Controlled remote implementation of quantum operations with high-dimensional systems*

Zhan You-Bang(詹佑邦)[a)†], Li Xiao-Wei(李晓薇)[a)], Ma Peng-Cheng(马鹏程)[a)], and Shi Jin(施 锦)[b)]

[a)] School of Physics and Electronic Electrical Engineering, Huaiyin Normal University, Huaian 223300, China
[b)] National Laboratory of Solid State Microstructures, Department of Physics, Nanjing University, Nanjing 210093, China

(Received 30 July 2012; revised manuscript received 26 September 2012)

We present two protocols for the controlled remote implementation of quantum operations between three-party high-dimensional systems. Firstly, the controlled teleportation of an arbitrary unitary operation by bidirectional quantum state teleportaion (BQST) with high-dimensional systems is considered. Then, instead of using the BQST method, a protocol for controlled remote implementation of partially unknown operations belonging to some restricted sets in high-dimensional systems is proposed. It is shown that, in these protocols, if and only if the controller would like to help the sender with the remote operations, the controlled remote implementation of quantum operations for high-dimensional systems can be completed.

Keywords: controlled remote implementation, quantum operation, teleportation, high-dimensional entangled state

PACS: 03.67.Hk, 03.65.Ud, 03.67.–a **DOI:** 10.1088/1674-1056/22/4/040306

1. Introduction

Quantum entanglement is one of the most striking features of quantum mechanics. A canonical example of entanglement-assisted processes is provided by quantum-state teleportation (QST), where an unknown quantum state is sent from a sender to a receiver at a distance without transferring the state carrier itself. In 1993, Bennett *et al.*[1] first proposed a protocol for teleporting an unknown quantum state of one-particle using the Einstein–Podolsky–Rosen (EPR) pair with the aid of some classical information. Since then, QST has always interested a lot of researchers in both theoretical and experimental respects.[2–20]

Another important application of the entanglement, which correlates closely to QST, is quantum operation teleportation (QOT). QOT may be understood as an unknown quantum operation being transferred from a local system to a remote system without physically sending the device. In the last decade, Huelga *et al.*[21] have investigated QOT for implementation of an arbitrary unitary operation upon a distant quantum system. In their paper,[21] the operation is completely unknown, and this remote implementation has to be realized by bidirectional QST (henceforth BQST), i.e., in which the receiver first teleports an unknown target state to the sender, then the sender performs an unknown operation on the received state, and finally the sender teleports the state back to the receiver. This QOT can be called "quantum remote control". Later, Huegla *et al.*[22] proposed two nontrivial protocols for the perfect remote implementation of certain continuous sets of quantum operations. In these protocols, the operations are partially unknown, that is, the quantum operations belong to some restricted sets that satisfy some given restricted conditions. In particular, the nonlocal operations in Ref. [22] can be implemented remotely by consuming less overall resources than BQST.[21] Since then, the QOT has attracted a lot of attention in both theory[23–36] and experiment.[37,38] For instance, Zou *et al.*[23] presented a feasible protocol to implement the nondeterministic quantum logic operation by a teleportation technique; Wang[29,31] extended the original QOT protocol which deals with single-qubit operations to those involving multi-qubits, and proposed some protocols of combined and controlled remote implementation of partially unknown quantum operations belonging to the restricted sets using GHZ states; Fan and Liu[34] presented a protocol for controlled remote implementation of a partially unknown operation. Recently, Zhang and Cheung[36] proposed several protocols for three-party sharing of unitary operations with the help of shared entanglement and LOCC. However, one can notice that most previous protocols of QOT have dealt with the problem by using a qubit system.

With the realization of preparing a high-dimensional quantum state,[39] during the last few years, the high-dimensional aspects of various topics in a quantum information system have been proposed.[40–48] Compared with qubits, high-dimensional quantum systems are better suited for certain purposes. For instance, Braβ and Macchiavello[41] showed that, in the presence of noise, qudit schemes for quan-

