Quantum discord dynamics of two qubits in single-mode cavities*

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The dynamics of quantum discord for two identical qubits in two independent single-mode cavities and a common single-mode cavity are discussed. For the initial Bell state with correlated spins, while the entanglement sudden death can occur, the quantum discord vanishes only at discrete moments in the independent cavities and never vanishes in the common cavity. Interestingly, quantum discord and entanglement show opposite behavior in the common cavity, unlike in the independent cavities. For the initial Bell state with anti-correlated spins, quantum discord and entanglement behave in the same way for both independent cavities and a common cavity. It is found that the detunings always stabilize the quantum discord.

Keywords: quantum information, cavity quantum electrodynamics, entanglement and quantum nonlocality

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1. Introduction

Quantum entanglement, originated from nonlocal quantum correlation,[1] is fundamental in quantum physics both for understanding the nonlocality of quantum mechanics[2–4] and plays an important role in almost all efficient protocols for quantum computations and communications.[5–7] Due to the unavoidable interaction with the quantum systems surrounding it, an initially entangled two-qubit system becomes totally disentangled after evolving for a finite time. This phenomenon is called entanglement sudden death (ESD)[6–14] and has been recently demonstrated experimentally.[15] However, the entanglement may fail to capture the existence of the quantum correlation in some mixed separate states, in which the entanglement is considered not a good measure.[16,17]

During the remarkable advance of quantum information measurements, quantum discord (QD) emerges as an alternative route to quantify quantum correlation.[18] The interpretation of the QD can be described as the deviation of the total information of the two sub-systems A and B before and after local detection is performed on one of them. The QD shows apparent robustness against decoherence, even in some mixed separable states, under which the entanglement vanishes.[16,19–24] The capacity for quantum algorithms to supersede classical equivalents in DQC1 has caused speculation that it can induce quantum computational speed-up.[25,26] However, the fact that almost every state had a discord may indicate this is pure coincidence.[27] The operational meaning of quantum discord was further clarified by relating it to quantum state merger, [28,29] and the mother protocol in quantum information processing.[30–32] Moreover, it has recently been demonstrated experimentally as a quantum resource that only quantum processors can harness.[33]

Some works have been devoted to the QD dynamics of two qubits coupled to Markovian[19,34,35] and non-Markovian[36] decoherence environments. Comparisons with entanglement dynamics have been also performed. The QD shows apparent robustness against dissipations, in sharp contrast to the entanglement. Such novel features make the QD more appropriate than entanglement to characterize quantum correlation.[18] Considering the fertile phenomena exhibited in the previous works with various decoherence channels,[19–24,34–36] we believe that the QD dynamics in these qubit systems are also of fundamental interest. In addition, some essential pictures can be clearly described and unfolded in the framework of the simple model where the exact solutions are available. Actually, the entanglement dynamics for two independent Jaynes–Cummings (JC) atoms have been well studied previously.[10–14,37] The ESD feature would prevent the application of the entanglement as a basic resource for quantum information processing. What is the consequence for the QD in this kind of the qubit system? This is the main topic of the present study.

In the present paper, we will study the QD dynamics of two identical qubits in both two independent identical single-mode cavities and one common single-mode cavity with perfect quality, where the cavity decoherence can be ignored. Comparisons with the corresponding pairwise entanglement, i.e. concurrence, are also given. The paper is organized as follows. In Section 2, we briefly review the general definition of the quantum correlation. In Sections 3 and 4, we derive

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the time-dependent QD in these two systems if initiated from two typical Bell states. In Section 5, the results are given and discussions are made. The conclusion is presented in the last section.

2. General expression of quantum correlation

During the remarkable advance of quantum information science, the quantum correlation of two arbitrary sub-systems A and B can be determined by measuring the mutual information of the two sub-systems, the total correlation $\mathscr{I}(A,B)$, and the classical correlation $\mathscr{J}(A,B)$. The definition of the total correlation reads

$$\mathscr{I}(A,B) = S(A) + S(B) - S(A,B). \tag{1}$$

$S(A,B) = -\mathrm{Tr}\{\rho_{AB}\ln\rho_{AB}\}$ is the joint von Neumann entropy with ρ_{AB} as the joint density operator of two sub-systems, and $S(A(B)) = -\mathrm{Tr}\{\rho_{A(B)}\ln\rho_{A(B)}\}$ is the von Neumann entropy of sub-system $A(B)$ with $\rho_{A(B)} = \mathrm{Tr}_{B(A)}\{\rho_{AB}\}$. The classical correlation relies on the minimum of the conditional entropy $S(A|\Pi_k^B)$,[18,38,39] shown as

$$\mathscr{J}(\rho_{AB}) = S(A) - \min_{\{\Pi_k^B\}} S(A|\Pi_k^B). \tag{2}$$

The quantum conditional entropy describes the information loss of A after measuring B, and $\{\Pi_k^B\}$ is a complete set of orthogonal basis projected on sub-system B, with the projecting state k. Specifically, the conditional density operator can be obtained by

