ij}(x_{j,n} - x_{i,n}), \quad i \neq j \tag{4}$$

through an electrical synapse with g_c being the coupling strength mimicking the maximal conductance. The matrix (b_{ij}) determines the network structure: $b_{ij} = b_{ji} = 1$ when a connection exists between neurons i and j, otherwise $b_{ij} = b_{ji} = 0$ and $b_{ii} = 0$ for all i. A previous study[24] using high-definition functional magnetic resonance imaging suggests that connectivity graphs formed out of all cortical and sub-cortical voxels have small-world properties. In this study, a small-world network is constructed as follows:[25] it starts with a one-dimensional regular ring which comprises $M = 100$ neurons, and each neuron is connected to its two nearest neighbors. Then we add shortcuts between pairs of non-nearest nodes with probability p. For $p = 0$, it reduces to the originally nearest neighbor coupled ring network, and for $p = 1$, we have a globally coupled network.

3. Spatiotemporal order and rhythm dynamics of the complex neuronal networks with mixed bursting neurons

Spatiotemporal order, that is, temporal coherence and spatial synchronization, is one of the most attractive phenomena of coupled nonlinear elements. Ordering spatiotemporal patterns and the rhythm dynamics of small-world neuronal networks were found in Ref. [13]. However, the results were based on networks with long bursting neurons or short bursting neurons. In fact, the types of bursting pattern of neurons in a neuronal network are not always the same. It is important to study the spatiotemporal order and the rhythm dynamics of the neuronal networks consisting of mixed bursting neurons. In order to evaluate the effect of the bursting type on the rhythm dynamics of the neuronal networks, an r parameter is introduced to describe the ratio of short bursting neurons in the neuronal network.

3.1. Spatiotemporal order of the complex neuronal networks

To show the spatiotemporal patterns of the neuronal networks with mixed bursting neurons, we fix the coupling strength at $g_c = 0.05$ and the probability at $p = 0.01$. The spatiotemporal evolutions of the membrane potentials of 100 electrically coupled mixed bursting neurons on the small-world networks for different r ratios are shown in Fig. 2. The initial values of the neurons' membrane potentials are chosen randomly. The black regions stand for the firing states of individual spikes inside a burst, and the white ones the quiescent states between two bursts.

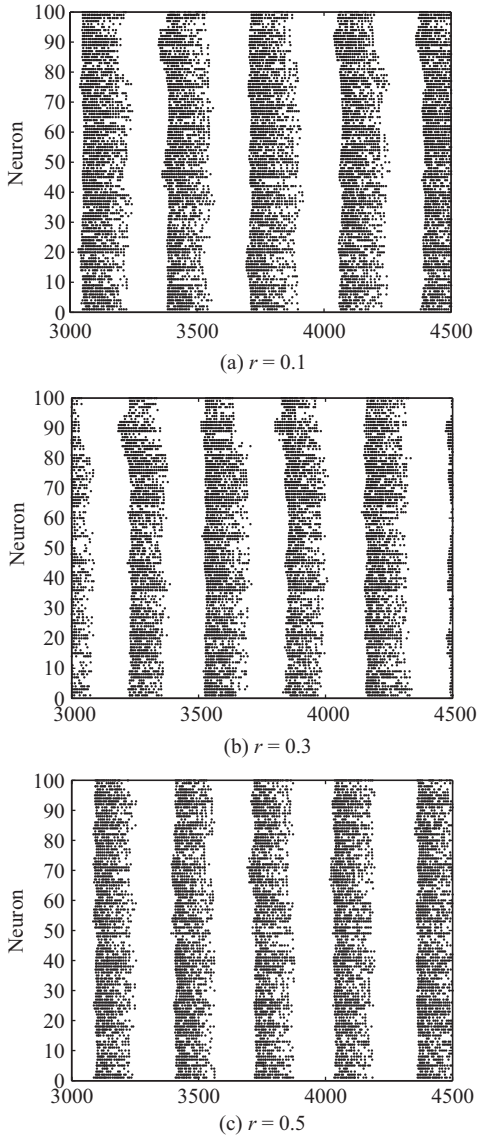


Fig. 2. Spatiotemporal patterns of the electrically coupled neuronal networks with mixed bursting neurons for various r ratios with $p = 0.01$ and $g_c = 0.05$.

From Fig. 2, it is clear that the networks with different ratios of the short bursting neurons show regular burst synchronization with spatiotemporal order. As the ratio r increases, the firing pattern of the neurons changes from long bursting to short bursting. Figure 3(b) shows that when $r = 0.3$ with 30 short bursting neurons in the network, then the width factor of the neurons is nearly 1, which means that $r = 0.3$ is the critical value such that the neurons in the network change from long bursting to short bursting when $p = 0.01$ and $g_c = 0.05$. The result also shows that short bursting plays a leading position in neuronal network.

