The dynamics of one-dimensional random quantum XY system with Dzyaloshinskii–Moriya interaction*

Li Yin-Fang(李银芳) and Kong Xiang-Mu(孔祥木)†

Shandong Provincial Key Laboratory of Laser Polarization and Information Technology, Department of Physics, Qufu Normal University, Qufu 273165, *China*

(Received 22 June 2012; revised manuscript received 26 September 2012)

In this paper, the effects of random variables on the dynamics of the $s = 1/2$ XY model with the Dzyaloshinskii–Moriya interaction are studied. By means of the recurrence relation method in the high-temperature limit, we calculate the spin autocorrelation functions as well as the corresponding spectral densities for the cases that the exchange couplings between spins or external magnetic fields satisfy the double-Gaussian distribution. It is found that when the standard deviation of random exchange coupling δ_J (or the standard deviation of random external field δ_B) is small, the dynamics of the system undergoes a crossover from a collective-mode behavior to a central-peak one. However, when δ_J (or δ_B) is large, the crossover vanishes, and the system shows a central-peak behavior or the most disordered one. We also analyze the cases in which the exchange couplings or the external fields satisfy the bimodal and the Gaussian distributions. Our results show that for all the cases considered, the dynamics of the above system is similar to that of the one-dimensional random XY model.

Keywords: correlation function, spectral density, Dzyaloshinskii–Moriya interaction, recurrence relation method

PACS: 75.10.Pq, 75.10.Jm, 75.40.Gb, 75.50.Lk **DOI:** 10.1088/1674-1056/22/3/037502

1. Introduction

Recently, much attention has been paid to the study of random quantum systems.[1–3] We study qualitatively the behaviors of both the time-dependent spin-correlation function and the corresponding spectral density function in the present of disorder.

One of the typical examples of quantum spin systems is the spin-1/2 XY model, which has been successfully related to some quasi-one-dimensional compounds, such as Cs_2CoCl_4 and $PrCl_3$.[4,5] Lieb *et al.* have shown that the XY model can be transformed into a set of noninteracting fermions and made a detailed study of its ground-state $(T = 0)$ properties.[6] Florencio and Lee have investigated the dynamics of the one dimensional (1D) transverse Ising (TI) model and the spin-1/2 isotropic XY model in infinite temperature by using the method of recurrence relations, and found that the realized dynamical Hilbert spaces of the two models have the same structure, which leads to a similar dynamical behavior apart from the time scale, and obtained the transverse spin correlation functions for both systems with $B = J$, $\left\langle S_j^x(t) S_j^x \right\rangle = \mathrm{e}^{-\Delta t^2}/4$, where J and B are the nearest-neighbor spin exchange coupling and the external field, respectively, and $\Delta = J^2/2$ for the TI model, J^2 for the XY model.[7] Regarding the dynamics of the XY model, Niemeijer has found that the time-dependent longitudinal spin correlation function at any temperature can be expressed as the square of the Bessel function, $(\mathrm{J}_0(2Jt))^2$, where J_0 is the 0-fold Bessel function.[8] McCoy and Barouch have made a detailed investigation of various space-dependent spin-correlation functions at different temperatures.[9,10]

On the other hand, there are some studies on the random quantum spin system, which can be used for describing more real materials such as ferroelectric crystals[11–14] and spin glasses.[15] For the random isotropic XY chain, the dynamics of the system at long time is governed by the stronger couplings even if only a very small amount of disorder is present. In the long-time limit, the correlation functions oscillate at the cutoff frequency of the disorder-less stronger-coupling case, with the spectral densities displaying tails that end at the stronger-coupling cutoff frequency.[16] Furthermore, a series of spin systems, such as the TI model, the transverse XY model with Gaussian or double-Gaussian distribution, and the two-dimensional TI model, have also been studied.[3,17–20]

In recent years, some other magnetic properties have been discovered in a variety of quasi-one-dimensional materials that are known to belong to the class of the Dzyaloshinskii–Moriya (DM) magnet. The form of the interaction is $\boldsymbol{D} \cdot (\boldsymbol{S}_i \times \boldsymbol{S}_j)$, where $\boldsymbol{S}_i$ are the spin operators. Jafari *et al.* have studied the phase diagram and the entanglement of a 1D Ising model with the DM interaction and found that the DM interaction drives the quantum fluctuation resulting in a phase transition in the model.[21] The Heisenberg chain with the DM interaction has also been extensively studied.[22,23]

In this work, we investigate the effects of random vari-

*Project supported by the National Natural Science Foundation of China (Grant No. 10775088), the Shandong Natural Science Foundation, China (Grant No. Y2006A05), and the Science Foundation of Qufu Normal University, China.