$$\rho_{A|\Pi_k^B} = \Pi_k^B \rho_{AB} \Pi_k^B / p_k, \tag{3}$$

where the corresponding conditional probability $p_k = \mathrm{Tr}_{A,B}\{\Pi_k^B \rho_{AB}\}$. The difference between these two measurement results in the QD, which is obtained by $\mathscr{D} = \mathscr{I}(A,B) - \mathscr{J}(A,B)$. Finally, the QD between A and B is given by[16,18,38,39]

$$\mathscr{D} = S(B) - S(A,B) + \min_{\{\Pi_k^B\}} S(A|\Pi_k^B). \tag{4}$$

3. QD in two identical Jaynes–Cummings atoms

The Hamiltonian of two identical Jaynes–Cummings atoms under rotating wave approximation (RWA) is given by

$$\begin{aligned} H_{\mathrm{JC}} &= \frac{\Delta}{2}(\sigma_z^A + \sigma_z^B) + \omega(a^\dagger a + b^\dagger b) \\ &\quad + g(a^\dagger \sigma_-^A + \sigma_+^A a) + g(b^\dagger \sigma_-^B + \sigma_+^B b), \end{aligned} \tag{5}$$

where $\sigma_k^{A(B)}$ $(k = x,y,z)$ is the Pauli operator of the atom $A(B)$, shown as $\sigma_z = |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|$ and $\sigma_x = |\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|$, with $|\uparrow\rangle$ ($|\downarrow\rangle$) the excited (ground) state of the two-level atom, $a^\dagger$ ($b^\dagger$) and a (b) are the creator and annihilator of the cavity $A(B)$, respectively, Δ and ω are the frequencies of the atom and the cavity, g is the atom–cavity coupling strength. Here we set $\hbar = 1$ and the detuning $\delta = \Delta - \omega$.

We first study the evolution of the QD initiated from the Bell state with anti-correlated spins, which has the following form

$$|\Psi_{\mathrm{Bell}}^{(1)}\rangle = \sin\alpha|\downarrow\uparrow\rangle + \cos\alpha|\uparrow\downarrow\rangle. \tag{6}$$

Initially, the vacuum state of the cavity is considered, so the initial state of the whole system can be written as

$$\begin{aligned} |\Psi(0)\rangle &= (\sin\alpha|\downarrow\uparrow\rangle + \cos\alpha|\uparrow\downarrow\rangle)\otimes|00\rangle \\ &= \sin\alpha|\downarrow\uparrow 00\rangle + \cos\alpha|\uparrow\downarrow 00\rangle. \end{aligned} \tag{7}$$

The time-dependent wave function can be generally expressed as[10]

$$|\Psi(t)\rangle = x_1|\uparrow\downarrow 00\rangle + x_2|\downarrow\uparrow 00\rangle + x_3|\downarrow\downarrow 10\rangle + x_4|\downarrow\downarrow 01\rangle, \tag{8}$$

where the coefficients are

$$\begin{aligned} x_1 &= (A\,\mathrm{e}^{-\mathrm{i}\lambda_+ t} + B\,\mathrm{e}^{-\mathrm{i}\lambda_- t})\cos\alpha, \\ x_2 &= (A\,\mathrm{e}^{-\mathrm{i}\lambda_+ t} + B\,\mathrm{e}^{-\mathrm{i}\lambda_- t})\sin\alpha, \\ x_3 &= C(\mathrm{e}^{-\mathrm{i}\lambda_+ t} - \mathrm{e}^{-\mathrm{i}\lambda_- t})\cos\alpha, \\ x_4 &= C(\mathrm{e}^{-\mathrm{i}\lambda_+ t} - \mathrm{e}^{-\mathrm{i}\lambda_- t})\sin\alpha, \end{aligned} \tag{9}$$

with the eigenfrequencies as

$$\lambda_\pm = \omega + \frac{\delta}{2} \pm \frac{\sqrt{\delta^2 + G^2}}{2}, \tag{10}$$

here $G = 2g$. The auxiliary parameters are shown as

$$\begin{aligned} A &= \frac{1}{2}\left(1 + \frac{\delta}{\sqrt{\delta^2 + G^2}}\right), \\ B &= \frac{1}{2}\left(1 - \frac{\delta}{\sqrt{\delta^2 + G^2}}\right), \\ C &= \frac{G}{2\sqrt{\delta^2 + G^2}}. \end{aligned} \tag{11}$$

The pairwise density matrix from Eq. (8) under the standard basis $\{|\downarrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\uparrow\uparrow\rangle\}$ is thus expressed by tracing the freedoms of the cavities $\rho_{AB}(t) = \mathrm{Tr}_{\mathrm{cav}}\{\rho(t)\} = \mathrm{Tr}_{\mathrm{cav}}\{|\Psi(t)\rangle\langle\Psi(t)|\}$ as the X form[40–42]

$$\rho_{AB}(t) = \begin{pmatrix} |x_3|^2 + |x_4|^2 & 0 & 0 & 0 \\ 0 & |x_2|^2 & x_1^* x_2 & 0 \\ 0 & x_2^* x_1 & |x_1|^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{12}$$

