The quantum nonthermal radiation and horizon surface gravity of an arbitrarily accelerating black hole with electric charge and magnetic charge*

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Using a new tortoise coordinate transformation, we discuss the quantum nonthermal radiation characteristics near an event horizon by studying the Hamilton–Jacobi equation of a scalar particle in curved space–time, and obtain the event horizon surface gravity and the Hawking temperature on that event horizon. The results show that there is a crossing of particle energy near the event horizon. We derive the maximum overlap of the positive and negative energy levels. It is also found that the Hawking temperature of a black hole depends not only on the time, but also on the angle. There is a problem of dimension in the usual tortoise coordinate, so the present results obtained by using a correct-dimension new tortoise coordinate transformation may be more reasonable.

Keywords: tortoise coordinate transformation, black hole, quantum nonthermal radiation, event horizon surface gravity

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1. Introduction

Black holes are extensively studied in physics and astronomy. In 1974, Hawking discovered that black holes have thermal properties and can emit thermal radiation.[1,2] Since then, four laws of black hole thermodynamics have been successfully established. Black hole thermodynamics is a cross discipline theory based on quantum mechanics, statistic physics, general relativity, and so forth. In the past few decades, the thermodynamic properties of black holes have attracted increasing interest, and a considerable amount of work related to the thermal radiation of black holes has been done.[3–17] Starobinsky and Unruh have demonstrated that for a rotating black hole or a charged black hole, not only is there thermal radiation, but also nonthermal radiation.[18,19] In recent years, the study of the nonthermal radiation of black holes has also achieved a great deal of progress.[20–23]

In this paper, the quantum nonthermal radiation characteristics of a black hole near the event horizon are discussed based on the Hamilton–Jacobi equation. By using an improved tortoise coordinate transformation, the event horizon surface gravity and the Hawking temperature of the black hole are obtained as well.

This paper is organized as follows. In Section 2, we give the line element of an arbitrarily accelerating black hole with electric charge and magnetic charge, and the surface equation of the horizon. In Section 3, the quantum nonthermal radiation of the black hole is calculated by using the improved tortoise coordinate transformation. In Section 4, the event horizon surface gravity and the Hawking temperature of the black hole are derived. The last section is devoted to a summary of the conclusions.

2. An arbitrarily accelerating black hole with electric charge and magnetic charge

The line element of an arbitrarily accelerating black hole with electric charge and magnetic charge can be expressed in the advanced Eddington–Finkelstein time coordinate as (adopting signature (–, +, +, +))[24]

$$\begin{aligned} \mathrm{d}s^2 &= g_{00}\mathrm{d}\upsilon^2 + 2g_{01}\mathrm{d}\upsilon\mathrm{d}r + 2g_{02}\mathrm{d}\upsilon\mathrm{d}\theta \\ &\quad + 2g_{03}\mathrm{d}\upsilon\mathrm{d}\varphi + g_{22}\mathrm{d}\theta^2 + g_{33}\mathrm{d}\varphi^2, \end{aligned} \tag{1}$$

where

$$\begin{aligned} g_{00} &= -\Big[1 - \frac{2m}{r} + \frac{E^2+Q^2}{r^2} - 2ar\cos\theta \\ &\quad - \frac{4a\left(E^2+Q^2\right)\cos\theta}{r} \\ &\quad - r^2(f^2 + h^2\sin^2\theta) - \frac{1}{3}\lambda r^2\Big], \\ g_{01} &= 1, \quad g_{02} = r^2 f, \quad g_{03} = r^2 h\sin^2\theta, \\ g_{22} &= r^2, \quad g_{33} = r^2\sin^2\theta, \\ f &= -a\sin\theta + b\sin\varphi + c\cos\varphi, \\ h &= \cot\theta\,(b\cos\varphi - c\sin\varphi). \end{aligned}$$

Parameters $m = m(\upsilon)$, $E = E(\upsilon)$, $Q = Q(\upsilon)$, $a = a(\upsilon)$, $b = b(\upsilon)$, and $c = c(\upsilon)$ are functions of the advanced Eddington–Finkelstein time coordinate υ. Here m is the mass of the black

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hole, E and Q are the electric charge and the magnetic charge of the black hole, a is the magnitude of acceleration, b and c refer to the change rates of the acceleration's direction.

