Non-Markovian dynamics of two non-coupled qubits interacting with two separate reservoirs with different spectral densities*

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The dynamics of two non-coupled qubits independently interacting with their reservoirs is solved by the time convolutionless projection operator method. We study two-qubit quantum correlation dynamics for two different types of spectral densities, which are a Lorentzian distribution and an Ohmic spectral density with a Lorentzian–Drude cutoff function. For two qubits initially prepared in the initial Bell state, quantum discord can keep longer time and reach larger values in non-Markovian reservoirs for the first spectral distribution or by reducing the cutoff frequency for the second case. For the initial Bell-like state, the dynamic behaviors of quantum discord and entanglement are compared. The results show that a long time of quantum correlation can be obtained by adjusting some parameters in experiment and further confirm that the discord can capture quantum correlation in addition to entanglement.

Keywords: Lorentzian and Ohmic spectral densities, the time convolution-less projection operator method, non-Markovian and Markovian regime, quantum discord and entanglement

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1. Introduction

Entanglement is a basic feature of quantum mechanics, which is used in many fields such as quantum information and communication theory,[1] quantum cryptography,[2] quantum teleportation,[3] and universal quantum computing.[4] However, it is not the only method of measuring the quantum correlation. There exists another non-classical correlation which is more important than entanglement, and it is defined as quantum discord from the view of Ollivier and Zurek.[5–10] Quantum discord can capture the fundamental features of quantum states, however, the relation between discord and entanglement is complicated. Though many scholars paid more attention to the study of entanglement in the past few years,[11–13] recent work involved the discord.[14,15] Some study of quantum discord has shown that it was more practical than entanglement in describing quantum correlation. In addition, quantum discord can be employed to improve the efficiency of quantum Carnot engine[16] and understand the quantum phase transition[17] or process of Grover search.[18]

In quantum information science, the discord is very important and has been studied widely.[19–21] Some papers dealt with quantum discord dynamics of a dissipative system under a Markovian environment,[19] but these results are not suitable for a non-Markovian environment.[20,21] For example, quantum discord is robuster than entanglement for general quantum states. Especially for non-coupled two-body quantum discord dynamics, most scholars firstly used the probability amplitude method to solve the dynamics of a system, and then obtain dynamics of two systems by the Kraus operator method.

In this paper, our study focuses on quantum discord dynamics, including the possible application which is important to understand quantum discord of multi-body interaction. We use two spin models interacting with reservoirs respectively,[22] which were previously analyzed in Refs. [23] and [24]. This model is composed of two non-coupled qubits interacting with two separate bosonic reservoirs. Our main purpose is to study evolution behaviors of quantum discord dynamics in different coupling regimes. First, the dynamics of two qubits is solved by the time convolutionless (TCL) projection operator method, and then we present analytical expressions of non-Markovian master equation for two-qubit reduced system, which is solved exactly at zero temperature limit. Finally, the two-qubit correlation dynamics is analyzed under the condition of Lorentzian spectral distribution and Ohmic spectral density with a Lorentzian–Drude cut-off function. This method can be extended to solve the dynamics of many bodies interacting with their reservoirs.

2. Physical model and solutions

Our system consists of a pair of two non-coupled qubits equally interacting with two separate bosonic reservoirs, which is displayed in Fig. 1.

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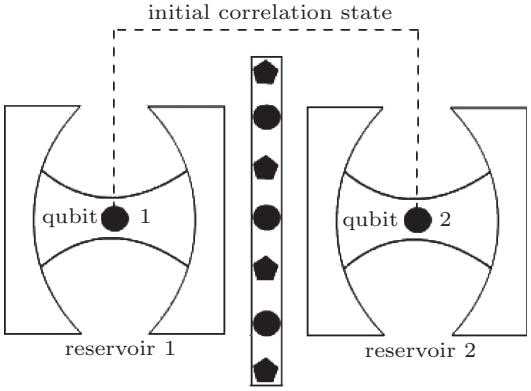


Fig. 1. The schematic plot of two non-coupled qubits in the influence of two independent reservoirs, denoted as 1 and 2.