*Project supported by the National Natural Science Foundation of China (Grant No. 11074088).
†Corresponding author. E-mail: ybzhan@hytc.edu.cn
© 2013 Chinese Physical Society and IOP Publishing Ltd

tum cryptography are more secure than protocols using qubit states. Klimov *et al.*[47] pointed out that the quantum computation with qutrits instead of a qubit is the exponential increase of the available Hilbert space with the same amount of physical resources. Therefore, extending the local implementation of the nonlocal quantum operation to a high-dimensional system is important. The purpose of the present paper is to present two protocols for the controlled remote implementation of unitary operations between three separated high-dimensional systems. Firstly, the controlled QOT of an arbitrary unitary operation by BQST with high-dimensional systems is considered. Then, instead of using the BQST method, a protocol for controlled remote implementation of partially unknown operations belonging to some restricted sets in high-dimensional systems is proposed. In these protocols, if and only if the controller would like to help the sender with the remote operations, the controlled remote implementation of quantum operations for high-dimensional systems can be completed. The present paper is organized as follows. In Section 2, the controlled QOT of an arbitrary unitary operations by BQST is studied. Section 3 presents the protocol for controlled remote implementation of partially unknown operations with high-dimensional systems. Conclusions are given in Section 4.

2. Controlled teleportation of arbitrary unitary operations in high-dimensional systems

To present our protocol more clearly, let us first begin with controlled QOT in three dimensions. Suppose there are three legitimate participants, customarily called Alice, Bob, and Charlie, in a quantum network, and they are spatially separated in different sites. Let Alice be the sender, Bob be the receiver, and Charlie be the controller. Assume Bob has a qutrit s in an unknown state

$$|p\rangle_s = \alpha_0|0\rangle_s + \alpha_1|1\rangle_s + \alpha_2|2\rangle_s, \tag{1}$$

with $|\alpha_0|^2 + |\alpha_1|^2 + |\alpha_2|^2 = 1$, all partners do not know the state $|p\rangle_s$, and Alice wishes to perform on this qutrit an arbitrary unitary operation $U^{(3)}$ which is unknown to Alice, Bob, and Charlie, where the superscript (3) in U denotes the operation in three dimensions. We also suppose that Alice shares a two-qutrit entangled state

$$|\psi'\rangle_{a'b'} = \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle)_{a'b'} \tag{2}$$

with Bob, and a three-qutrit entangled state

$$|\psi\rangle_{abc} = \frac{1}{\sqrt{3}}(|000\rangle + |111\rangle + |222\rangle)_{abc} \tag{3}$$

with Bob and Charlie, respectively. Here qutrits a' and a belong to Alice, qutrits b' and b to Bob, and qutrit c to Charlie, respectively. Our protocol can be expressed in the following steps.

Step 1 Bob teleports the state $|p\rangle_s$ to Alice by using the state $|\psi'\rangle_{a'b'}$, then Alice can obtain the state

$$|p\rangle_{a'} = \alpha_0|0\rangle_{a'} + \alpha_1|1\rangle_{a'} + \alpha_2|2\rangle_{a'} \tag{4}$$

by the usual teleportation technique.

Step 2 Alice applies the operation $U^{(3)}$ on the state $|p\rangle_a$, and the state of the whole system may be written as

$$\begin{aligned} |\xi\rangle &= |\psi\rangle_{abc} \otimes (U^{(3)}|p\rangle)_{a'} \\ &= \frac{1}{3\sqrt{3}} \sum_{nm} |\Psi_{nm}\rangle_{aa'} \mathrm{e}^{2\pi \mathrm{i} nm/3} \\ &\quad \times \left[\sum_{l=0}^{2} (U_{nm} U^{(3)}|p\rangle)_b |x_f\rangle_c \right], \end{aligned} \tag{5}$$

where $|\Psi_{nm}\rangle$ is the generalized Bell basis of the Hilbert space of two qutrits[43]

$$|\Psi_{nm}\rangle = \sum_j \mathrm{e}^{2\pi \mathrm{i} jn/3} |j\rangle \otimes |(j+m) \bmod 3\rangle/\sqrt{3} \tag{6}$$

with $n, m, j = 0, 1, 2$, U_{nm} is a single-qutrit transformation[31]

$$U_{nm} = \sum_j \mathrm{e}^{2\pi \mathrm{i} jn/3} |j\rangle\langle (j+m) \bmod 3| \tag{7}$$

with $n, m, j = 0, 1, 2$, $|x_f\rangle$ is a single-qutrit measuring basis

$$|x_f\rangle = \sum_j \mathrm{e}^{2\pi i jf/3} |j\rangle/\sqrt{3} \tag{8}$$

with $f, j = 0, 1, 2$.