The metric determinant is

$$g = -r^4 \sin^2\theta, \tag{2}$$

and the non-zero contravariant components of the metric are

$$\begin{aligned} g^{10} &= g^{01} = 1, \\ g^{11} &= 1 - \frac{2m}{r} + \frac{E^2+Q^2}{r^2} \\ &\quad - 2ar\cos\theta - \frac{4a\left(E^2+Q^2\right)\cos\theta}{r} - \frac{1}{3}\lambda r^2, \\ g^{12} &= g^{21} = -f, \quad g^{13} = g^{31} = -h, \\ g^{22} &= \frac{1}{r^2}, \quad g^{33} = \frac{1}{r^2\sin^2\theta}. \end{aligned} \tag{3}$$

Now let us find the horizon equation of the space–time represented by Eq. (1). The surface equation of the event horizon can be written as

$$F(\upsilon, r, \theta, \varphi) = 0 \quad \text{or} \quad r_{\mathrm{H}} = r_{\mathrm{H}}(\upsilon, \theta, \varphi), \tag{4}$$

which should satisfy the null surface condition

$$g^{\mu\nu}\frac{\partial F}{\partial x^{\mu}}\frac{\partial F}{\partial x^{\nu}} = 0. \tag{5}$$

Substituting Eq. (3) into Eq. (5), we can obtain

$$\begin{aligned} &2\frac{\partial F}{\partial \upsilon}\frac{\partial F}{\partial r} + 2(-f)\frac{\partial F}{\partial r}\frac{\partial F}{\partial \theta} + 2(-h)\frac{\partial F}{\partial r}\frac{\partial F}{\partial \varphi} \\ &+ \frac{1}{r^2}\left(\frac{\partial F}{\partial \theta}\right)^2 + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial F}{\partial \varphi}\right)^2 \\ &+ \left(1 - \frac{2m}{r} + \frac{E^2+Q^2}{r^2} - 2ar\cos\theta \right. \\ &\left. - \frac{4a\left(E^2+Q^2\right)\cos\theta}{r} - \frac{1}{3}\lambda r^2\right)\left(\frac{\partial F}{\partial r}\right)^2 = 0. \end{aligned} \tag{6}$$

From Eq. (4), we can get

$$\begin{aligned} &\frac{\partial F}{\partial r}\frac{\partial r}{\partial \theta} + \frac{\partial F}{\partial \theta} = 0, \quad \frac{\partial F}{\partial r}\frac{\partial r}{\partial \varphi} + \frac{\partial F}{\partial \varphi} = 0, \\ &\frac{\partial F}{\partial r}\frac{\partial r}{\partial \upsilon} + \frac{\partial F}{\partial \upsilon} = 0. \end{aligned} \tag{7}$$

Substituting Eq. (7) into Eq. (6), we have

$$\begin{aligned} &1 - \frac{2m}{r_{\mathrm{H}}} + \frac{E^2+Q^2}{r_{\mathrm{H}}^2} - 2ar_{\mathrm{H}}\cos\theta_0 - \frac{4a\left(E^2+Q^2\right)\cos\theta_0}{r_{\mathrm{H}}} \\ &- \frac{1}{3}\lambda r_{\mathrm{H}}^2 - 2\dot{r}_{\mathrm{H}} + 2f_{\mathrm{H}}r'_{\mathrm{H}} + 2h_{\mathrm{H}}r'_{\mathrm{H}\varphi} \\ &+ \frac{r'^2_{\mathrm{H}}}{r_{\mathrm{H}}^2} + \frac{r'^2_{\mathrm{H}\varphi}}{r_{\mathrm{H}}^2\sin^2\theta_0} = 0, \end{aligned} \tag{8}$$

where

$$\dot{r}_{\mathrm{H}} = \frac{\partial r_{\mathrm{H}}}{\partial \upsilon}, \quad r'_{\mathrm{H}} = \frac{\partial r_{\mathrm{H}}}{\partial \theta}, \quad r'_{\mathrm{H}\varphi} = \frac{\partial r_{\mathrm{H}}}{\partial \varphi}. \tag{9}$$

Surface r_{H} that satisfies Eq. (8) is the event horizon of the arbitrarily accelerating black hole with electric charge and magnetic charge.