†Corresponding author. E-mail: kongxm@mail.qfnu.edu.cn

© 2013 Chinese Physical Society and IOP Publishing Ltd

ables (magnetic field or exchange coupling) in the 1D $s = 1/2$ quantum XY model with the DM interaction in the high-temperature limit by using the recurrence relation method. The paper is organized as follows. In Section 2, we give a simple introduction to the 1D quantum XY model with the DM interaction and the recurrence relation method. In Section 3, we discuss the results. A summary is given in Section 4.

2. Model and method

The 1D quantum spin-1/2 XY model with the DM interaction has been of considerable theoretical interest in recent years as a many-body system. The Hamiltonian of this system under an external field B_i can be written as

$$H = -\frac{1}{2}\sum_i [J_i\sigma_i^x\sigma_{i+1}^x + K_i\sigma_i^y\sigma_{i+1}^y + B_i\sigma_i^z + D_i\left(\sigma_i^x\sigma_{i+1}^y - \sigma_i^y\sigma_{i+1}^x\right)], \tag{1}$$

where σ_i^α $(\alpha = x,y,z)$ are the Pauli spin operators, J_i and K_i are the x and the y direction exchange couplings between nearest-neighbor spins, respectively, and D_i indicates the z direction of $\boldsymbol{D}$. We introduce the exchange couplings or the external fields as independent random variables, which satisfy the double-Gaussian distribution

$$P(\beta_i) = pP_1(\beta_i) + (1-p)P_2(\beta_i), \tag{2}$$

where

$$P_1(\beta_i) = \frac{1}{\sqrt{2\pi}\delta_\beta}\exp\left[-\frac{(\beta_i-\beta_a)^2}{2\delta_\beta^2}\right],$$
$$P_2(\beta_i) = \frac{1}{\sqrt{2\pi}\delta_\beta}\exp\left[-\frac{(\beta_i-\beta_b)^2}{2\delta_\beta^2}\right] \tag{3}$$

are the Gaussian distributions, β_a and β_b are the corresponding mean values of the random variables β_i, δ_β is the standard deviation that denotes the deviation of random variables β_i from the mean values β_a or β_b, and p represents the probability of the variable satisfying the Gaussian distribution $P_1(\beta_i)$, with $0 \le p \le 1$. The $P(\beta_i)$ means that β_i satisfies the Gaussian distribution $P_1(\beta_i)$ or $P_2(\beta_i)$ with the fraction of p or $1-p$, respectively.

Typically, when the standard deviation $\delta_\beta \to 0$, the Gaussian parts in Eq. (2) become δ functions, so the double-Gaussian distribution becomes a bimodal distribution

$$P(\beta_i) = p\delta(\beta_i-\beta_a) + (1-p)\delta(\beta_i-\beta_b). \tag{4}$$

When $p = 0$ or $p = 1$, equation (2) becomes the standard Gaussian distribution.

The spin autocorrelation function plays an important role in the study of the dynamics of quantum spin systems. In this paper, we are mainly interested in the average of the spin autocorrelation function in the high-temperature limit. It is defined as

$$C(t) = \overline{\langle\sigma_i^x(t)\,\sigma_i^x(0)\rangle}, \tag{5}$$

where $\overline{\langle\ldots\rangle}$ denotes an ensemble average followed by an average over the disorder variables. The corresponding spectral density $\Phi(\omega)$, which is the Fourier transformation of $C(t)$ and can be measured directly in experiments, is defined as

$$\Phi(\omega) = \int_{-\infty}^{+\infty} e^{i\omega t}C(t)\,dt. \tag{6}$$

For the mathematical simplicity, $\Phi(\omega)$ can be obtained as

$$\Phi(\omega) = \lim_{\varepsilon\to 0}\left[\mathrm{Re}\int_0^\infty dt C(t)\,e^{-zt}\right], \tag{7}$$

where $z = \varepsilon + i\omega$, and $\varepsilon > 0$.