Step 3 Alice performs a generalized Bell state measurement on her qutrits a' and a under the basis $\{|\Psi_{nm}\rangle\}$, and then informs Bob and Charlie of the result of measurement by two classical channels.

Step 4 If Charlie would like to help Alice with the QOT, he should measure his qutrit c in the $\{|x_f\rangle\}$ and announces publicly the outcome via a classical channel.

Step 5 In accordance with the measurement result of Alice and Charlie, the receiver Bob can obtain $U^{(3)}|\varphi\rangle$ by performing a corresponding local single-qutrit transformation which can easily be found from Eq. (5), so that the final state $U^{(3)}|\varphi\rangle$ can be realized at Bob's site.

Our controlled QOT protocol can be generalized to arbitrary high dimensions. We denote that dimension as d. Suppose that Alice, Bob, and Charlie share a three-qudit entangled state

$$|\phi\rangle_{ABC} = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} |jjj\rangle_{ABC}, \tag{9}$$

and Alice also shares a two-qudit entangled state

$$|\phi'\rangle_{A'B'} = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} |jj\rangle_{A'B'} \tag{10}$$

with Bob, where qudits A and A' are held by Alice, qudits B and B' by Bob, and qudit C by Charlie, respectively. Assuming Bob has a qudit s in an unknown state

$$|q\rangle_s = \sum_{j=0}^{d-1} \alpha_j |j\rangle_s \tag{11}$$

with $\sum_{j=0}^{d-1} |\alpha_j|^2 = 1$.

Similarly to the above protocol in three dimensions, using the state $|\phi'\rangle_{A'B'}$, Bob first teleports the state $|q\rangle_s$ to Alice, who then applies an arbitrary unitary operation $U^{(d)}$ on the state $|q\rangle$. The operation $U^{(d)}$ is unknown to all partners, and (d) denotes the operation in d dimensions. After that, the state of the whole system can be expressed as

$$\begin{aligned} |\Xi\rangle &= |\phi\rangle_{ABC} \otimes (U^{(d)}|q\rangle)_{A'} \\ &= \frac{1}{d\sqrt{d}} \sum_{nm} |\Phi_{nm}\rangle_{AA'} \mathrm{e}^{2\pi \mathrm{i} nm/d} \\ &\quad \times \left[\sum_{k=0}^{d-1} (U_{nm} U^{(d)}|q\rangle)_B |y_k\rangle_C \right], \end{aligned} \tag{12}$$

where $|\Phi_{nm}\rangle$ is the generalized Bell basis of the Hilbert space of two qudits

$$|\Phi_{nm}\rangle = \sum_j \mathrm{e}^{2\pi \mathrm{i} jn/d} |j\rangle |(j+m) \bmod d\rangle / \sqrt{d} \tag{13}$$

with $n, m, j = 0, 1, \ldots, d-1$, U_{nm} is a single-qudit transformation

$$U_{nm} = \sum_j \mathrm{e}^{2\pi \mathrm{i} jn/d} |j\rangle\langle (j+m) \bmod d| \tag{14}$$

with $n, m, j = 0, 1, \ldots, d-1$, $|y_k\rangle$ is a single-qudit measuring basis

$$|y_k\rangle = \sum_j \mathrm{e}^{2\pi \mathrm{i} jk/d} |j\rangle / \sqrt{d} \tag{15}$$

with $k, j = 0, 1, \ldots, d-1$. Next, Alice performs a generalized Bell state measurement on her qutrits A and A' in basis $\{|\Phi_{nm}\rangle\}$, and announces publicly the result by classical channels. To help Alice complete the QOT, Charlie measures the qudit C under the basis $\{|y_k\rangle\}$ and then informs Bob of his outcome by a classical channel. According to Alice's and Charlie's results, Bob can acquire $U^{(d)}|q\rangle$ by performing a suitable single-qudit transformation.