Under the dipolar and rotating wave approximations, the Hamiltonian of total system can be written as $H = H_0 + V_I$,[22–24] where H_0 is the non-perturbation part containing the free Hamiltonian of two qubits and separate reservoirs, which is given as

$$H_0 = \varepsilon\sigma_+^1\sigma_-^1 + \varepsilon\sigma_+^2\sigma_-^2 + \sum_{j=1,2}\sum_k \omega_k^j b_k^{j\dagger} b_k^j. \tag{1}$$

Here, $\sigma_\pm^j$ $(j = 1,2)$ are the Pauli rising and lowering operators of the j-th qubit. For simplicity, we assume that two qubits have the same Bohr frequency and are denoted by ε. b_k^j and $b_k^{j\dagger}$ are the annihilation and creation operators for mode k of the j-th reservoir, which is characterized by frequency ω_k^j. The interaction Hamiltonian V_I reads

$$V_I = \sum_{j=1,2}\left[\sigma_+^j \otimes \sum_k g_k^j b_k^j + \sigma_-^j \otimes \sum_k g_k^{j*} b_k^{j\dagger}\right]. \tag{2}$$

Here, $g_k^j = -\mathrm{i}\sqrt{\omega_k^j/(2V\varepsilon_0)}\langle e|\boldsymbol{d}^j \cdot \boldsymbol{e}_k^j|g\rangle$ is coupling constant, with $\boldsymbol{e}_k^j$ and $\boldsymbol{d}^j$ being unit polarization vector of the j-th reservoir and dipole moment of the j-th qubit respectively, $|e\rangle$ $(|g\rangle)$ being the excited (ground) state of qubit. In the interaction picture $V_I(t)$ becomes

$$\begin{aligned} V_I(t) = &\sum_{j=1,2}\Big\{\sigma_+^j \otimes \sum_k g_k^j b_k^j \exp[\mathrm{i}(\varepsilon-\omega_k^j)t] \\ &+ \sigma_-^j \otimes \sum_k g_k^{j*} b_k^{j\dagger} \exp[-\mathrm{i}(\varepsilon-\omega_k^j)t]\Big\}. \end{aligned} \tag{3}$$

We use the TCL projection operator method to solve the dynamics of two qubits.[25] The master equation for this complex system is described by the reduced density operator $\rho_s(t)$ with lowercase letter s to represent by using the second approximation. The reduced density operator satisfies the following differential equation:

$$\begin{aligned} \frac{\mathrm{d}\rho_s(t)}{\mathrm{d}t} = &-\sum_{j=1,2}\Big\{C^{(j)}(t)[\sigma_+^j, \sigma_-^j\rho_s(t)] \\ &- D^{(j)}(t)[\sigma_+^j, \rho_s(t)\sigma_-^j] + D^{(j)*}(t)[\sigma_-^j, \sigma_+^j\rho_s(t)] \\ &- C^{(j)*}(t)[\sigma_+^j, \sigma_-^j\rho_s(t)]\Big\}. \end{aligned} \tag{4}$$

In Eq. (4), the quantities $C^{(j)}(t)$ and $D^{(j)}(t)$ are often called correlation functions, in the continuum limit they are written as

$$\begin{aligned} C^{(j)}(t) &= \int_0^t \mathrm{d}t' \sum_k |g_k^j|^2 \exp[\mathrm{i}(\varepsilon-\omega_k^j)(t-t')]\,\mathrm{tr}_{B_j}[b_k^j b_k^{j\dagger}] \\ &= \mathrm{i}\int \mathrm{d}\omega^j J_j(\omega^j)\,\mathrm{tr}_{B_j}[b_k^j b_k^{j\dagger}]\frac{1-\exp[\mathrm{i}(\varepsilon-\omega^j)t]}{\varepsilon-\omega^j}, \\ D^{(j)}(t) &= \int_0^t \mathrm{d}t' \sum_k |g_k^j|^2 \exp[\mathrm{i}(\varepsilon-\omega_k^j)(t-t')]\,\mathrm{tr}_{B_j}[b_k^{j\dagger} b_k^j] \\ &= \mathrm{i}\int \mathrm{d}\omega^j J_j(\omega^j)\,\mathrm{tr}_{B_j}[b_k^{j\dagger} b_k^j]\frac{1-\exp[\mathrm{i}(\varepsilon-\omega^j)t]}{\varepsilon-\omega^j}, \end{aligned} \tag{5}$$

where tr_{B_j} and $J_j(\omega^j)$ are trace and spectral density of the j-th reservoir .