3. The quantum nonthermal radiation of the black hole

The motion of a scalar particle with mass μ, electric charge e, and magnetic charge q in curved space–time obeys the Hamilton–Jacobi equation

$$g^{\mu\nu}\left(\frac{\partial S}{\partial x^{\mu}} - eA_{\mu} - qB_{\mu}\right)\left(\frac{\partial S}{\partial x^{\nu}} - eA_{\nu} - qB_{\nu}\right) - \mu^2 = 0, \tag{10}$$

where S is the Hamilton master function, and A_{μ} and B_{μ} are the four-dimensional electromagnetic potentials.

Substituting Eq. (3) into Eq. (10), we can obtain

$$\begin{aligned} &2g^{10}\left[\frac{\partial S}{\partial \upsilon}\frac{\partial S}{\partial r} - (eA_1 + qB_1)\frac{\partial S}{\partial \upsilon} - (eA_0 + qB_0)\frac{\partial S}{\partial r}\right. \\ &\left. + (eA_0 + qB_0)(eA_1 + qB_1)\right] \\ &+ 2g^{12}\left[\frac{\partial S}{\partial r}\frac{\partial S}{\partial \theta} - (eA_2 + qB_2)\frac{\partial S}{\partial r} - (eA_1 + qB_1)\frac{\partial S}{\partial \theta}\right. \\ &\left. + (eA_1 + qB_1)(eA_2 + qB_2)\right] \\ &+ 2g^{13}\left[\frac{\partial S}{\partial r}\frac{\partial S}{\partial \varphi} - (eA_3 + qB_3)\frac{\partial S}{\partial r} - (eA_1 + qB_1)\frac{\partial S}{\partial \varphi}\right. \\ &\left. + (eA_1 + qB_1)(eA_3 + qB_3)\right] \\ &+ g^{11}\left[\left(\frac{\partial S}{\partial r}\right)^2 - 2(eA_1 + qB_1)\frac{\partial S}{\partial r} + (eA_1 + qB_1)^2\right] \\ &+ g^{22}\left[\left(\frac{\partial S}{\partial \theta}\right)^2 - 2(eA_2 + qB_2)\frac{\partial S}{\partial \theta} + (eA_2 + qB_2)^2\right] \\ &+ g^{33}\left[\left(\frac{\partial S}{\partial \varphi}\right)^2 - 2(eA_3 + qB_3)\frac{\partial S}{\partial \varphi} + (eA_3 + qB_3)^2\right] \\ &- \mu^2 = 0. \end{aligned} \tag{11}$$

A new tortoise coordinate can be written as[25]

$$\begin{aligned} r_* &= r + \frac{1}{2\kappa(\upsilon_0, \theta_0, \varphi_0)}\ln\left[\frac{r - r_{\mathrm{H}}(\upsilon, \theta, \varphi)}{r_{\mathrm{H}}(\upsilon, \theta, \varphi)}\right], \\ \upsilon_* &= \upsilon - \upsilon_0, \quad \theta_* = \theta - \theta_0, \quad \varphi_* = \varphi - \varphi_0, \end{aligned} \tag{12}$$

where κ is an adjustable parameter, and κ, υ_0, θ_0, and φ_0 are all constants under the tortoise coordinate transformation.

Through the tortoise coordinate transformation of Eq. (12), Eq. (11) can be rewritten as

$$\frac{A'}{2\kappa r_{\mathrm{H}}^2(r - r_{\mathrm{H}})}\left(\frac{\partial S}{\partial r_*}\right)^2 + \frac{B'}{r_{\mathrm{H}}}\frac{\partial S}{\partial r_*} + 2\kappa(r - r_{\mathrm{H}})C' = 0, \tag{13}$$

where

$$\begin{aligned} A' &= g^{11}r_{\mathrm{H}}^2[1 + 2\kappa(r - r_{\mathrm{H}})]^2 + g^{22}r^2r'^2_{\mathrm{H}} + g^{33}r^2r'^2_{\mathrm{H}\varphi} \\ &\quad - 2rr_{\mathrm{H}}[1 + 2\kappa(r - r_{\mathrm{H}})]\left(g^{10}\dot{r}_{\mathrm{H}} + g^{12}r'_{\mathrm{H}} + g^{13}r'_{\mathrm{H}\varphi}\right), \end{aligned}$$