With this bipartite density matrix, the routine to derive the QD is formally given in Appendix A. The von Neumann entropy for the two atoms in Eq. (A2) is given by

$$\begin{aligned} S(A,B) &= -(|x_3|^2 + |x_4|^2)\ln(|x_3|^2 + |x_4|^2) \\ &\quad - (|x_1|^2 + |x_2|^2)\ln(|x_1|^2 + |x_2|^2), \end{aligned} \tag{13}$$

and the sub-system entropies in Eq. (A3) are shown as

$$S(A) = -(1 - |x_1|^2)\ln(1 - |x_1|^2) - |x_1|^2 \ln|x_1|^2,$$

$$S(B) = -(1-|x_2|^2)\ln(1-|x_2|^2) - |x_2|^2\ln|x_2|^2.$$

From Appendix A, one can find that the expressions of the elements in Eq. (A1) are

$$\begin{aligned} v_+ &= |x_3|^2+|x_4|^2, \quad v_- = 0, \quad w = |x_2|^2, \\ x &= |x_1|^2, \quad y = x_1x_2^*, \quad u = 0. \end{aligned} \tag{14}$$

Moreover,

$$\begin{aligned} X_{1,+} &= (|x_3|^2+|x_4|^2)\cos^2\theta + |x_2|^2\sin^2\theta, \\ X_{1,-} &= |x_1|^2\cos^2\theta, \\ |Y_1|^2 &= \frac{\sin^2 2\theta}{4}|x_1|^2|x_2|^2, \end{aligned}$$

and

$$\begin{aligned} X_{2,+} &= (|x_3|^2+|x_4|^2)\sin^2\theta + |x_2|^2\cos^2\theta, \\ X_{2,-} &= |x_1|^2\sin^2\theta, \\ |Y_2|^2 &= \frac{\sin^2 2\theta}{4}|x_1|^2|x_2|^2. \end{aligned}$$

Therefore these parameters are independent of ϕ. It follows that we can search the minimum of the conditional von Neumann entropy by only varying θ in the regime $[0,\pi/2]$. Following the procedures outlined in Appendix A, we can finally derive the quantum discord. Since α is limited to $(0,\pi/2)$, it can be numerically checked that $\theta=\pi/4$ corresponds to the minimum of the conditional entropy in the following calculations. The minimum of the conditional von Neumann entropy reads

$$S(A|\Pi^B) = -\sum_{\varepsilon=\pm}\eta_\varepsilon\ln\eta_\varepsilon, \tag{15}$$

where

$$\eta_\pm = \{1\pm[(1-2|x_1|^2)^2+4|x_1x_2|^2]^{1/2}\}/2. \tag{16}$$

As a result, the quantum discord is finally given by

$$\begin{aligned} \mathscr{D} = &-(1-|x_2|^2)\ln(1-|x_2|^2) - |x_2|^2\ln|x_2|^2 \\ &+(|x_3|^2+|x_4|^2)\ln(|x_3|^2+|x_4|^2) \\ &+(|x_1|^2+|x_2|^2)\ln(|x_1|^2+|x_2|^2) \\ &-\sum_{\varepsilon=\pm}\eta_\varepsilon\ln\eta_\varepsilon. \end{aligned} \tag{17}$$

For later use, we also list the expression for concurrence derived in Ref. [10] as

$$C_{AB}(t) = |\sin 2\alpha|[1-4C^2\sin^2(\sqrt{\delta^2+G^2}t/2)]. \tag{18}$$

Next, we consider the Bell state with correlated spin as the initial atomic state, which is

$$|\Psi^{(2)}_{\rm Bell}\rangle = \sin\alpha|\downarrow\downarrow\rangle + \cos\alpha|\uparrow\uparrow\rangle. \tag{19}$$

Including the initial vacuum cavities, the wave function of the whole system can be expressed as

$$\begin{aligned} |\Psi\rangle(t) = &\, x_1|\uparrow\uparrow 00\rangle + x_2|\downarrow\downarrow 11\rangle + x_3|\uparrow\downarrow 01\rangle \\ &+x_4|\downarrow\uparrow 10\rangle + x_5|\downarrow\downarrow 00\rangle, \end{aligned} \tag{20}$$

where the coefficients are

$$\begin{aligned} x_1 &= (A\,\mathrm{e}^{-\mathrm{i}\lambda_+ t} + B\,\mathrm{e}^{-\mathrm{i}\lambda_- t})^2\cos\alpha, \\ x_2 &= AB(\mathrm{e}^{-\mathrm{i}\lambda_+ t} - \mathrm{e}^{-\mathrm{i}\lambda_- t})^2\cos\alpha, \\ x_3 &= C(\mathrm{e}^{-\mathrm{i}\lambda_+ t} - \mathrm{e}^{-\mathrm{i}\lambda_- t})(A\,\mathrm{e}^{-\mathrm{i}\lambda_+ t} + B\,\mathrm{e}^{-\mathrm{i}\lambda_- t})\cos\alpha, \\ x_4 &= C(\mathrm{e}^{-\mathrm{i}\lambda_+ t} - \mathrm{e}^{-\mathrm{i}\lambda_- t})(A\,\mathrm{e}^{-\mathrm{i}\lambda_+ t} + B\,\mathrm{e}^{-\mathrm{i}\lambda_- t})\cos\alpha, \\ x_5 &= \sin\alpha. \end{aligned} \tag{21}$$