In order to solve the solution of master equation (4), we choose two-qubit standard computational basis $Tb = \{|1\rangle = |gg\rangle, |2\rangle = |ge\rangle, |3\rangle = |eg\rangle, |4\rangle = |ee\rangle\}$. Therefore, the reduced density matrix elements of two qubits are obtained, the diagonal elements satisfy

$$\begin{aligned} \frac{\mathrm{d}\rho_s^{11}(t)}{\mathrm{d}t} = &-[D^{(1)}(t) + D^{(1)*}(t) + D^{(2)}(t) + D^{(2)*}(t)]\rho_s^{11}(t) \\ &+ [C^{(2)}(t) + C^{(2)*}(t)]\rho_s^{22}(t) \\ &+ [C^{(1)}(t) + C^{(1)*}(t)]\rho_s^{33}(t), \\ \frac{\mathrm{d}\rho_s^{22}(t)}{\mathrm{d}t} = &[D^{(2)}(t) + D^{(2)*}(t)]\rho_s^{11}(t) \\ &- [D^{(1)}(t) + D^{(1)*}(t) + C^{(2)}(t) + C^{(2)*}(t)]\rho_s^{22}(t) \\ &+ [C^{(1)}(t) + C^{(1)*}(t)]\rho_s^{44}(t), \\ \frac{\mathrm{d}\rho_s^{33}(t)}{\mathrm{d}t} = &[D^{(1)}(t) + D^{(1)*}(t)]\rho_s^{11}(t) \\ &- [C^{(1)}(t) + C^{(1)*}(t) + D^{(2)}(t) + D^{(2)*}(t)]\rho_s^{33}(t) \\ &+ [C^{(2)}(t) + C^{(2)*}(t)]\rho_s^{44}(t), \\ \frac{\mathrm{d}\rho_s^{44}(t)}{\mathrm{d}t} = &[D^{(1)}(t) + D^{(1)*}(t)]\rho_s^{22}(t) \\ &+ [D^{(1)}(t) + D^{(1)*}(t)]\rho_s^{33}(t) - [C^{(1)}(t) + C^{(1)*}(t) \\ &+ C^{(2)}(t) + C^{(2)*}(t)]\rho_s^{44}(t). \end{aligned} \tag{6}$$

By using the same method, the anti-diagonal elements are obtained as follows:

$$\begin{aligned} \frac{\mathrm{d}\rho_s^{12}(t)}{\mathrm{d}t} = &-[D^{(1)}(t) + D^{(1)*}(t) + D^{(2)*}(t) + C^{(2)*}(t)]\rho_s^{12}(t) \\ &+ [C^{(1)}(t) + C^{(1)*}(t)]\rho_s^{34}(t), \\ \frac{\mathrm{d}\rho_s^{13}(t)}{\mathrm{d}t} = &-[D^{(1)}(t) + C^{(1)*}(t) + D^{(2)*}(t) + D^{(2)*}(t)]\rho_s^{13}(t) \\ &+ [C^{(2)}(t) + C^{(2)*}(t)]\rho_s^{24}(t), \\ \frac{\mathrm{d}\rho_s^{14}(t)}{\mathrm{d}t} = &-[D^{(1)*}(t) + C^{(1)*}(t) + D^{(2)*}(t) + C^{(2)*}(t)]\rho_s^{14}(t), \\ \frac{\mathrm{d}\rho_s^{23}(t)}{\mathrm{d}t} = &-[D^{(1)*}(t) + C^{(1)*}(t) + D^{(2)}(t) + C^{(2)}(t)]\rho_s^{23}(t), \end{aligned}$$

$$\begin{aligned}\frac{\mathrm{d}\rho_s^{24}(t)}{\mathrm{d}t} &= -[D^{(2)}(t)+D^{(2)*}(t)]\rho_s^{13}(t)\\ &\quad -[D^{(1)*}(t)+C^{(1)*}(t)+C^{(2)}(t)+C^{(2)*}(t)]\rho_s^{24}(t),\\ \frac{\mathrm{d}\rho_s^{34}(t)}{\mathrm{d}t} &= -[D^{(1)}(t)+D^{(1)*}(t)]\rho_s^{12}(t)-[C^{(1)}(t)+C^{(1)*}(t)\\ &\quad +D^{(2)}(t)+C^{(2)*}(t)]\rho_s^{34}(t). \end{aligned} \tag{7}$$