The recurrence relation method has been proved to be very powerful in the calculation of dynamic correlation function.[2,24–29] Next we will give a brief introduction to this method. The Kubo inner product in the Hilbert space is defined as

$$(X,Y) = \beta^{-1}\int_0^\infty d\lambda\,\langle X(\lambda)Y^+\rangle - \langle X\rangle\langle Y^+\rangle, \tag{8}$$

where $X(\lambda) = e^{\lambda H}Xe^{-\lambda H}$, $\langle XY\rangle = \mathrm{Tr}\left(e^{-\beta H}XY\right)/\mathrm{Tr}\,e^{-\beta H}$, $\beta = (k_BT)^{-1}$, and H is the Hamiltonian of the system. Considering Hermitian operator $\sigma_i^x(t)$ as a dynamical variable and then expanding it with an orthogonal set in the Hilbert space, we have

$$\sigma_i^x(t) = \sum_{\nu=0}^\infty a_\nu(t)f_\nu, \tag{9}$$

where $a_\nu(t)$ are the time-dependent coefficients, and $\{f_\nu\}$ is a complete set of orthogonalized basis vectors that spans the Hilbert space, that is $(f_\nu, f_\mu) = 0$ if $\nu \ne \mu$. There exists the following set of recurrence relations for the basis vectors f_ν:

$$f_{\nu+1} = iLf_\nu + \Delta_\nu f_{\nu-1},\quad \nu \ge 0, \tag{10}$$

$$\Delta_\nu = \frac{(f_\nu, f_\nu)}{(f_{\nu-1}, f_{\nu-1})},\quad \nu \ge 1, \tag{11}$$

where L is the quantum Liouvillian operator $LA \equiv [H,A] = HA - AH$, and $f_{-1} \equiv 0$, $\Delta_0 \equiv 1$.

We insert Eq. (9) into equation $d\sigma_i^x(t)/dt = i[H,\sigma_i^x(t)] = iL\sigma_i^x(t)$, the differential operator acts on $a_\nu(t)$, and the Liouvillian operator acts on f_ν with the result given by Eqs. (10) and (11). A term-by-term comparison of the coefficients of each vector f_ν in the equation of motion then yields the coefficients $a_\nu(t)$ satisfying the relation

$$\Delta_{\nu+1}a_{\nu+1}(t) = -\dot{a}_\nu(t) + a_{\nu-1}(t),\quad \nu \ge 0, \tag{12}$$

with $\dot{a}_\nu(t) = da_\nu(t)/dt$, $a_{-1}(t) \equiv 0$, and $a_k(0) = \delta_{k,0}$.

Taking the inner product of $\sigma_j^x(t)$ and $\sigma_j^x(0)$, from Eq. (9), we find that the inner product can be expressed as

$\left(\sigma_j^x(t), \sigma_j^x(0)\right) = \overline{\langle \sigma_j^x(t)\,\sigma_j^x(0)\rangle}$ when considering the average over the disorder variables in the high-temperature limit. We then find that $a_0(t) = \left(\sigma_j^x(t), \sigma_j^x(0)\right)$ is the spin autocorrelation function

$$a_0(t) = \overline{\langle \sigma_j^x(t)\,\sigma_j^x(0)\rangle} = C(t).$$

By applying the Laplace transform $a_\nu(z) = \int_0^\infty \mathrm{e}^{-zt} a_\nu(t)\,\mathrm{d}t$ to Eq. (12), $a_0(z)$ in the continued-fraction representation can be obtained as

$$a_0(z) = \cfrac{1}{z + \cfrac{\Delta_1}{z + \cfrac{\Delta_2}{z + \cdots}}}. \tag{13}$$

The spin autocorrelation function $C(t)$ can be expressed in the form of moment expansion

$$C(t) = \sum_{\nu=0}^{\infty} \frac{(-1)^\nu}{(2\nu)!} M_{2\nu} t^{2\nu}, \tag{14}$$

with

$$M_{2\nu} = \frac{1}{Z}\overline{\mathrm{Tr}\sigma_i^x[H,[H,\cdots[H,\sigma_i^x]\cdots]]}, \tag{15}$$

where $M_{2\nu}$ is the 2ν-th moment of $C(t)$.