The eigenfrequencies and the auxiliary parameters are the same as those in Eqs. (10) and (11). Then under the standard basis $\{|\downarrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\uparrow\uparrow\rangle\}$, the pairwise density matrix is shown as

$$\rho_{AB}(t) = \begin{pmatrix} |x_2|^2+|x_5|^2 & 0 & 0 & x_1^*x_5 \\ 0 & |x_4|^2 & 0 & 0 \\ 0 & 0 & |x_3|^2 & 0 \\ x_1x_5^* & 0 & 0 & |x_1|^2 \end{pmatrix}. \tag{22}$$

Hence, the joint von Neumann entropy is derived as

$$S(A,B) = -|x_3|^2\ln|x_3|^2 - |x_4|^2\ln|x_4|^2 - \sum_{\varepsilon=\pm}\Omega_\varepsilon\ln\Omega_\varepsilon, \tag{23}$$

where

$$\begin{aligned} \Omega_\pm = &\{(|x_1|^2+|x_2|^2+|x_5|^2) \\ &\pm\sqrt{(|x_2|^2+|x_5|^2-|x_1|^2)^2+4|x_1|^2|x_5|^2}\}/2. \end{aligned} \tag{24}$$

The sub-system entropy can be derived as

$$\begin{aligned} S(A) = &-(|x_2|^2+|x_4|^2+|x_5|^2)\ln(|x_2|^2+|x_4|^2+|x_5|^2) \\ &-(|x_1|^2+|x_3|^2)\ln(|x_1|^2+|x_3|^2), \end{aligned} \tag{25}$$

$$\begin{aligned} S(B) = &-(|x_2|^2+|x_3|^2+|x_5|^2)\ln(|x_2|^2+|x_3|^2+|x_5|^2) \\ &-(|x_1|^2+|x_4|^2)\ln(|x_1|^2+|x_4|^2). \end{aligned} \tag{26}$$

Similar to the above Bell state with anti-correlated spins, if we focus on $\alpha\in(0,\pi/2)$, $\theta=\pi/4$ also corresponds to the minimum of the conditional von Neumann entropy. Hence, the minimum of the conditional entropy is given by

$$S(A|\Pi^B) = -\sum_{\varepsilon=\pm}\eta_\varepsilon\ln\eta_\varepsilon, \tag{27}$$

where

$$\eta_\pm = \{1\pm\sqrt{(1-2|x_1|^2-2|x_3|^2)^2+4|x_1|^2|x_5|^2}\}/2. \tag{28}$$

As a result, the quantum discord can be derived from Eqs. (25), (26), and (27) as

$$\mathscr{D} = S(B) - S(A,B) + S(A|\Pi^B). \tag{29}$$

The concurrence in this case has been also derived previously,[10] and is also collected here

$$C_{AB}(t) = \max\{0, f(t)\}, \tag{30}$$

$$\begin{aligned} f(t) &= [1-4C^2\sin^2(\sqrt{\delta^2+G^2}t/2)] \\ &\times [|\sin 2\alpha| - 8C^2\sin^2(\sqrt{\delta^2+G^2}t/2)\cos^2\alpha]. \end{aligned} \tag{31}$$

Especially at resonance ($\delta = 0$), the entanglement sudden transition occurs only for $\alpha < \alpha_c$, where $\alpha_c = \pi/4$.

4. QD in two identical qubits in one common single-mode cavity

The Hamiltonian of two identical qubits interacting with one common single-mode cavity under RWA reads

$$\begin{aligned} H_{\rm DN2} &= \frac{\Delta}{2}(\sigma_z^A + \sigma_z^B) + \omega\left(a^\dagger a + \frac{1}{2}\right) \\ &+ g\sum_{k=A,B}(a\sigma_k^+ + \sigma_k a^\dagger), \end{aligned} \tag{32}$$

where $a^\dagger$ and a are the creator and annihilator of the common cavity. Actually, it is just the $N = 2$ Dicke model.[43] The detuning is also set as $\delta = \Delta - \omega$.