3. Controlled implementation of partially unknown quantum operations on a distant qudit

Now we consider controlled remote implementation of partially unknown quantum operations in high-dimensional systems. Here, "partially unknown" means that the quantum operations belong to some restricted sets that satisfy some given restricted conditions. For the sake of clarity, we first consider the three-dimensional case. Suppose the sender Alice shares a two-qutrit entangled state

$$|\psi\rangle_{ab} = \frac{1}{\sqrt{3}} (|00\rangle + |11\rangle + |22\rangle)_{ab} \tag{16}$$

with the receiver Bob, and another

$$|\psi\rangle_{a'c} = \frac{1}{\sqrt{3}} (|00\rangle + |11\rangle + |22\rangle)_{a'c} \tag{17}$$

with the controller Charlie, where qutrits a and a' belong to Alice, qutrit b to Bob, and qutrit c to Charlie, respectively. In addition, Bob holds a qutrit s in an unknown state $|p\rangle_s$ given by Eq. (1), and Alice wants to perform an operation on his qutrit. The quantum operation to be remotely implemented by Alice belongs to one of the following three restricted sets:

$$\begin{aligned} U_0^{(3)} &= \begin{pmatrix} u_{00} & 0 & 0 \\ 0 & u_{11} & 0 \\ 0 & 0 & u_{22} \end{pmatrix}, \\ U_1^{(3)} &= \begin{pmatrix} 0 & 0 & u_{02} \\ u_{10} & 0 & 0 \\ 0 & u_{21} & 0 \end{pmatrix}, \\ U_2^{(3)} &= \begin{pmatrix} 0 & u_{01} & 0 \\ 0 & 0 & u_{12} \\ u_{20} & 0 & 0 \end{pmatrix}. \end{aligned} \tag{18}$$

We will use $U_m^{(3)}$ $(m = 0, 1, 2)$ to denote the operation to be remotely implemented, and say that they are partially unknown in the sense that the values of their matrix elements are unknown, but their structures, that is, the positions of their nonzero matrix elements, are known by Alice. The whole protocol can be described in the following steps.

Step A Bob first performs a generalized C-NOT gate C_{bs} using qutrit b as the control and qutrit s as the target, where the generalized C-NOT gate is defined as[49]

$$C_{bs}|i\rangle_b |j\rangle_s = |i\rangle_b |(i-j) \bmod 3\rangle_s. \tag{19}$$

The joint state of Alice and Bob is transformed to

$$\begin{aligned} |\lambda\rangle &= C_{bs}|\psi\rangle_{ab} \otimes |p\rangle_s \\ &= \frac{1}{\sqrt{3}} [(\alpha_0|00\rangle + \alpha_1|11\rangle + \alpha_2|22\rangle)_{ab}|0\rangle_s \\ &\quad + (\alpha_0|11\rangle + \alpha_1|22\rangle + \alpha_2|00\rangle)_{ab}|1\rangle_s \\ &\quad + (\alpha_0|22\rangle + \alpha_1|00\rangle + \alpha_2|11\rangle)_{ab}|2\rangle_s]. \end{aligned} \tag{20}$$

Then, Bob measures the qutrit s in the basis $\{|0\rangle, |1\rangle, |2\rangle\}$ and announces the result publicly. If the outcome is $|0\rangle_s$, they do nothing; if it is $|1\rangle_s$ ($|2\rangle_s$), they both carry out the transformation U_{01} (U_{02}) (see Eq. (7)) on their own qutrit locally. After that, the resulting state they share becomes

$$|\psi'\rangle_{ab} = (\alpha_0|00\rangle + \alpha_1|11\rangle + \alpha_2|22\rangle)_{ab}. \tag{21}$$

Step B Alice performs the unitary operation $U^{(3)}$ on her qutrit and obtains

$$\begin{aligned}|\psi''\rangle_{ab} &= (U^{(3)})_a|\psi'\rangle_{ab} \\ &= [\alpha_0(U^{(3)}|0\rangle)_a|0\rangle_b + \alpha_1(U^{(3)}|1\rangle)_a|1\rangle_b \\ &\quad + \alpha_2(U^{(3)}|2\rangle)_a|2\rangle_b]. \end{aligned} \tag{22}$$

