Efficient generation of two-dimensional cluster states in a cavity QED*

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We propose a scheme to achieve a kind of nontrivial multipartite pair-wise controlled phase operation in a cavity QED setup. The operation implemented is of geometrical nature and is not sensitive to the thermal state of the cavity. In particular, we have managed to avoid the conventional dispersive coupling so that high speed gate operation is achieved which is very important in view of decoherence. We show that this multipartite pair-wise controlled phase operation makes the generation of two-dimensional cluster states very efficient.

Keywords: controlled phase gate, cluster state, cavity QED

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1. Introduction

Physical realization of quantum computers has attracted much attention recently as it is generally believed to be more efficient than the classical computer for certain hard tasks.[1] Despite exciting experimental progress, this remains a great challenge since the infamous decoherence effect will ruin the encoded quantum information. One promising possibility is the so-called one-way quantum computation,[2] which constructs quantum logic gates by single-qubit measurements on the cluster states. The distinct advantage of this quantum computing strategy is that it separates the processes of entanglement generation and computation processing. Therefore, one can tolerate failures during the entanglement generation process simply by repeating it, provided that the failures are heralded. As shown in Ref. [2], a two-dimensional (2D) space is a universal resource for quantum computation, i.e., a universal set of quantum gates can be imprinted on the cluster states providing the ability of single-qubit measurements. Conventionally, the qubits of the 2D cluster states are in a square lattice configuration. Here, we also adhere to this convention and propose to generate 2D square lattice cluster states. After the generation of the 2D cluster states, the remaining needs for universal quantum computation are single-qubit measurements, which can be experimentally achieved with very high fidelity. Due to its novel application, the generation of cluster states has also been proposed in many contexts, e.g., in cavity QED with atomic qubits,[3–17] superconducting qubits coupled by a cavity,[18–22] and others.[23–25]

It is well known that any quantum circuit can be partitioned into a sequence of one-qubit rotations and nontrivial two-qubit gates.[26] Multi-qubit gates may also play an important role in constructing quantum computational circuits, e.g., the quantum Toffoli gate is very useful in complex quantum algorithms.[27–29] This type of quantum gate consists of multiple control qubits acting on a single qubit. Meanwhile, a multipartite phase gate with one control qubit simultaneously controlling N target qubits is also found in applications in many quantum tasks.[30,31] Recently, the implementation of the later gate was proposed in two alternative approaches:[32] one approach is based on tuning the qubit frequency, while in the other method one tunes the cavity (or resonator) frequency. In order to fulfill the task, they must tune the detuning between the cavity mode frequency and the qubit frequency. As is well known, tuning of the systematic parameters is not favorable during a quantum operation as this will introduce additional decoherence. Recently, the implementation of this multipartite phase gate has been proposed with superconducting flux qubits coupled to a microwave cavity via a two-channel Raman transition,[33] which works with fixed systematical parameters. However, they use complex interactions in a Lamda configuration of qubits. In this paper, we propose to achieve such an operation in a cavity QED setup. The operation implemented is of a geometrical nature and is not sensitive to the thermal state of the cavity, and thus greatly suppress the decoherence effect from the cavity decay. Meanwhile, our scheme removes the requirement of large detuning usually adopted in

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two-qubit operations, and thus enables fast gate operation. In comparison with Ref. [32], we have managed to work with fixed systematic parameters, which is a more efficient way of controlling the system dynamics as it is much easier for experimental realization in principle. Compared with Ref. [33], we only involve the interaction of the qubit levels with a simple interaction scenario, which will reduce the control-induce decoherence effect. In particular, we show that this multipartite controlled phase gate enables us to efficiently generate 2D cluster states, which is a universal resource for quantum computation.[2]