If the initial atom state is selected as the Bell state with anti-correlated spins, we can obtain the time-dependent wavefunction as

$$|\Psi(t)\rangle = x_1|\uparrow\downarrow 0\rangle + x_2|\downarrow\uparrow 0\rangle + x_3|\downarrow\downarrow 1\rangle, \tag{33}$$

with

$$\begin{aligned} x_1 &= \langle\uparrow\downarrow 0|\,{\rm e}^{-{\rm i}H_{\rm DN2}t}|\Psi^{(1)}_{\rm BellII}\rangle, \\ x_2 &= \langle\downarrow\uparrow 0|\,{\rm e}^{-{\rm i}H_{\rm DN2}t}|\Psi^{(1)}_{\rm BellII}\rangle, \\ x_3 &= \langle\downarrow\downarrow 1|\,{\rm e}^{-{\rm i}H_{\rm DN2}t}|\Psi^{(1)}_{\rm BellII}\rangle. \end{aligned}$$

Then the pairwise density matrix under the standard basis $\{|\downarrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\uparrow\uparrow\rangle\}$ is given by

$$\rho_{AB}(t) = \begin{pmatrix} |x_3|^2 & 0 & 0 & 0 \\ 0 & |x_2|^2 & x_1^* x_2 & 0 \\ 0 & x_1 x_2^* & |x_1|^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{34}$$

For the resonant case ($\delta = 0$), we specify the coefficients of the wavefunction in Eq. (33) as

$$\begin{aligned} x_1 &= \frac{\cos\alpha}{2}(\cos\sqrt{2}\lambda t + 1) + \frac{\sin\alpha}{2}(\cos\sqrt{2}\lambda t - 1), \\ x_2 &= \frac{\cos\alpha}{2}(\cos\sqrt{2}\lambda t - 1) + \frac{\sin\alpha}{2}(\cos\sqrt{2}\lambda t + 1), \\ x_3 &= \frac{-{\rm i}}{\sqrt{2}}(\cos\alpha + \sin\alpha)\sin\sqrt{2}\lambda t. \end{aligned}$$

After the numerical checks, we find that $\theta = \pi/4$ corresponds to the minimum of the conditional entropy at an arbitrary time. Hence the QD is described as

$$\begin{aligned} \mathscr{D} &= -(1-|x_2|^2)\ln(1-|x_2|^2) - |x_2|^2\ln|x_2|^2 \\ &+ (|x_3|^2)\ln(|x_3|^2) + (1-|x_3|^2)\ln(1-|x_3|^2) \\ &- \sum_{\varepsilon=\pm}\eta_\varepsilon\ln\eta_\varepsilon, \end{aligned} \tag{35}$$

with

$$\eta_\pm = \{1\pm[(1-2|x_1|^2)^2 + 4|x_1x_2|^2]^{1/2}\}/2. \tag{36}$$

Besides, the concurrence of the two atoms can also be given as

$$C_{AB}(t) = 2\max\{0, |x_1x_2|\}. \tag{37}$$

If starting from the initial Bell state with correlated spins, the wavefunction can be expressed as

$$\begin{aligned} |\Psi(t)\rangle &= x_1|\uparrow\uparrow 0\rangle + x_2|\downarrow\downarrow 2\rangle + x_3|\uparrow\downarrow 1\rangle \\ &+ x_4|\downarrow\uparrow 1\rangle + x_5|\downarrow\downarrow 0\rangle. \end{aligned} \tag{38}$$

Hence the pairwise density matrix can be derived as

$$\rho_{AB}(t) = \begin{pmatrix} |x_2|^2+|x_5|^2 & 0 & 0 & x_1^*x_5 \\ 0 & |x_4|^2 & x_3^*x_4 & 0 \\ 0 & x_3x_4^* & |x_3|^2 & 0 \\ x_1x_5* & 0 & 0 & |x_1|^2 \end{pmatrix}. \tag{39}$$

The coefficients at resonant condition are shown as

$$\begin{aligned} x_1 &= \frac{\cos\alpha}{3}\,{\rm e}^{-{\rm i}\omega t}(2+\cos\sqrt{6}\lambda t), \\ x_2 &= \frac{\sqrt{2}\cos\alpha}{3}\,{\rm e}^{-{\rm i}\omega t}(\cos\sqrt{6}\lambda t - 1), \\ x_3 &= \frac{-{\rm i}\cos\alpha}{\sqrt{6}}\,{\rm e}^{-{\rm i}\omega t}\sin\sqrt{6}\lambda t, \\ x_4 &= \frac{-{\rm i}\cos\alpha}{\sqrt{6}}\,{\rm e}^{-{\rm i}\omega t}\sin\sqrt{6}\lambda t, \\ x_5 &= \sin\alpha. \end{aligned}$$

Then we can obtain the QD straightforwardly by following Refs. [40]– [42].