For a given set of $\Delta_\nu = M_{2\nu}^{(\nu)}$, $M_{2\nu}^{(-1)} = 0$, and $\Delta_{-1} = \Delta_0 = 1$, the moments $M_{2\nu}^{(0)} = M_{2\nu}$ result from the relation

$$M_{2\nu}^{(\nu-1)} = \Delta_{\nu-1} M_{2\nu}^{(\nu)} + \frac{\Delta_{\nu-1}}{\Delta_{\nu-2}} M_{2\nu-2}^{(\nu-2)}. \tag{16}$$

These simple transformation relations establish the link between the recursion method and the various techniques in use of the analysis of moment expansions. The computational effort for calculating the moments (Eq. (15)) is similar to that for calculating the same number of Δ_k's by using the recursion method.[24] Supposing that the first eight recurrants and the corresponding sixteen moments have been calculated by Eqs. (10) and (16), we can obtain $C(t)$ by constructing the padé approximate. Due to the mathematical complexity, a finite number of moments can be obtained, thus we can just discuss the short time behavior of $C(t)$.

However, we will see that working with continued-fraction coefficients Δ_k's are more suitable for the direct reconstruction of the spectral density $\Phi(\omega)$. Because we can only get a finite number of recurrants, it is necessary to terminate it with some schemes. Here, we use the Gaussian terminator,[24,30] which is consistent with the property that the Δ_ν sequence has an approximate linear growth rate. With the help of the Padé approximate and the Gaussian terminator, we can obtain the spin autocorrelation function $C(t)$ and the corresponding spectral density $\Phi(\omega)$ of the system, respectively.

3. Results and discussion

In order to investigate the spin autocorrelation function $C(t)$, we choose the zeroth basis vector $f_0 = \sigma_i^x$. With the recurrence relation Eq. (10), the first basis vector can be obtained as

$$\begin{aligned} f_1 = {} & B_i\sigma_i^y - D_{i-1}\sigma_{i-1}^x\sigma_i^z + D_i\sigma_{i+1}^x\sigma_i^z - K_{i-1}\sigma_{i-1}^y\sigma_i^z \\ & - K_i\sigma_{i+1}^y\sigma_i^z. \end{aligned}$$

The norms of the above vectors are obtained as follows:

$$(f_0, f_0) = 1,$$

$$(f_1, f_1) = \overline{B_i^2} + \overline{D_{i-1}^2} + \overline{D_i^2} + \overline{K_{i-1}^2} + \overline{K_i^2}.$$

We have exactly calculated the basis vectors up to f_6. Then the continued fraction coefficients can be obtained from Eq. (11). It is observed that the Δ_ν sequence has an approximate linear growth rate, we obtain Δ_7 and Δ_8 by $\Delta_\nu = (\Delta_N/N)\,\nu$ for $\nu > N$.

Next, we discuss the numerical results of the spin autocorrelation function $C(t)$ and the spectral density $\Phi(\omega)$ when the exchange couplings or the external fields satisfy three types of distributions: double-Gaussian, bimodal, and Gaussian distributions with the special values of the isotropy ($J_i = K_i$) XY system. The DM interaction takes $D_i = J_i/2$. The effects of the randomness on the dynamics of the given system are investigated as follows.

3.1. Gaussian distribution

First we consider the case that the exchange coupling satisfies the Gaussian distribution (3). For simplicity, we choose the external fields $B_i = B = 1.0$, which sets the energy scale. When the mean exchange couplings in the distribution take the value $\overline{J} = 0.00, 0.50, 1.00, 1.50, 2.00$, the spin autocorrelation functions $C(t)$ and the corresponding spectral densities $\Phi(\omega)$ for two cases in which the standard deviation δ_J takes a small value (0.3) and a larger one (3.0) respectively are shown in Fig. 1. The continued-fraction coefficients are presented in the insets of Figs. 1(b) and 1(d).