2. The cavity QED model

We first proceed to the implementation of the controlled phase gate in a cavity QED setup. We consider N identical two-level atoms simultaneously interacting with a single-mode cavity, meanwhile being driven by a classical field. Then the interaction between the single-mode cavity and the atoms can be described, in the rotating-wave approximation, as follows:[34,35]

$$H = \omega_0 a^\dagger a + \sum_{j=1}^{N}\left[\frac{\omega_j}{2}\sigma_j^z + g(a^\dagger\sigma_j^- + a\sigma_j^+) + \frac{\Omega}{2}(\sigma_j^+ \mathrm{e}^{-\mathrm{i}\omega_\mathrm{d}t} + \sigma_j^- \mathrm{e}^{\mathrm{i}\omega_\mathrm{d}t})\right], \quad (1)$$

where $a^\dagger$ and a are the creation and annihilation operators for the cavity mode, respectively; $\sigma_j^z = (|e\rangle_{j,j}\langle e| - |g\rangle_{j,j}\langle g|)$, $\sigma_j^+ = |g\rangle_{j,j}\langle e|$, $\sigma_j^- = |e\rangle_{j,j}\langle g|$, and $|e\rangle_j$, $|g\rangle_j$ are the excited and ground states of the j-th atom, respectively; g is the coupling constant between the atomic system and the cavity, Ω is the Rabi frequency of the driven laser, ω_j, ω_0, and ω_d are the atomic transition frequency, cavity frequency, and the frequency of the driven classical field, respectively.

While in the case of resonant driving $\omega_j = \omega_\mathrm{d}$, in the rotate frame of the frequency of ω_d, the above Hamiltonian reads

$$H_\mathrm{RF} = \delta a^\dagger a + \frac{\Omega}{2}J_x + g\sum_{j=1}^{N}\left(a\sigma_j^\dagger + a^\dagger\sigma_j^-\right), \quad (2)$$

where $\delta = \omega_0 - \omega_\mathrm{d}$ and $J_x = \sum_{j=1}^{N}\sigma_j^x$. In the interaction picture with respect to $H_0 = \delta a^\dagger a + \Omega/2J_x$, the interacting Hamiltonian reads[34]

$$H_1 = \frac{g}{2}\sum_{j=1}^{N} a\mathrm{e}^{-\mathrm{i}\delta t}\left(\widetilde{\sigma}_z^j + \mathrm{e}^{\mathrm{i}\Omega t}|+\rangle\langle -| - \mathrm{e}^{-\mathrm{i}\Omega t}|-\rangle\langle +|\right) + \mathrm{H.c.}, \quad (3)$$

where $\widetilde{\sigma}_z$ denotes the Pauli matrix in the eigenbasis of σ_x with eigenequations as $\sigma_x|\pm\rangle = \pm|\pm\rangle$, where $|\pm\rangle = (|g\rangle \pm |e\rangle)/\sqrt{2}$. In the case of $\Omega \gg \{\delta, g\}$, we can omit the fast oscillation terms with frequencies $\Omega \pm \delta$, then the Hamiltonian (3) reduces to

$$H_I = \frac{g}{2}(a^\dagger \mathrm{e}^{\mathrm{i}\delta t} + a\mathrm{e}^{-\mathrm{i}\delta t})\widetilde{J_z} = \frac{g}{2}(a^\dagger \mathrm{e}^{\mathrm{i}\delta t} + a\mathrm{e}^{-\mathrm{i}\delta t})J_x. \quad (4)$$

The time-evolution operator for the Hamiltonian (4) can be expressed as[34–41]

$$U_I(t) = \exp\left[-\mathrm{i}A(t)J_x^2\right]\exp\left[-\mathrm{i}B(t)aJ_x\right] \times \exp\left[-\mathrm{i}B^*(t)a^\dagger J_x\right], \quad (5)$$

where

$$A_N(t) = \frac{g^2}{4\delta}\left[\frac{1}{\mathrm{i}\delta}\left(\mathrm{e}^{\mathrm{i}\delta t} - 1\right) - t\right], \quad (6)$$

$$B_N(t) = \mathrm{i}\frac{g}{2\delta}\left(\mathrm{e}^{-\mathrm{i}\delta t} - 1\right). \quad (7)$$