While for the concurrence at resonance, we know that the pairwise density matrix in Eq. (39) has the form

$$\rho_{AB}(t) = \begin{pmatrix} v_+ & 0 & 0 & u^* \\ 0 & y & y & 0 \\ 0 & y & y & 0 \\ u & 0 & 0 & v_- \end{pmatrix}. \tag{40}$$

Then we can derive it as

$$\begin{aligned} C_{AB}(t) &= 2\max\{0, |x_1x_5| - |x_3|^2, |x_3|^2 \\ &- |x_1|\sqrt{|x_2|^2+|x_5|^2}\}. \end{aligned} \tag{41}$$

From the definition, we find that there exists a critical bound for α. The ESD happens only for $\alpha < \alpha_c$. The α_c is determined by

$$\tan^2\alpha_c - 4\tan\alpha_c + 1 = 0, \tag{42}$$

resulting in $\alpha_c = \arctan(2-\sqrt{3}) = \pi/12$.

5. Results and discussions

First, we compare the QD with the concurrence in the two identical JC atoms with two initial atomic states, i.e. the Bell states with anti-correlated spins and correlated spins, for zero detuning. The results are collected in Fig. 1. The evolution of both QD and concurrence for the initial Bell state with anti-correlated spins displays similar behavior, as seen in Fig. 1(a). Yonac *et al.*[10] has shown that the ESD only occurs in the initial atomic Bell states with correlated spin, where the entanglement can fall abruptly to zero and vanish for a period of time before revival. It is interesting to note from Fig. 1(b) that during the period of ESD, QD becomes small but is always finite, except when it vanishes at discrete moments $t = (2k+1)\pi/G, (k = 0, 1, 2, \ldots)$.

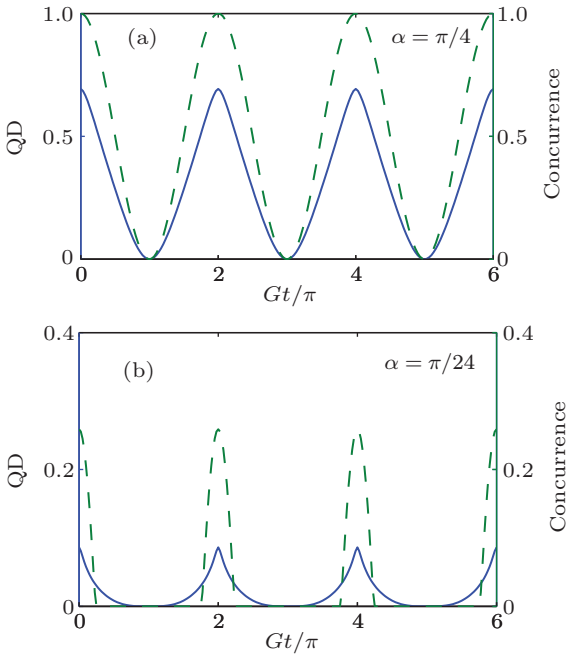


Fig. 1. (color online) Resonant dynamics of QD and concurrence of two identical JC atoms with the initial atomic Bell states with anti-correlated spins (a) and correlated spins (b). $\omega = 1.0$. The blue solid line and the green dashed line correspond to QD and concurrence respectively.

Then, we show the effects of detuning on the QD in independent cavities in Fig. 2, starting from these two Bell states. Interestingly, the amplitude of oscillation of the QD as a function of time is suppressed monotonically by the detuning for both initial Bell states. More importantly, the zeros of the QD at discrete instants shown in Fig. 1(b) disappear with finite detuning.

Next, we compare the QD with the concurrence in the two identical qubits coupled to the common cavity with two initial atomic Bell states. The results for zero detuning are presented in Fig. 3. For the initial atomic Bell state with anti-correlated spins, similar behaviors for both QD and concurrence are observed. For the initial atomic Bell state with correlated spins, one can find from Fig. 3(b) that the ESD can occur, but the QD never vanishes. Interestingly, QD and entanglement show opposite behaviors. In particular, during the period of ESD, the QD always becomes larger, in sharp contrast with that observed in the independent cavities (c.f. Fig. 1(b)).

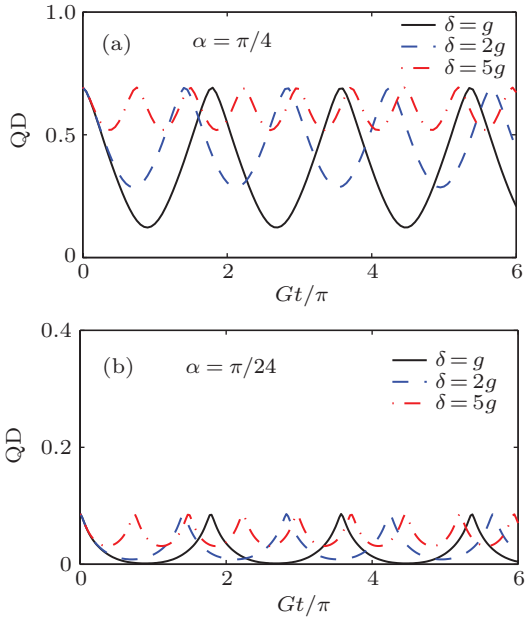


Fig. 2. (color online) Off-resonant dynamics of the QD for two identical JC atoms with the initial atomic Bell states with anti-correlated spins (a) and correlated spins (b) for different detunings $\delta = g$, $2g$, and $5g$. $\omega = 1.0$.