The other elements satisfy the relations $\dot{\rho}_s^{nm}(t)=[\dot{\rho}_s^{mn}(t)]^*$ $(m,n=1,2,3,4)$. We assume two reservoirs to be the same at zero temperature for simplicity, which means that $\mathrm{tr}_{B_j}[b_k^{j\dagger}b_k^j]=\mathrm{tr}_B[b_k^+b_k]=0$, $\mathrm{tr}_{B_j}[b_k^jb_k^{j\dagger}]=\mathrm{tr}_B[b_kb_k^+]=1$, and $J_j(\omega^j)=J(\omega)$. Therefore, the correlation functions $D^{(j)}(t)$ and $C^{(j)}(t)$ reduce as $D^{(j)}(t)=D(t)=0$ and

$$C^{(j)}(t)=C(t)=\mathrm{i}\int \mathrm{d}\omega J(\omega)\frac{1-\exp[\mathrm{i}(\varepsilon-\omega)t]}{\varepsilon-\omega}.$$

Solutions of Eqs. (6) and (7) have explicit forms under this supposition (see Appendix A). Here we define a function $F(t)$ appearing in expression of Appendix as $F(t)=\Theta(t)+\Theta^*(t)$, where the quantity $\Theta(t)$ is

$$\begin{aligned}\Theta(t) &= \int_0^t \mathrm{d}t' C(t')\\ &= \int \mathrm{d}\omega J(\omega)\frac{\mathrm{i}(\varepsilon-\omega)t+1-\exp[\mathrm{i}(\varepsilon-\omega)t]}{(\varepsilon-\omega)^2}. \end{aligned} \tag{8}$$

3. Quantum correlation measurement

Quantum states contain quantum and classical correlations. For quantum aspect, it is measured by entanglement or discord.[14,22] For two-body or multi-body quantum states, it is very difficult to obtain an analytical expression. However for a kind of X structure density matrix of two bodies denoted by q_1 and q_2, quantum correlation identifications have analytical expressions. In the two-body standard basis Tb, the X-type density matrix of two bodies is given by

$$\rho(t)=\begin{Bmatrix} \rho^{11}(t) & 0 & 0 & \rho^{14}(t)\\ 0 & \rho^{22}(t) & \rho^{23}(t) & 0\\ 0 & [\rho^{23}(t)]^* & \rho^{33}(t) & 0\\ [\rho^{14}(t)]^* & 0 & 0 & \rho^{44}(t)\end{Bmatrix}, \tag{9}$$

with $\rho^{11}(t)+\rho^{22}(t)+\rho^{33}(t)+\rho^{44}(t)=1$ which means that the sum of probability is 1. The entanglement measured by concurrence has a simple analytical form[26]

$$\begin{aligned}&\mathrm{Concurrence}(t)\\ &= 2\max\left\{\begin{matrix}0,|\rho^{14}(t)|-\sqrt{\rho^{22}(t)\rho^{33}(t)},\\ |\rho^{23}(t)|-\sqrt{\rho^{11}(t)\rho^{44}(t)}\end{matrix}\right\}. \end{aligned} \tag{10}$$

For quantum discord of Eq. (9), quantum discord also has a simple form.[27] Measuring the system q_2, the form of discord is

$$\mathrm{Discord}_{q_2}[\rho(t)]=\min\{Q_1[\rho(t)],Q_2[\rho(t)]\}, \tag{11}$$

where

$$\begin{aligned}Q_1[\rho(t)] &= S[\rho^{q_2}(t)]-S[\rho(t)]-M,\\ Q_2[\rho(t)] &= S[\rho^{q_2}(t)]-S[\rho(t)]-N,\end{aligned}$$

with capital letter S being the von Neumann entropy. Measuring the system q_1, the discord is

$$\mathrm{Discord}_{q_1}[\rho(t)]=\min\{Q_3[\rho(t)],Q_4[\rho(t)]\}, \tag{12}$$

with

$$\begin{aligned}Q_3[\rho(t)] &= S[\rho^{q_1}(t)]-S[\rho(t)]-R,\\ Q_4[\rho(t)] &= S[\rho^{q_1}(t)]-S[\rho(t)]-T.\end{aligned}$$

The defined four quantities M, N, R, T are given in Ref. [27], we only take the same values of discord between measuring systems q_1 and q_2. When the matrix elements satisfy $\rho^{11}(t)=\rho^{44}(t)$ or $\rho^{22}(t)=\rho^{33}(t)$, the discord measuring systems q_1 and q_2 satisfy $\mathrm{Discord}_{q_2}=\mathrm{Discord}_{q_1}$.