Step C Using the state $|\psi\rangle_{a'c}$, Alice performs a generalized Bell state measurement on her qutrits a and a'. As a result of this procedure, Bob and Charlie always end up holding the following state

$$\begin{aligned}|\varphi\rangle_{bc} &= [\alpha_0(U^{(3)}|0\rangle)_b|0\rangle_c + \alpha_1(U^{(3)}|1\rangle)_b|1\rangle_c \\ &\quad + \alpha_2(U^{(3)}|2\rangle)_b|2\rangle_c], \end{aligned} \tag{23}$$

which can be rewritten as

$$\begin{aligned}|\varphi'\rangle_{bc} &= \frac{1}{\sqrt{3}}[(U^{(3)}|p\rangle)_b|x_0\rangle_c + (U^{(3)}U_{20}|p\rangle)_b|x_1\rangle_c \\ &\quad + (U^{(3)}U_{10}|p\rangle)_b|x_2\rangle_c], \end{aligned} \tag{24}$$

where $|x_f\rangle$ is the single-qutrit measuring basis given by Eq. (8) and U_{n0} $(n=1,2)$ given by Eq. (7).

Step D If Charlie agrees to cooperate with Alice, he can measure his qutrit c under the basis $\{|x_f\rangle\}$ and announces the result. If Charlie's outcome is $|x_0\rangle_c$, the qutrit b can be found in the desired state $U^{(3)}|p\rangle$, and Bob needs to do nothing. Unfortunately, if Charlie's measurement result is $|x_1\rangle_c$ (or $|x_2\rangle_c$), Bob would hold the state $U^{(3)}U_{20}|p\rangle$ (or $U^{(3)}U_{10}|p\rangle$) and, provided that $U^{(3)}$ is completely arbitrary, he cannot restore this state to the correct form $U^{(3)}|p\rangle$. As a result, the protocol only succeeds with a probability $1/3$.

Now we consider another case. Assume that before the protocol starts, Alice tells Bob the information about the restricted sets, i.e., Bob has partial knowledge of $U^{(3)}$. One can find that, in this situation, if $U^{(3)}$ satisfies

$$VU^{(3)}U_{n0}|p\rangle = \mathrm{e}^{\mathrm{i}\delta}U^{(3)}|p\rangle, \tag{25}$$

where V is a universal correction operation, δ is a real parameter, Bob can acquire $U^{(3)}|p\rangle$ determinately. From Eq. (25), we have

$$VU^{(3)} = \mathrm{e}^{\mathrm{i}\delta}U^{(3)}U_{k0}, \tag{26}$$

where $k=3-n$ (here k mod 3). Evidently, one may explicitly identify a set of operations that can be remotely implemented. The universal correction operation V can be set as

$$V = V_m^{(k)} = \mathrm{e}^{\mathrm{i}2\pi m(3-k)/3}U_{k0}, \tag{27}$$

where $n,k=0,1,2$. The three possible solutions to Eq. (26) are given by

$$U^{(3)} = \sum_{n,m,j} \mathrm{e}^{\mathrm{i}(\phi+2\pi jn/3)}|j\rangle\langle(j+m) \bmod 3|, \tag{28}$$

where $m,n,j=0,1,2$, and ϕ is an arbitrary real parameter. By the above approach, any operations within the restricted set $U^{(3)}=\{U_1^{(3)},U_2^{(3)},U_3^{(3)}\}$ (see Eq. (18)) can be deterministically shared by Bob and Charlie. For instance, if Charlie's result is $|x_0\rangle_c$ when he measures qutirt c in $|\varphi'\rangle_{bc}$, Bob needs to do nothing; if Charlie's outcome is $|x_1\rangle_c$, by Eqs. (24)–(27), Bob should apply a correctional operation

$$V_m^{(2)} = \mathrm{e}^{\mathrm{i}2\pi m/3}U_{20}, \tag{29}$$

where $m=0,1,2$; and if Charlie reports $|x_2\rangle_c$, Bob can apply

$$V_m^{(1)} = \mathrm{e}^{\mathrm{i}4\pi m/3}U_{10}. \tag{30}$$

It is easily verified that, employing the corresponding operation $V_m^{(k)}$ in Eqs. (29) and (30), Bob can obtain $U^{(3)}|p\rangle$ deterministically.