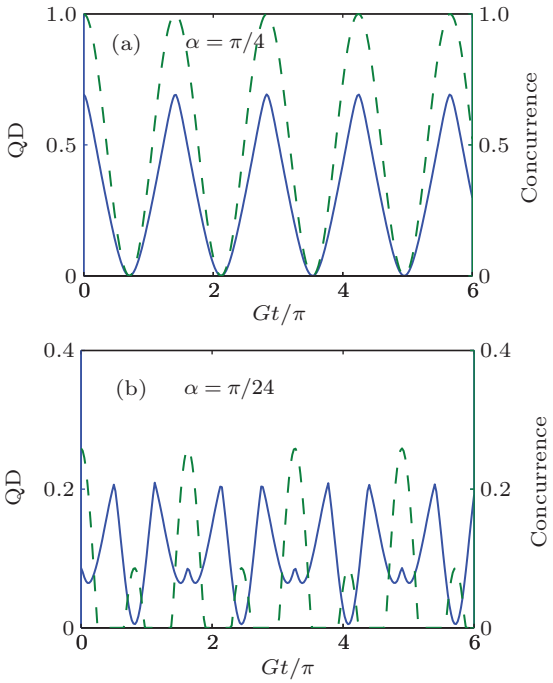


Fig. 3. (color online) Resonant dynamics of QD and concurrence of two qubits coupled to a common cavity with the initial atomic Bell states with anti-correlated spins (a) and correlated spins (b). $\omega = 1.0$. The blue solid line and the green dashed line correspond to QD and concurrence respectively.

Note that the critical parameter $\alpha_c = \pi/12$ below which the ESD can occur in the common cavity is smaller than

$\alpha_c = \pi/4$ in independent cavities. The instant vanish of QD is absent in the common cavity, implying that the common cavity enhances QD. So, it is suggested that the quantum correlation in the common cavity is stronger than that in independent cavities in some sense.

The effect of detuning on the QD of two qubits coupled to a common cavity is also studied. As shown in Fig. 4(a), for the initial Bell state with anti-correlated spins, the amplitude of oscillation of the QD as a function of time is also suppressed monotonically by the detuning and is a little bit larger than that in two cavities (c.f. Fig. 2(a)). While for the initial atomic Bell state with correlated spins with $\alpha = \pi/24$ where ESD can occur, the oscillation of the QD for two qubits in the common cavity is considerably suppressed with detuning, as shown in Fig. 4(b). In particular, for a large detuning $\delta = 5g$, the QD remains almost unchanged. In this case, we find that the components of the Bell states show slight variation with time for large detuning, due to the fact that the large detuning prevents the hopping for photons between different atomic levels to a certain degree.

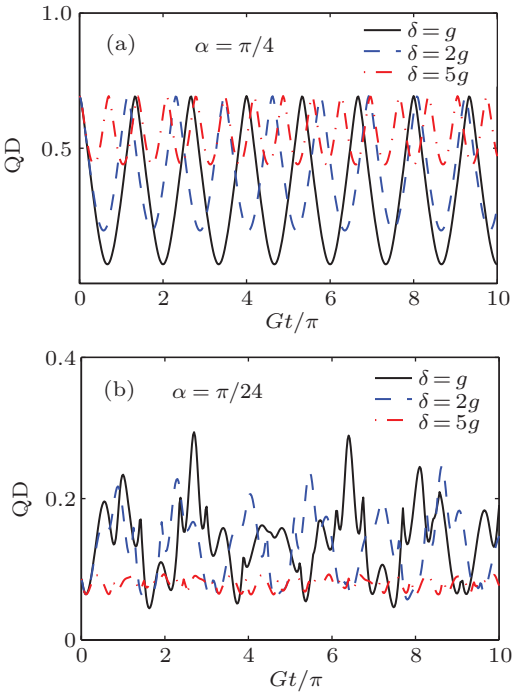


Fig. 4. (color online) Off-resonant dynamics of QD for two qubits coupled to a common cavity with the initial atomic Bell states with anti-correlated spins (a) and correlated spins (b) for different detunings $\delta = g, 2g$, and $5g$. $\omega = 1.0$.

6. Conclusions

In this paper, the QD dynamics of two qubits in both independent and common cavities are investigated. A comparison with the entanglement evolution is also performed. For the initial atomic Bell state with anti-correlated spins, the QD and entanglement show similar behavior for both cavities. However, for the initial atomic Bell state with correlated spins, the QD and entanglement behave in a remarkably different way. For the independent cavities, the QD vanishes only at discrete instants and can be lifted with finite detuning. In the common cavity the QD is always finite. In particular, the QD and entanglement display opposite behavior in the common cavity, different from those in independent cavities. The detunings are always found to stabilize the QD, rather than enhance the QD, which could provide insights into the real application of the QD in quantum information science and quantum computing.