3.2. Rhythm dynamics of the complex neuronal networks

From the above subsection, it is clear that the neurons in the network show regular bursting when the neuronal network achieves spatiotemporal order. However, the bursting rhythm of the neurons will change with different ratios between two

types of neuronal bursting. In order to study the change of the rhythm dynamics of the neuronal networks, we calculate the average width factor for various ratios in the bursting neuronal networks, which is defined as follows:

$$\overline{f} = \frac{1}{M}\sum_{j=1}^{M} f(j), \tag{5}$$

where $f(j)$ is the width factor of the j-th neuron. The results are shown in Fig. 3.

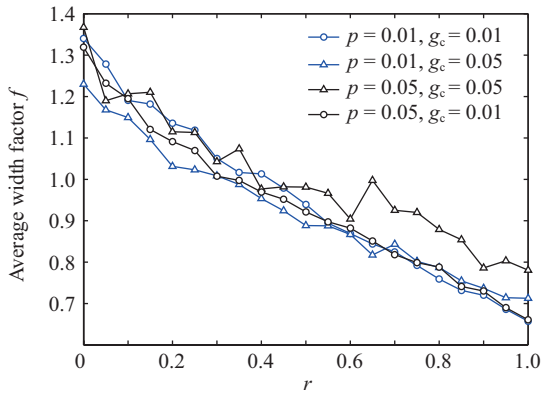


Fig. 3. The evolution of the average width factor $\overline{f}$ of the small world neuronal network with the change in ratio r for different coupling strength and probability.

We fix the value of the probability p at $p = 0.01$ and $p = 0.05$, and then change the value of the ratio r for two different coupling strengths, g_c, to evaluate the effect of ratio on the rhythm dynamics of the networks. From Fig. 3, we can see that the neurons in the networks show short bursting when the value of the ratio is larger than 0.4, whatever the values of the coupling strength and probability are. This means that short bursting plays a key role in shaping the rhythm dynamics of the neuronal network. However, it is clear that the critical value of the ratio is not the same for different coupling strengths and probability since the coupling and network topology also play important roles in shaping the rhythm dynamics of the network.

In order to show the change in the critical value of the ratio with which the neuronal network experiences long bursting to short bursting, we calculate the critical value of r with the change in the coupling strength and probability for fixed probability and coupling strength. We fix the probability at $p = 0.01$ and the coupling strength at $g_c = 0.01$ since the neuronal network with long bursting neurons would achieve short bursting with higher coupling strength and probability according to Ref. [13]. The results are presented in Fig. 4, and the results for other lower probability or coupling strength are the same. From the figure we can see that the critical value of the ratio is between 0.3 and 0.4, which means that the critical value of the ratio is robust to the coupling strength and network topology.

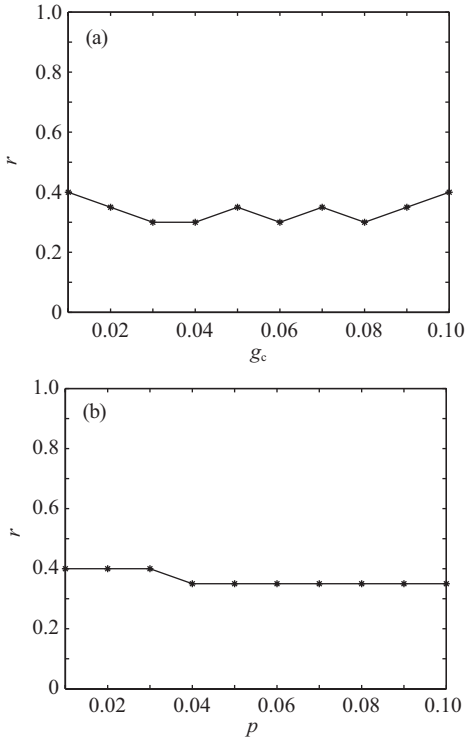


Fig. 4. The evolution of the critical value of the ratio with probability and coupling strength. (a) and (b) represent the situations when $p = 0.01$ and $g_c = 0.01$, respectively.

4. Conclusion

In summary, the spatiotemporal pattern and rhythm dynamics of a small-world neuronal network with mixed bursting neurons are studied.

We introduced the width factor to characterize the rhythm dynamics of a single neuron and calculated the average width factor of a neuronal network to show its rhythm dynamics. Since the types of bursting pattern of the neurons in a neuronal network are not always the same, an r parameter is introduced to describe the ratio of short bursting neurons in a neuronal network and to evaluate the effect of bursting type on the spatiotemporal pattern and rhythm dynamics of a neuronal network. The networks with different ratios of short bursting neurons show regular burst synchronization with spatiotemporal order. The neurons in the networks remain as short bursting when the value of the ratio is larger than 0.4, whatever the values of the coupling strength and the probability are. This means that short bursting plays a key role in shaping the rhythm dynamics of neuronal networks. Furthermore, the critical value of the ratio with which the neuronal network experiences long bursting to short bursting was presented, and the results show that the critical value of the ratio is robust to the coupling strength and network topology.

Synchronous neuronal firing has been suggested to be particularly relevant to neuronal signal transmission and coding. Our results may provide a useful tool for instructing electrophysiological experiments and understanding the rhythm

dynamics in coupled real neurobiological systems.

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