While the whole time evolution is nonperiodic, equation (7) shows that $B(t)$ is a periodic function of time, and it vanishes at $\delta T = 2k\pi$ where $k = 1, 2, 3, \ldots$. At these times, the time evolution operator in the interaction picture reduces to

$$U(\gamma) = \exp(\mathrm{i}\gamma J_x^2), \quad (8)$$

with $\gamma = [g^2/(4\delta)]T \equiv \lambda T$. It is notable that $U(\gamma)$ operation is insensitive to the thermal state of the cavity mode as the related influence represented by the last two exponents in Eq. (5) is completely removed. The operation also removes the constrain of large detuning. It is noted that $t \sim 1/g$ for $\delta \sim g$, which is comparable to the resonant coupling strategy. This shows that the fast gate operation can be achieved, which is very important in view of decoherence. It is obvious that the time needed for this gate operation is comparable to a two-qubit gate as $A(t)$ is independent of N. This is another distinct merit of our proposed gate operation: the gate speed is not slowed down with the increase of qubits.

3. Cluster states generation

For $\gamma_1 = A_2 T_k = \pi/8$, the two-qubit operation is

$$U(\gamma_1) = \exp\left(\mathrm{i}\frac{\pi}{8}J_x^2\right) = \exp\left[\mathrm{i}\frac{\pi}{4}(1 + \sigma_1^x\sigma_2^x)\right]. \quad (9)$$

It is straightforward to check that $U(\gamma_1)$ is a nontrivial two-qubit unconventional geometric phase gate,[42] which still depends only on global geometric features and is robust against random operation errors, and thus the high-fidelity two-qubit operation can be achieved. Note that this gate operation is first proposed in the trapped ions system[36–38] with later experimental verification.[43] Then, it was generalized to many cavity-QED-alike systems.[34,35,39–41] Supplied with two single-qubit rotations $\exp\left[\mathrm{i}\frac{\pi}{4}(\sigma_1^x + \sigma_2^x)\right]$, the combined gate is

$$U_2 = \exp\left(\mathrm{i}\pi\frac{1+\sigma_1^x}{2}\frac{1+\sigma_2^x}{2}\right), \quad (10)$$

which is a controlled-phase gate in the eigenbasis of σ_x for the two involved qubits.

This gate operation can be readily scale up to many of qubit scenario. For $N+1$ qubits labeled as 0, 1, 2, ... and $\gamma_2 = A_{(N+1)}T_k = \pi/8$. The operation is

$$U(\gamma_2) = \exp\left(\mathrm{i}\frac{\pi}{8}J_x^2\right) = \exp\left(\mathrm{i}\frac{\pi}{4}\sum_{j>i=0}^{N-1}\sigma_i^x\sigma_j^x\right), \quad (11)$$

where we have neglected an overall phase factor for the last equation. Supplied with two single-qubit rotations $\exp\left[\mathrm{i}\frac{N(N-1)\pi}{8}J_x\right]$, upon an overall phase factor, the operation reads

$$U_{N+1} = \exp\left(\mathrm{i}\pi\sum_{j>i=0}^{N-1}\frac{1+\sigma_i^x}{2}\frac{1+\sigma_j^x}{2}\right), \quad (12)$$

which is a multipartite pair-wised controlled-phase gate in the eigenbasis of σ_x for any pair of qubits from the $N+1$ involved qubits.

Because the inverse of a controlled-phase gate is itself, the net effect of two repeated phase gates is null events. This gives us the opportunity to simultaneously implement controlled-phase gates on many qubits with the help of operator (12). For example, we want to implement N controlled-phase gates with a qubit labeled as "0" to be the control qubit and the other N qubits ($j = 1,2,\ldots,N$) as the target qubits. The procedure goes as follows: in the first step, we perform controlled-phase gates to every qubit pairs from the $(N+1)$ qubits using operator (12) with $A_{(N+1)}T_k = \pi/8$. In the second step we perform controlled-phase gates to every qubit pairs of the N qubits except 0. The net effect of the above two steps is a simultaneous realization of the N controlled-phase gates with qubit "0" simultaneously controlling the other N qubits