When the exchange coupling satisfies the Gaussian distribution, the standard deviation δ_J is small ($\delta_J = 0.3$), and $\overline{J}$ equals to 0.00, the dynamics of the system shows a collective-mode behavior that is increasingly dominated by the external fields. At the same time, the spin autocorrelation function shows a damped oscillation behavior. With the increase of the mean exchange coupling $\left(\overline{J} = 2.00\right)$, the system shows a central-peak behavior instead of the collective-mode one when the values of the exchange couplings are much larger than those of the external fields. The peak of $\Phi(\omega)$ shown in Fig. 1(b) moves from $\omega = 1$ to 0 as $\overline{J}$ increases further, while the amplitude of $\Phi(\omega)$ decreases rapidly ($2.17 \to 0.38$). Due to the increase of the exchange coupling, the spin interactions are stronger and prevailing gradually, dominating the dynamics properties of the system, so that the collective-mode behavior disappears, and is replaced by the central-peak behavior.

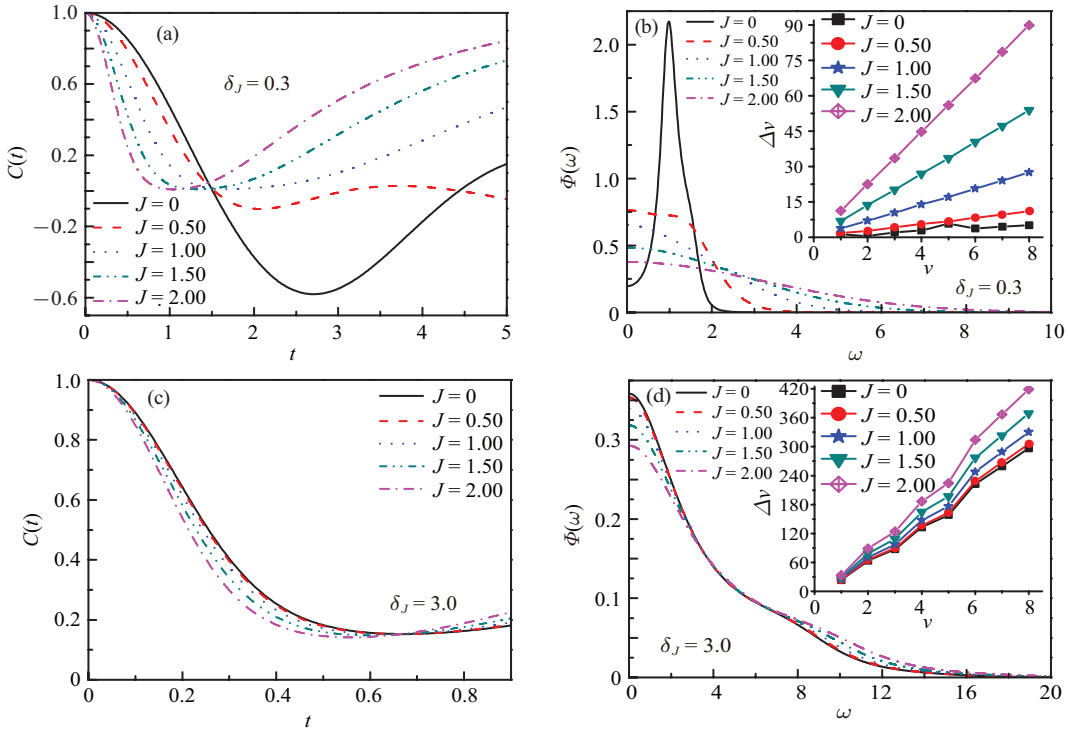


Fig. 1. (color online) Spin autocorrelation functions and corresponding spectral densities when the bonds satisfy the Gaussian distribution, and $B_i = B = 1$. The standard deviation is (a), (b) $\delta_J = 0.3$; and (c), (d) $\delta_J = 3.0$. The continued-fraction coefficients are presented in the insets.

On the other hand, for the case of a large standard deviation δ_J ($\delta_J = 3.0$), the system only presents a central-peak behavior regardless of the exchange couplings.