$$
\begin{aligned}
B' &= 2r[(eA_1+qB_1)(g^{10}\dot{r}_{\rm H}+g^{12}r'_{\rm H}+g^{13}r'_{{\rm H}\varphi})\\
&\quad +g^{22}(-l+eA_2+qB_2)r'_{\rm H}+g^{33}(-j+eA_3+qB_3)r'_{{\rm H}\varphi}]\\
&\quad -2[1+2\kappa(r-r_{\rm H})][g^{10}r_{\rm H}(\omega+eA_0+qB_0)\\
&\quad +g^{11}r_{\rm H}(eA_1+qB_1)+g^{12}r_{\rm H}(-l+eA_2+qB_2)\\
&\quad +g^{13}r_{\rm H}(-j+eA_3+qB_3)],\\
C' &= 2g^{10}(eA_1+qB_1)(\omega+eA_0+qB_0)\\
&\quad +2g^{12}(eA_1+qB_1)(-l+eA_2+qB_2)\\
&\quad +2g^{13}(eA_1+qB_1)(-j+eA_3+qB_3)\\
&\quad +g^{11}(eA_1+qB_1)^2+g^{22}(-l+eA_2+qB_2)^2\\
&\quad +g^{33}(-j+eA_3+qB_3)^2-\mu^2,\\
\omega &= -\frac{\partial S}{\partial \upsilon_*},\quad l=\frac{\partial S}{\partial \theta_*},\quad j=\frac{\partial S}{\partial \varphi_*},
\end{aligned}
$$

where ω is the energy of the particle, l and j are the angular momenta of the particle.

The solution of Eq. (13) is

$$
\frac{\partial S}{\partial r_*}=\frac{\kappa r_{\rm H}(r-r_{\rm H})}{A'}(-B'\pm\sqrt{B'^2-4A'C'}). \tag{14}
$$

Because S and $\partial S/\partial r_*$ are real numbers, the energy ω of the particle in the curve space–time described by Eq. (1) satisfies

$$
\begin{aligned}
&4(g^{10})^2r_{\rm H}^2[1+2\kappa(r-r_{\rm H})]^2\omega^2\\
&-\{4B'_1g^{10}r_{\rm H}[1+2\kappa(r-r_{\rm H})]\\
&+8g^{10}A'(eA_1+qB_1)\}\omega+B'^2_1-4A'C'_1\geq 0,
\end{aligned}\tag{15}
$$

where $B'_1=B'+2g^{10}\omega r_{\rm H}[1+2\kappa(r-r_{\rm H})]$, and $C'_1=C'-2g^{10}(eA_1+qB_1)\omega$. Then we have

$$
\begin{aligned}
\omega^{\pm} &= (B'_1r_{\rm H}[1+2\kappa(r-r_{\rm H})]+2A'(eA_1+qB_1)\\
&\quad \pm2\{B'_1r_{\rm H}A'(eA_1+qB_1)[1+2\kappa(r-r_{\rm H})]\\
&\quad +A'^2(eA_1+qB_1)^2+r_{\rm H}^2A'C'_1[1+2\kappa(r-r_{\rm H})]^2\}^{1/2})\\
&\quad \times\{2g^{10}r_{\rm H}^2[1+2\kappa(r-r_{\rm H})]^2\}^{-1},
\end{aligned}\tag{16}
$$

$$
\begin{cases}
\omega\geq\omega^+,\\
\omega\leq\omega^-.
\end{cases}\tag{17}
$$

The particle is in the positive energy state when $\omega\geq\omega^+$, in the negative state when $\omega\leq\omega^-$, while $\omega^-\leq\omega\leq\omega^+$ is the energy restricted zone. The width of the forbidden region is

$$
\Delta\omega=\omega^+-\omega^-=\frac{2\sqrt{B'_1r_{\rm H}A'(eA_1+qB_1)[1+2\kappa(r-r_{\rm H})]+A'^2(eA_1+qB_1)^2+r_{\rm H}^2A'C'_1[1+2\kappa(r-r_{\rm H})]^2}}{g^{10}r_{\rm H}^2[1+2\kappa(r-r_{\rm H})]^2}. \tag{18}
$$

Equations (16) and (17) both describe the distribution characteristics of the particle energy levels in the curve space–time described by Eq. (1).