$$U_{\mathrm{CP}} = \exp\left(\mathrm{i}\pi\frac{1+\sigma_0^x}{2}\sum_{j=1}^{N}\frac{1+\sigma_j^x}{2}\right), \quad (13)$$

which is used for constructing graph states. If the $(N+1)$ qubits are initialized at the state $|0\rangle = (|+\rangle + |-\rangle)/\sqrt{2}$, then the net effect of the above two steps is a star graph state with qubit "0" at the center.[44] In Figs. 1(a) and 1(b), we have illustrated the generated cluster states for a 5-qubits case with the connected qubits indicating that there is a controlled phase gate operation between them. After the pair-wised interaction for the 5 qubits, each pair of them is connected by a controlled phase gate operation, and is thus entangled. After another pair-wised interaction without the center qubit, the 4 edge qubits are operated by 2 controlled phase gates, which equal a unit operation. Therefore, all the 4 connected edges, indicated as black line in Fig. 1(a), of the rectangle are now broken, and thus result in a cluster of states in a cross shape as shown in Fig. 1(b).

Furthermore, as the unit contains 5 qubits as illustrated in Fig. 1(a), we may construct a 2D cluster state in an efficient way. Figure. 1(c) shows the example of generation of a 4×4 square lattice configuration of cluster states. In the generation process, we need 24 qubits in 8-unit configurations. The edges indicated in blue lines belong to two units, i.e., have already been operated with controlled phase gate twice, and thus conceal themselves. To obtain our goal, we next need to conceal the controlled phase gates on the boundary qubits in a closed zig–zag chain indicated in solid black lines. After this, we obtain the 2D cluster states with the 8 edges indicated by dotted black lines. This state is already usable for one-way quantum computation, if we measure out these qubits, we get the exact square lattice configuration of cluster states as indicated with red line in Fig. 1(c).

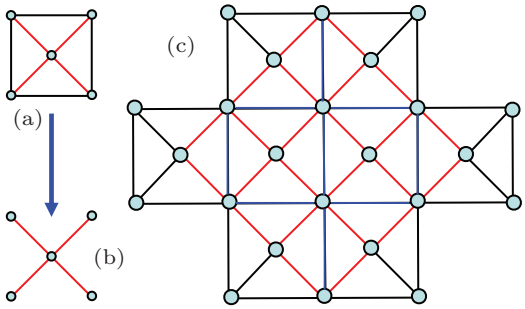


Fig. 1. (color online) The generation of 2D cluster states. (a) The result of the pair-wised interaction for 5-qubits case. (b) After another pair-wised interaction without the center qubit, the result is a cluster state in a cross shape. (c) With the unit as illustrated in panel (a), we may construct the 2D cluster state in an efficient way.

4. Discussion and conclusion

Finally, we briefly discuss the experimental feasibility of our scheme in a specific implementation of microwave cavity QED setup.[45] Our scheme removes the requirement of large detuning, thus for $\delta = g = 2\pi\times25$ kHz,[45] the pair-wise gate operation time of $\gamma = \pi/8$ is $t_o = 10$ μs. Note that this time will not be longer for a many qubit scenario. With a typical decoherence time of atom and cavity decays being 10 ms and 1 ms, many operations are possible. After each operation, the atomic qubits are disentangled with the cavity state and then the cavity decay will no longer affect the operation. Meanwhile, we also need some single-qubit rotations. With a Raman-type Rabi frequency of 1 MHz in typical experiments, the interaction time between the atoms and a classical field for single-qubit rotations is in the order of 0.1 μs, which is negligibly small in comparison with two-qubit operations. Therefore, we believe that the proposed scheme is within current experimental accessibility.

In summary, we have proposed a scheme to achieve a kind of nontrivial multipartite pair-wise controlled phase operation in cavity QED. This fast operation implemented is of geometrical nature and insensitive to the thermal state of the cavity. In particular, we show that this multipartite pair-

wise controlled phase operation makes the generation of two-dimensional cluster states very efficient.

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