Now we generalize the above protocol to arbitrary high dimensions and donote the dimension as d. Suppose the sender Alice shares an entangled qudit pair

$$|\Phi\rangle_{AB} = \frac{1}{\sqrt{d}}\sum_{j=0}^{d-1}|jj\rangle_{AB} \tag{31}$$

with receiver Bob, and another

$$|\Phi\rangle_{A'C} = \frac{1}{\sqrt{d}}\sum_{j=0}^{d-1}|jj\rangle_{A'C} \tag{32}$$

with controller Charlie, where qudits A and A' belong to Alice, B to Bob, and C to Charlie, respectively. Assuming that a qudit s in an unknown state $|q\rangle_s$ that Bob holds is still in state (11), and Alice wants to employ an operation on this qudit. The quantum operation to be remotely implemented by Alice belongs to one of the following d restricted sets

$$U_m^{(d)} = \sum_{j=0}^{d-1} u_{j,j+m \pmod d}, \tag{33}$$

where $m=0,1,\ldots,d-1$. Assume that Alice knows the operation (33) partly, that is, she knows the positions of their nonzero matrix elements, but does not know the values of the elements.

Now let Bob first carry out a generalized C-NOT gate C_{Bs} using qudit B as the control and qudit s as the target, here C-NOT gate C_{Bs} is defined as[49]

$$C_{Bs}|i\rangle_B|j\rangle_s = |i\rangle_B|(i-j) \bmod d\rangle_s. \tag{34}$$

The joint state of the qudits A,B and s is transformed to

$$|\eta\rangle = C_{Bs}|\Phi\rangle_{AB}\otimes|q\rangle_s = \frac{1}{\sqrt{d}}\sum_{j=0}^{d-1}|\zeta_j\rangle_{AB}\otimes|j\rangle_s, \tag{35}$$

where

$$|\zeta_j\rangle_{AB} = \sum_{j=0}^{k-1} |j+k,\ j+k(\mathrm{mod}\ d)\rangle_{AB}, \tag{36}$$

where $j = 0,1,\ldots,d-1$. Then, Bob measures the qudit s on the basis $\{|j\rangle\}$ $(j = 0,1,\ldots,d-1)$ and publicly announces the outcome. Without a loss of generality, if Bob's result is $|l\rangle_s$, the state of qudits A and B will be collapsed into

$$|\zeta_l\rangle_{AB} = \sum_{l=0}^{k-1} |l+k,\ l+k(\mathrm{mod}\ d)\rangle_{AB}. \tag{37}$$

Alice and Bob both perform the transformation U_{0l} on their own qudits A and B respectively, here U_{0l} is the single-qubit transformation given by Eq. (14). As a result of this procedure, Alice and Bob always obtain the shared state

$$|\mu\rangle_{AB} = \sum_{j=0}^{d-1} \alpha_j |jj\rangle_{AB}. \tag{38}$$

Next, Alice carries out the unitary operation $U^{(d)}$ on her qudit A and gets

$$|\mu'\rangle_{AB} = (U^{(d)})_A |\mu\rangle_{AB} = \sum_{j=0}^{d-1} \alpha_j (U^{(d)}|j\rangle)_A |j\rangle_B. \tag{39}$$

Using the remaining Bell state $|\Phi\rangle_{A'C}$ she shares with Charlie, Alice teleports the state of her qudit A to Charlie. After that, Bob and Charlie always end up holding the state

$$|\xi\rangle_{BC} = \sum_{j=0}^{d-1} \alpha_j (U^{(d)}|j\rangle)_B |j\rangle_C, \tag{40}$$

which can be rewritten as

$$|\xi'\rangle_{BC} = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} (U^{(d)} U_{r0}|q\rangle)_B |y_j\rangle_C, \tag{41}$$

where $r = d-1-j$, $|y_j\rangle$ is the single-qudit measuring basis given by Eq. (15) and U_{r0} given by Eq. (14). If Charlie agrees to cooperate with Alice, he should measure his qudit C under the basis $\{|y_j\rangle\}$ and announces publicly the outcome. If Charlie's result is $|y_0\rangle_C$, the qudit B can be found in the desired state $U^{(d)}|q\rangle$ and Bob needs to do nothing. If Charlie obtains other outcomes $|y_f\rangle_C$ $(f = 1,2,\ldots,d-1)$, qudit B will be collapsed into the state $U^{(d)}U_{d-1-f}|q\rangle$, and Bob cannot recover this state to the correct form $U^{(d)}|q\rangle$ since he has no knowledge of the exact form of $U^{(d)}$. Obviously, the protocol only succeeds with a probability of $1/d$.