4. Quantum correlation dynamics

The correlation dynamics depends on the initial condition and spectral properties of reservoir through function $F(t)$ in our model. Firstly, we consider the case of initial pure Bell-like states[22]

$$\begin{aligned}|\Phi\rangle &= \alpha|01\rangle+\sqrt{1-\alpha^2}|10\rangle,\\ |\Psi\rangle &= \alpha|00\rangle+\sqrt{1-\alpha^2}|11\rangle,\end{aligned} \tag{13}$$

with $\alpha\in[0,1]$. When α is $1/\sqrt{2}$, these states become the four famous Bell states which are the maximum entanglement states. Secondly, we choose different reservoir spectral densities and analyze the time evolution of quantum correlation dynamics.

The first spectral density is of a Lorentzian distribution, which is written as[25,28]

$$J(\omega)=\frac{\gamma_0}{2\pi}\frac{\gamma^2}{(\omega-\varepsilon)^2+\gamma^2}. \tag{14}$$

Here, γ_0 and γ are the decay rates of qubits and reservoirs respectively. The Markovian and non-Markovian regime are distinguished by the relation between two parameters γ_0 and γ. In the Markovian regime there is $\gamma_0<\gamma/2$, and in the non-Markovian regime there exists $\gamma_0>\gamma/2$. For this choice, exploiting residue theorem, we can obtain the correlation function $C(t)=\{\gamma_0[1-\exp(-\gamma t)]\}/2$. The function becomes

$$\Theta(t)=\frac{\gamma_0\left[t+\dfrac{\exp(-\gamma t)-1}{\gamma}\right]}{2}. \tag{15}$$

Hence, the function $F(t)$ is $F(t)=\Theta(t)+\Theta^*(t)=2\Theta(t)$. We choose initial Bell state $|\Phi\rangle=[|01\rangle+|10\rangle]/\sqrt{2}$. As indicated in the Appendix, the reduced density matrix elements are

$$\begin{aligned}\rho_s^{11}(t) &= 1-\exp[-F(t)],\\ \rho_s^{22}(t) &= \rho_s^{33}(t)=\frac{1}{2}\exp[-F(t)]\end{aligned}$$

$$\rho_s^{23}(t) = \frac{1}{2}\exp[-F(t)]. \tag{16}$$

All other elements are zero. Using the quantum discord to measure, we analyze the two-qubit quantum discord dynamics in different coupled regimes. Figure 2 shows the time evolution behaviors of quantum discord for different values of γ. When two qubits interact with respective reservoirs, four curves in Fig. 2 decrease to zero finally.

The main reason is the dissipation effect of two reservoirs. In non-Markovian regime such as $\gamma = 0.1\gamma_0$ or $\gamma = 0.5\gamma_0$, the two-qubit quantum discord dynamics decreases more slowly than in Markovian regime such as $\gamma = 2.5\gamma_0$ or $\gamma = 12.5\gamma_0$. Because the non-Markovian reservoir has memory effect, it can feed information back to two qubits, while the Markovian regime is a strong dissipative reservoir. Therefore, we obtain a long-time quantum correlation by adjusting the value of parameter γ.

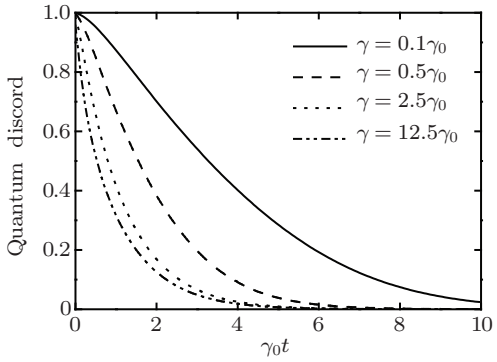


Fig. 2. Quantum discord dynamic behaviors of two qubits varying with time in the initial state $|\Phi\rangle = [|01\rangle + |10\rangle]/\sqrt{2}$ for a Lorentzian spectral density.