3.2. Bimodal distribution

Now we consider the cases that the external fields B_i or the exchange couplings J_i satisfy the bimodal distribution (4). Two cases are considered. In one case, the exchange couplings are uniform ($J_i = J = 1.0$), while the mean external fields in the distribution are $B_a = 1.8$ ($q = 1$) to $B_b = 0.2$ ($q = 0$) where $0 \leq q \leq 1$, (Figs. 2(a) and 2(b)). In the other case, the external fields are constants ($B_i = B = 1.0$), whereas the exchange couplings satisfy the bimodal distribution and vary from $J_b = 0.4$ ($p = 0$) to $J_a = 1.0$ ($p = 1$) (Figs. 2(c) and 2(d)).

To summarize, the system has a crossover from a collective-mode behavior to a central-peak one as J/B increases in both cases. Generally, when $J < B$, the system is dominated by the collective-mode behavior. However, when $J > B$, the main mode shows the central-peak behavior. So the dynamics of the system changes with B and J.

From the above discussion, we obtain that the dynamical properties of the system are affected by the relative strength between the spin–spin interactions and the external fields, not by the different disordered distributions. The physical interpretation of these behaviors can be obtained by analyzing the root-mean square internal field associated with the exchange coupling and the transverse frequency. The field due to the exchange coupling in the infinite temperature limit is

$$G_{ij}^2 = \sum_{j,k} J_{ij} J_{jk} \left\langle \sigma_i^x \sigma_j^x \right\rangle = \sum_j J_{ij}^2 \left\langle (\sigma_i^x)^2 \right\rangle = J^2,$$

so the root-mean-square goes as J. Roughly speaking, for a large B ($B > J$), the system behaves as independent spins precessing about the field B. The fluctuating internal field causes the damping of the precessing spins and gives the spectral line its width. The resonant structure (collective-mode) disappears when the transverse field is smaller than the root-mean-square internal field ($B < J$). For the disordered situations, the general dynamic behavior of the system is neither central-peak-like nor collective-mode-like, but something in between.[2]

It is easy to find that the dynamics of the system is similar to that of the 1D quantum XY model when the DM interaction is small $D_i = J_i/2$. We have studied the case in which the DM interaction is smaller $D_i = J_i/5$ and find that the results are the same as those of the XY model.

3.3. Double-Gaussian distribution

The double-Gaussian distribution is a common form of bimodal distribution and Gaussian distribution, which can be used to describe both the discrete distribution and the continuous one. For a small standard deviation ($\delta_J = 0.1$), the dynamics of the system (see Figs. 3(a) and 3(b)) is analogous to that for the case in which the exchange couplings satisfy the bimodal distribution. To the contrary, when the standard deviation δ_J is large ($\delta_J = 2.0$), the dynamics of the system (see Figs. 3(a) and 3(b)) is similar to that for the case in which the exchange couplings satisfy the Gaussian distribution.