Now we assume that Bob has partial knowledge of $U^{(d)}$, i.e., he knows the information about the restricted sets (33). In this case, if $U^{(d)}$ satisfies

$$V_m^{(k)} U_{nm}^{(d)} U_{d-k,0}|q\rangle = \mathrm{e}^{\mathrm{i}\delta} U_{nm}^{(d)}|q\rangle, \tag{42}$$

where $V_m^{(k)}$ is a universal correction operation, $k,d,n,m = 0,1,\ldots,d-1$; δ is a real parameter, Bob can obtain $U^{(d)}|q\rangle$ determately. From Eq. (42), we have

$$V_m^{(k)} U_{nm}^{(d)} = U_{nm}^{(d)} U_{k0}. \tag{43}$$

By Eq. (43), we can identify a set of transformations that can be remotely implemented. If we set

$$V_m^{(k)} = \mathrm{e}^{\mathrm{i}2\pi m(d-k)/d} U_{k0}, \tag{44}$$

where $m,k = 0,1,\ldots,d-1$, the d possible solutions to Eq. (44) can be given by

$$U_m^{(d)} = \sum_{n,m,j} \mathrm{e}^{\mathrm{i}(\phi+2\pi jn/d)} |j\rangle\langle (j+m) \bmod d|, \tag{45}$$

where $m,n,j = 0,1,\ldots,d-1$; ϕ is an arbitrary real parameter. We will see in the following that any operations within the restricted set $\{U_m^{(d)}\}$ (see Eq. (33)) can be deterministically shared by Bob and Charlie. For example, if Charlie's outcome is $|y_0\rangle_C$ when he measures qudit C in $|\xi'\rangle_{BC}$, the qudit B is automatically in the desired state $U_m^{(d)}|q\rangle$ and Bob needs to do nothing; if Charlie's result is $|y_f\rangle_C$ $(f = 1,2,\ldots,d-1)$, by Eqs. (41)–(44), Bob should apply a correctional operation

$$V_m^{(f)} = \mathrm{e}^{\mathrm{i}2\pi m(d-f)/d} U_{f0}, \tag{46}$$

then he can obtain $U_m^{(d)}|q\rangle$ deterministically. Thus, the controlled remote implementation of the partial unknown quantum operation has been successfully completed.

4. Conclusion

In this paper, the controlled remote implementation of quantum operations with a high-dimensional system has been considered. We have presented two protocols for quantum operation teleportation and remote implementation of a quantum operation in some given restricted sets with a controller, respectively. In the first protocol, the controlled teleportation of an arbitrary unitary operation by bidirectional quantum state teleportation has been studied. In the second protocol, the quantum operation to be remotely implemented belongs to one of the d restricted sets, which are partially unknown in the sense that the values of their matrix elements are unknown, but the positions of their nonzero matrix elements are known. It is well known that entanglement consumption and classical communication cost are important factors in quantum information processing. Compared with the first protocol, the obvious advantage of the second protocol is more economical in terms of entanglement consumption and classical communication cost. To present these protocols more clearly, we first investigated each protocol in a three-dimensional system,

then generalized the protocol to an arbitrary high-dimensional case. Furthermore, we point out that, with a similar method, our protocols can be generalized to multi-particle operations and multi-party control systems. We hope our work can stimulate further research on quantum information processing and distributed quantum computing.