When the initial condition is Bell-like state $|\Psi\rangle = \alpha|00\rangle + \sqrt{1-\alpha^2}|11\rangle$, the non-zero density matrix elements are

$$\begin{aligned}
\rho_s^{11}(t) &= 1 - 2(1-\alpha^2)\exp[-F(t)] + (1-\alpha^2)\exp[-2F(t)],\\
\rho_s^{22}(t) &= \rho_s^{33}(t) = (1-\alpha^2)\exp[-F(t)]\{1-\exp[-F(t)]\},\\
\rho_s^{44}(t) &= (1-\alpha^2)\exp[-2F(t)],\\
\rho_s^{14}(t) &= \alpha\sqrt{1-\alpha^2}\exp[-F(t)].
\end{aligned} \tag{17}$$

In non-Markovian regime such as $\gamma = 0.1\gamma_0$ figure 3 shows the evolutions of quantum discord and entanglement for the initial Bell-like $|\Psi\rangle$. It can be seen that the discord is asymptotically reduced and keeps non-zero value.

On the contrary, the entanglement measured by concurrence displays a phenomenon of entanglement sudden death (ESD) under the same condition in some ranges of parameter α. In Markovian regime, there exists the same phenomenon. That is to say, quantum discord can measure the other non-classical correlation expect for entanglement for concrete physical model.

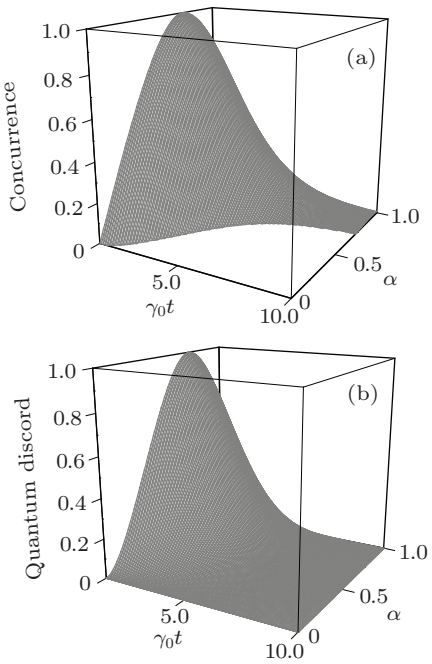


Fig. 3. Variations of the concurrence (a) and discord (b) evolve with $\gamma_0 t$ and α in non-Markovian case such as $\gamma = 0.1\gamma_0$ from the initial state $|\Psi\rangle$.

Next, we suppose that two separate reservoirs are characterized by an Ohmic spectral density with a Lorentzian Drude cut-off function[28]

$$J(\omega) = \frac{2\omega}{\pi}\frac{\omega_c^2}{\omega^2 + \omega_c^2}, \tag{18}$$

where ω is the frequency of reservoir and ω_c is the cut-off frequency. Setting $\omega_0 = \varepsilon$, the correlation function is $C(t) = -i2\omega_c^2\{1-\exp[-(\omega_c - i\omega_0)t]\}/(\omega_c - i\omega_0)$, then the defined function $\Theta(t)$ becomes

$$\Theta(t) = -\frac{2\omega_c^2}{\omega_c - i\omega_0}\left\{it + \frac{\exp[-i(\omega_c - i\omega_0)t] - 1}{\omega_c - i\omega_0}\right\}. \tag{19}$$

The function $F(t)$ can be obtained as $F(t) = 2\mathrm{Re}[\Theta(t)]$, where Re denotes the real part. We consider the initial condition to be in maximal entanglement state such as $|\Phi\rangle = (|10\rangle - |01\rangle)/\sqrt{2}$, thus non-zero density elements are similar to those in Eq. (16) expect for $\rho_s^{23}(t) = -\frac{1}{2}\exp[-F(t)]$. In Fig. 4, we plot the quantum discords of different values of cut-off frequency. Comparing these four curves, when the transition frequency of two qubits is $\omega_c < \omega_0$, the evolution time of quantum discord is longer than that for $\omega_c > \omega_0$. It means that two-qubit quantum correlation for $\omega_c < \omega_0$ decreases slower than that for $\omega_c > \omega_0$. Therefore, in order to acquire long time quantum correlation for the Ohmic spectral density with a Lorentzian–Drude cut-off function, we can reduce the value of cut-off frequency.