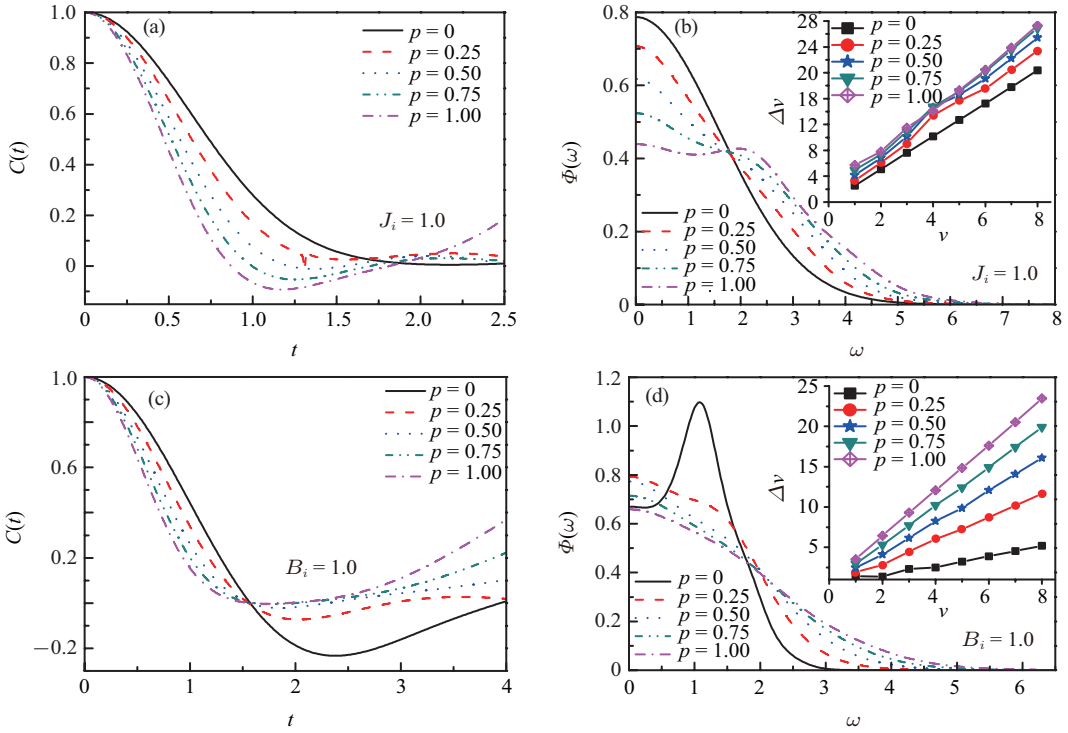


Fig. 2. (color online) Autocorrelation functions and spectral densities for the cases that (a), (b) the external fields satisfy the bimodal distribution; and (c), (d) the exchange couplings satisfy the bimodal distribution. The continued-fraction coefficients are presented in the insets.

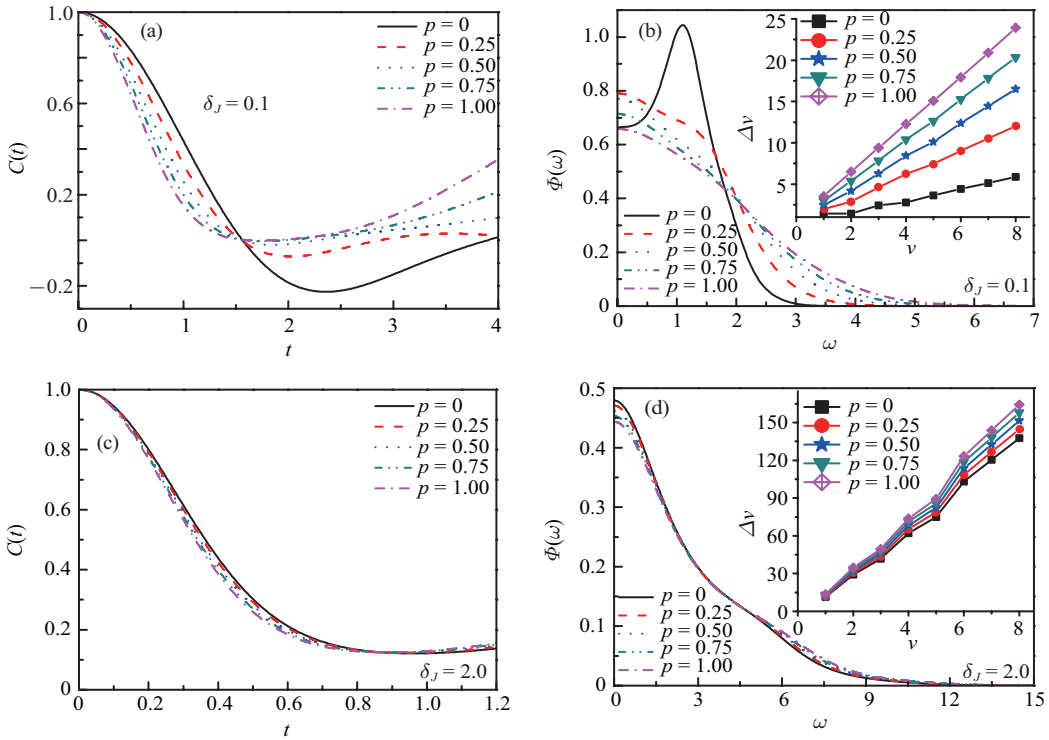


Fig. 3. (color online) Spin autocorrelation functions and spectral densities for the case that the exchange couplings satisfy the double-Gaussian distribution, the mean values are $J_a = 1.0$ and $J_b = 0.4$ with probabilities p and $(1-p)$, respectively. The standard deviation is $(\delta_J = 0.1)$ in panels (a) and (b); $(\delta_J = 2.0)$ in panels (c) and (d). The continued-fraction coefficients are presented in the insets.