Appendix A: Derivation of the quantum correlation

In the present paper, the general pairwise density matrix under the standard basis $\{|\downarrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\uparrow\uparrow\rangle\}$, is shown as

$$\rho_{AB}(t) = \begin{pmatrix} v_+ & 0 & 0 & u^* \\ 0 & w & y^* & 0 \\ 0 & y & x & 0 \\ u & 0 & 0 & v_- \end{pmatrix}. \tag{A1}$$

The von Neumann entropy of the two atoms is

$$S(A,B) = \sum_{\varepsilon=\pm,k=1,2} \Omega_{k,\varepsilon} \ln \Omega_{k,\varepsilon} \tag{A2}$$

with

$$\Omega_{1,\pm} = \frac{(v_+ + v_-) \pm \sqrt{(v_+ - v_-)^2 + 4|u|^2}}{2},$$
$$\Omega_{2,\pm} = \frac{(w+x) \pm \sqrt{(w-x)^2 + 4|y|^2}}{2}.$$

The reduced sub-system density matrices for A and B are obtained as

$$\rho_A(t) = (v_+ + w)|\downarrow\rangle_A\langle\downarrow| + (x + v_-)|\uparrow\rangle_A\langle\uparrow|,$$
$$\rho_B(t) = (v_+ + x)|\downarrow\rangle_B\langle\downarrow| + (w + v_-)|\uparrow\rangle_B\langle\uparrow|.$$

Hence, we derive the corresponding von Neumann entropies as

$$\begin{aligned} S(A) &= -(v_+ + w)\ln(v_+ + w) \\ &\quad - (x + v_-)\ln(x + v_-), \\ S(B) &= -(v_+ + x)\ln(v_+ + x) \\ &\quad - (w + v_-)\ln(w + v_-). \end{aligned} \tag{A3}$$

While for the conditional density matrix $\rho_{A|\Pi^B}$, projection bases are considered as

$$|\Phi_1\rangle_B = \cos\theta|\downarrow\rangle_B + e^{i\phi}\sin\theta|\uparrow\rangle_B,$$
$$|\Phi_2\rangle_B = e^{-i\phi}\sin\theta|\downarrow\rangle_B - \cos\theta|\uparrow\rangle_B.$$

The conditional density operator is expressed as

$$\rho_{A|\Pi_k^B} = \Pi_k^B \rho_{AB} \Pi_k^B / p_k, \tag{A4}$$

where $\Pi_k^B = I_A \otimes |\Phi_k\rangle_B\langle\Phi_k|$ and $p_k = \mathrm{Tr}_{AB}\{\rho_{A|\Pi_k^B}\}$. Specifically under the projections in Eq. (A4),

$$\rho_{A|\Pi_k^B} = |\Phi_k\rangle_B\langle\Phi_k| \otimes \{|\downarrow\rangle_A\langle\downarrow|X_{k,+} + |\uparrow\rangle_A\langle\uparrow|X_{k,-} + |\downarrow\rangle_A$$

$$\times \langle \uparrow | Y_k + | \uparrow \rangle_A \langle \uparrow | Y_k^* \} / p_k. \tag{A5}$$

For $k = 1$, we show

$$X_{1,+} = v_+ \cos^2\theta + w \sin^2\theta,$$
$$X_{1,-} = x \cos^2\theta + v_- \sin^2\theta, \quad Y_1 = (y^* \mathrm{e}^{-\mathrm{i}\phi} + u^* \mathrm{e}^{\mathrm{i}\phi}) \sin\theta \cos\theta.$$

For $k = 2$,

$$X_{2,+} = v_+ \sin^2\theta + w \cos^2\theta,$$
$$X_{2,-} = x \sin^2\theta + v_- \cos^2\theta,$$
$$Y_2 = -(y^* \mathrm{e}^{-\mathrm{i}\phi} + u^* \mathrm{e}^{\mathrm{i}\phi}) \sin\theta \cos\theta.$$

Then the eigenvalues of the conditional density matrix reads

$$\eta_{k,\pm} = \frac{1}{2p_k} \{ (X_{k,+} + X_{k,-}) \pm [(X_{k,+} - X_{k,-})^2 + 4|Y_k|^2]^{1/2} \}. \tag{A6}$$

The conditional von Neumann entropy is described as

$$S(A|\Pi^B) = \sum_{k=1,2} -p_k \mathrm{Tr}_A \{ \rho_{A|\Pi_k^B} \ln \rho_{A|\Pi_k^B} \} = -\sum_{\varepsilon=\pm} \sum_{k=1,2} p_k \eta_{k,\varepsilon}(\theta,\phi) \ln \eta_{k,\varepsilon}(\theta,\phi). \tag{A7}$$

As a result, the quantum discord can be obtained by

$$\mathscr{D} = \min\{S(B) - S(A,B) + S(A|\Pi^B)\}, \tag{A8}$$

and the classical correlation is given as

$$\mathscr{C} = \max\{S(A) - S(A|\Pi^B)\}. \tag{A9}$$

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