When the radius $r\to\infty$, the space–time starts to become flat one and the electromagnetic field can be neglected, from Eq. (16), we can obtain

$$
\lim_{r\to\infty}\omega^{\pm}\to\pm\mu. \tag{19}
$$

This shows that the particle energy spectrum is just the Dirac energy spectrum in the Minkowski space–time when $r\to\infty$.

When the radius $r\to r_{\rm H}$ near the event horizon, from Eqs. (16) and (8), we can obtain

$$
\begin{aligned}
\lim_{r\to r_{\rm H},\upsilon\to\upsilon_0,\theta\to\theta_0,\varphi\to\varphi_0}\omega^+ &= \lim_{r\to r_{\rm H},\upsilon\to\upsilon_0,\theta\to\theta_0,\varphi\to\varphi_0}\omega^-\\
&= \omega_0=\left.\frac{B'_1}{2g^{10}r_{\rm H}}\right|_{r\to r_{\rm H}},\\
\lim_{r\to r_{\rm H},\upsilon\to\upsilon_0,\theta\to\theta_0,\varphi\to\varphi_0}\Delta\omega &= 0.
\end{aligned}\tag{20}
$$

This shows that the energy width of the forbidden region of a particle is $\Delta\omega=0$ on the event horizon. When $\mu<\omega_0$ near the event horizon, both the positive and the negative energy levels will show a cross phenomenon. The energy of some negative energy particles can be larger than μ, and also larger than the energy of nearby positive energy levels. When the energy satisfies $\mu<\omega\leq\omega_0$, a negative energy particle can travel across the forbidden region because of the quantum tunneling effect and becomes a positive energy particle that can move out. This is a spontaneous radiation not related to the temperature, which can be called the Starobinsky–Unruh process.[9]

4. Event horizon surface gravity and the Hawking temperature of the black hole

In Eq. (13), the coefficient of $(\partial S/\partial r_*)^2$ is 0/0 type as $r\to r_{\rm H}$, therefore we can calculate the limit of the coefficient using the l'Hôpital law. When we select the adjustable parameter κ as

$$
\kappa=\frac{\dfrac{2m}{r_{\rm H}^2}-\dfrac{r'^2_{\rm H}}{2r_{\rm H}^3}-\dfrac{r'^2_{{\rm H}\varphi}}{2r_{\rm H}^3\sin^2\theta_0}-\dfrac{1}{2r_{\rm H}}-\dfrac{3\left(E^2+Q^2\right)}{2r_{\rm H}^3}+\dfrac{8a\left(E^2+Q^2\right)\cos\theta_0}{2r_{\rm H}^2}-\dfrac{1}{6}\lambda r_{\rm H}}{2r_{\rm H}\left(a\cos\theta_0+\dfrac{m}{r_{\rm H}^2}+\dfrac{r'^2_{\rm H}}{2r_{\rm H}^3}+\dfrac{r'^2_{{\rm H}\varphi}}{2r_{\rm H}^3\sin^2\theta_0}-\dfrac{E^2+Q^2}{2r_{\rm H}^3}+\dfrac{2a\left(E^2+Q^2\right)\cos\theta_0}{r_{\rm H}^2}+\dfrac{1}{6}\lambda r_{\rm H}\right)}, \tag{21}
$$

then we have $\lim_{r\to r_{\rm H}} \dfrac{A'}{2\kappa r_{\rm H}^2 (r-r_{\rm H})} = 1$. The κ is just the event horizon surface gravity, and is identical to the κ obtained by taking the limitation of the coefficient of $\partial^2\Phi/\partial r_*^2$ in the Klein–Gordon equation under the same tortoise coordinate transformation. So the value of κ can be obtained in different ways.