References

[1] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
[2] Cirac J I and Parkins A S 1994 *Phys. Rev. A* **50** R4441
[3] Moussa M H Y 1997 *Phys. Rev. A* **55** R3287
[4] Li W L, Li C F and Guo G C 2000 *Phys. Rev. A* **61** 034301
[5] Lee J and Kim M S 2000 *Phys. Rev. Lett.* **84** 4236
[6] Bowen G and Bose S 2001 *Phys. Rev. Lett.* **87** 267901
[7] Rigolin G 2005 *Phys. Rev. A* **71** 032303
[8] Yao Y and Chua W K 2006 *Phys. Rev. Lett.* **96** 060502
[9] Gordon G and Rigolin G 2006 *Phys. Rev. A* **73** 042309
[10] Muralidharan S and Panigrahi P K 2008 *Phys. Rev. A* **77** 032321
[11] Sun Y, Man Z X and Xia Y J 2009 *Chin. Phys. B* **18** 1742
[12] Mei F, Yu Y F and Zhang Z M 2010 *Chin. Phys. B* **19** 020308
[13] Wang Z J, Zhang K and Fan C Y 2010 *Chin. Phys. B* **19** 110311
[14] Zhan Y B, Zhang Q Y, Wang Y W and Ma P C 2010 *Chin. Phys. Lett.* **27** 010307
[15] Wang M Y and Yan F L 2011 *Chin. Phys. Lett.* **28** 060301
[16] Tang J W, Zhao G X and He X H 2011 *Chin. Phys. B* **20** 050312
[17] Wang M Y and Yan F L 2011 *Chin. Phys. B* **20** 120309
[18] Guo Y and Luo X B 2012 *Chin. Phys. Lett.* **29** 060303
[19] Bouwmeester D, Pan J W, Kmattle, Eibl M, Weinfurter H and Zeilinger A 1997 *Nature* **390** 575
[20] Furusawa A, Sorensen J L, Braustein S L, Fuchs C A, Kimble H J and Polzik E S 1998 *Science* **282** 706
[21] Huelga S F, Vaccaro J A, Chefles A and Plenio M B 2001 *Phys. Rev. A* **63** 042303
[22] Huelga S F, Plenio M B and Vaccaro J A 2002 *Phys. Rev. A* **65** 042316
[23] Zou X B, Pahlke K and Mathis W 2002 *Phys. Rev. A* **65** 064305
[24] Dür W, Vidal G and Cirac J I 2002 *Phys. Rev. Lett.* **89** 057901
[25] Reznik B, Aharonov Y and Groisman B 2002 *Phys. Rev. A* **65** 032312
[26] Zheng Y Z, Gu Y J and Guo G C 2002 *Chin. Phys. Lett.* **19** 623
[27] Zheng Y Z, Ye P and Guo G C 2004 *Chin. Phys. Lett.* **21** 9
[28] Zhang Y S, Ye M Y and Guo G C 2005 *Phys. Rev. A* **71** 062331
[29] Wang A M 2006 *Phys. Rev. A* **74** 032317
[30] Yao C M 2006 *Chin. Phys. Lett.* **23** 545
[31] Wang A M 2007 *Phys. Rev. A* **75** 062323
[32] Zhao N B and Wang A M 2007 *Phys. Rev. A* **76** 062317
[33] Zhao N B and Wang A M 2008 *Phys. Rev. A* **78** 014305
[34] Fan Q B and Liu D D 2008 *Sci. China G: Phys. Mech. Astron.* **51** 1661
[35] Chen L B, Jin R B and Lu H 2009 *Chin. Phys. B* **18** 30
[36] Zhang Z J and Cheung C Y 2011 *J. Phys. B: At. Mol. Opt. Phys.* **44** 165508
[37] Huang Y F, Ren X F, Zhang Y S, Duan L M and Guo G C 2004 *Phys. Rev. Lett.* **93** 240501
[38] Xiang G Y, Li J and Guo G C 2005 *Phys. Rev. A* **71** 044304
[39] Mair A, Vaziri A, Weihs G and Zeilinger A 2001 *Nature* **412** 313
[40] Son W, Lee J, Kim M S and Park Y J 2001 *Phys. Rev. A* **64** 064304
[41] Bruβ D and Macchiavello C 2002 *Phys. Rev. Lett.* **88** 127901
[42] Cabello A 2002 *Phys. Rev. Lett.* **89** 100402
[43] Liu X S, Long G L, Tong D M and Li F 2002 *Phys. Rev. A* **65** 022304
[44] Karimipour V, Bahraminasab A and Bagherinezhad S 2002 *Phys. Rev. A* **65** 052331
[45] Zeng B and Zhang P 2002 *Phys. Rev. A* **65** 022316
[46] Thew R T, Nemoto K, White A G and Munro W J 2002 *Phys. Rev. A* **66** 012303
[47] Klimov A B, Guzmán R, Retamal J C and Saavedra C 2003 *Phys. Rev. A* **67** 062313
[48] Cheong Y W, Lee S W, Lee J and Lee H W 2007 *Phys. Rev. A* **76** 042314
[49] Alber G, Delgado A, Gisin N and Jex I 2001 *J. Phys. A: Math. Gen.* **34** 8821