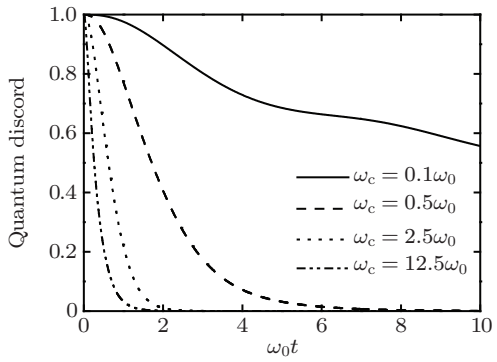


Fig. 4. Discord dynamic behaviors of two qubits varying with time parameter $\omega_0 t$ at different cut-off frequencies for the initial state $|\Phi\rangle = (|10\rangle - |01\rangle)/\sqrt{2}$.

5. Conclusions

In this paper, we have presented a procedure to obtain the dynamic behaviors of two identical non-coupled qubits, each of which locally interacts with a bosonic reservoir. Exploiting the TCL projection operator method, the second order TCL master equation for our model is derived. Then we obtain the analytical solutions for the reservoir temperature $T = 0$ K. For a Lorentzian spectral distribution, the discord dynamics is analyzed in Markovian and non-Markovian reservoirs in initial conditions $|\Phi\rangle = [|10\rangle + |01\rangle]/\sqrt{2}$. For initial Bell-like state $|\Psi\rangle = \alpha|00\rangle + \sqrt{1-\alpha^2}|11\rangle$, we compare quantum discord and entanglement dynamics in non-Markovian reservoir such as $\gamma = 0.1\gamma_0$ and observed different phenomena of them. For an Ohmic spectral density with a Lorentzian–Drude cut-off function, we can obtain long-time discord by reducing the value of cut-off frequency. When $\omega_c < \omega_0$, quantum correlation between two qubits reduces slower. Therefore, long-time correlation for these two spectral distributions is developed by our method and we find that the quantum discord indeed captures the other non-classical correlation except for entanglement. Further applications of our expressions involving good analytic approximations will enable more detailed comparisons to be made.

Appendix A

In arbitrary initial state and two reservoirs at temperature $T = 0$ K, the full solutions of two-qubit reduced density operator are obtained as follows:

$$\begin{aligned}
\rho_s^{11}(t) &= 1 - [\rho_s^{22}(0) + \rho_s^{33}(0)]\exp[-F(t)] \\
&\quad - 2\rho_s^{44}(0)\exp[-F(t)] + \rho_s^{44}(0)\exp[-2F(t)], \\
\rho_s^{22}(t) &= \rho_s^{22}(0)\exp[-F(t)] \\
&\quad + \rho_s^{44}(0)\exp[-F(t)]\{1 - \exp[-F(t)]\}, \\
\rho_s^{33}(t) &= \rho_s^{33}(0)\exp[-F(t)] \\
&\quad + \rho_s^{44}(0)\exp[-F(t)]\{1 - \exp[-F(t)]\}, \\
\rho_s^{44}(t) &= \rho_s^{44}(0)\exp[-2F(t)]. \qquad \text{(A1)}
\end{aligned}$$

The solution of non-diagonal elements can be also obtained as follows:

$$\begin{aligned}
\rho_s^{12}(t) &= \rho_s^{12}(0)\exp[-\Theta^*(t)] \\
&\quad + \rho_s^{34}(0)\{1 - \exp[-F(t)]\}\exp[-\Theta^*(t)], \\
\rho_s^{13}(t) &= \rho_s^{13}(0)\exp[-\Theta^*(t)] \\
&\quad + \rho_s^{24}(0)\{1 - \exp[-F(t)]\}\exp[-\Theta^*(t)], \\
\rho_s^{14}(t) &= \rho_s^{14}(0)\exp[-F(t)], \\
\rho_s^{23}(t) &= \rho_s^{23}(0)\exp[-F(t)], \\
\rho_s^{24}(t) &= \rho_s^{24}(0)\exp[-F(t) - \Theta^*(t)], \\
\rho_s^{34}(t) &= \rho_s^{34}(0)\exp[-F(t) - \Theta^*(t)]. \qquad \text{(A2)}
\end{aligned}$$

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