In addition, from the relation $T = \kappa/2\pi\kappa_{\rm B}$, we can get the Hawking temperature of the black hole on the event horizon

$$T = \frac{\kappa}{2\pi\kappa_{\rm B}} = \frac{1}{2\pi\kappa_{\rm B}} \frac{\dfrac{2m}{r_{\rm H}^2} - \dfrac{r_{\rm H}'^2}{2r_{\rm H}^3} - \dfrac{r_{{\rm H}\varphi}'^2}{2r_{\rm H}^3\sin^2\theta_0} - \dfrac{1}{2r_{\rm H}} - \dfrac{3\left(E^2+Q^2\right)}{2r_{\rm H}^3} + \dfrac{8a\left(E^2+Q^2\right)\cos\theta_0}{2r_{\rm H}^2} - \dfrac{1}{6}\lambda r_{\rm H}}{2r_{\rm H}\left[a\cos\theta_0 + \dfrac{m}{r_{\rm H}^2} + \dfrac{r_{\rm H}'^2}{2r_{\rm H}^3} + \dfrac{r_{{\rm H}\varphi}'^2}{2r_{\rm H}^3\sin^2\theta_0} - \dfrac{E^2+Q^2}{2r_{\rm H}^3} + \dfrac{2a\left(E^2+Q^2\right)\cos\theta_0}{r_{\rm H}^2} + \dfrac{1}{6}\lambda r_{\rm H}\right]}. \tag{22}$$

We can see that the Hawking temperature of the arbitrarily accelerating black hole with electric charge and magnetic charge depends on not only the time, but also the angle.

5. Conclusions

Using a correct-dimension new tortoise coordinate transformation, we have studied the Hamilton–Jacobi equation of scalar particles, and obtained the quantum nonthermal radiation of an arbitrarily accelerating black hole with electric charge and magnetic charge. The energy condition for the occurrence of the Starobinsky–Unruh process has been obtained, and the event horizon surface gravity and the Hawking temperature on the event horizon have also been given. Studying the thermal and nonthermal radiations of the dynamic black hole has contributed to the generalization of the thermodynamic rule of black holes from static and stationary black holes to dynamic ones. Currently, researchers usually discuss simple dynamic black holes, such as the Vaidya black hole, and research into more complex dynamic black holes is required. The conclusions are summarized as follows.

1) The nonthermal radiation of a black hole has nothing to do with its temperature, it is a quantum effect, and it would cost the rotational and the electromagnetic energies of the black hole. There exists a crossing of the particle's energy level when $\mu < \omega_0$ near the event horizon. For some negative energy particles, their energy can be larger than the energy of the nearby positive energy levels, then these negative energy particles can travel across the forbidden region because of the quantum tunneling effect and become positive energy particles that can move out. The particle radiation probability in the Starobinsky–Unruh process is related to ω_0 and $\omega^+ - \omega^-$ near the event horizon. From Eqs. (18) and (20), we know that the nonthermal radiation probability not only depends on the mass, electric charge, magnetic charge, and acceleration of the black hole, but also is related to the variations of the event horizon and the four-dimensional electromagnetic potential. The nonthermal radiation will not stop until the rotation angular momentum, electric charge, and magnetic charge are all zero.

2) The event horizon surface gravity κ has been obtained by analyzing the coefficient of $(\partial S/\partial r_*)^2$ in the Hamilton–Jacobi equation with $r \to r_{\rm H}$, and the Hawking temperature has been given, which are identical to the results obtained by the Klein–Gordon equation. The event horizon surface gravity κ and the Hawking temperature are connected with the time and position on the event horizon, which is a characteristic of dynamic and non-spherically symmetric black holes. The Hawking radiation of a black hole is different from its nonthermal radiation; it reflects the thermal properties of the black hole. The discovery of Hawking radiation shows that the black hole has thermodynamic properties, and the laws of black hole thermodynamics may be built in its true sense. The Hawking temperature of a stationary black hole is a constant, and the Hawking radiation spectrum is a black body spectrum. However, the Hawking temperature of a dynamical black hole is not a constant, and the Hawking radiation spectrum with only a spectrum of black body form is not the black body spectrum indeed.

3) There is a problem of dimension in the past tortoise coordinate transformation, we can obtain a more reasonable result by using the improved correct dimension new tortoise coordinate transformation of Eq. (12).

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