We also analyze the case of the random field and uniform bond. It is not difficult to find that for the three disorders, the dynamics of the XY chain with the DM interaction is similar to that of the 1D transverse XY model.[31] This behavior can be understood from the work of Schotte *et al.*[32–35] In some materials, such as $CsCuCl_3$, copper benzoate, and quantum dot of GaAs, they estimated that the DM coupling constants are $|D|/J = 0.18$, 0.11, and 0.01, respectively. The DM interac-

tion is very small, so it cannot influence the dynamic behavior of the XY system.

4. Summary

We have studied the dynamics of the 1D spin-1/2 quantum XY model with random disorder at the high-temperature limit by using the recurrence relation method. We find that the dynamics of the system with three types of random distributions are affected by the mean values of the external field B, the exchange coupling J, and the standard deviation δ_B or δ_J. For the cases of Gaussian disorder and double-Gaussian distribution, when δ_J (δ_B) is small (e.g., $\delta_J = 0.3$ or $\delta_B = 0.1$), the dynamics of the system undergoes a crossover from a collective-mode behavior to a central-peak one. In contrary, when the standard deviation δ_J (δ_B) becomes large, the dynamics of the system changes to a central-peak behavior or only a most disordered behavior which is intervenient between a central-peak one and a collective-mode one. In the case of bimodal disorder, the dynamics of the system changes according to the strength of B or J, when $J < B$, the system is dominated by a collective-mode behavior. However, for $J > B$, the main mode shows the central-peak behavior.

References

[1] Honarasa G R, Tavassoly M K and Hatami M 2012 *Chin. Phys. B* **21** 054208
[2] Florencio J and Sá Barreto F C 1999 *Phys. Rev. B* **60** 9555
[3] Boechat B, Cordeiro C, Florencio J, Sá Barreto F C and de Alcantara Bonfim O F 2000 *Phys. Rev. B* **61** 14327
[4] Basak R and Chatterjee I 1989 *Phys. Rev. B* **40** 4627
[5] Goovaerts E, De Raedt H and Schoemaker D 1984 *Phys. Rev. Lett.* **52** 1649
[6] Lieb E, Schultz T D and Mattis D C 1961 *Ann. Phys.* **16** 407
[7] Florencio J and Lee M H 1987 *Phys. Rev. B* **35** 1835
[8] Niemeijer T 1967 *Physica* **36** 377
[9] McCoy B M 1968 *Phys. Rev.* **173** 531
[10] Barouch E and McCoy B M 1971 *Phys. Rev. A* **3** 786
[11] Plascak J A, Pires A S T and Sá Barreto F C 1982 *Solid State Commun.* **44** 787
[12] Plascak J A, Sá Barreto F C, Pires A S T and Goncalves L L 1983 *J. Phys. C* **16** 49
[13] Watarai S and Matsubara T 1984 *J. Phys. Soc. Jpn.* **53** 3648
[14] Levitsky R R, Grigas J, Zachek I R, Mits Y V and Paprotny W 1986 *Ferroelectrics* **67** 109
[15] Wu W, Ellman B, Rosenbaum T F, Aeppli G and Reich D H 1991 *Phys. Rev. Lett.* **67** 2076
[16] Nunes M E S and Florencio J 2003 *Phys. Rev. B* **68** 014406
[17] Yuan X J, Zhao B Y, Chen S X and Kong X M 2010 *Acta Phys. Sin.* **59** 1499 (in Chinese)
[18] Li Y F, Shen Y Y and Kong X M 2012 *Acta Phys. Sin.* **61** 107501 (in Chinese)
[19] Boechat B, Cordeiro C, de Alcantara Bonfim O F, Florencio J and Sá Barreto F C 2000 *Braz. J. Phys.* **30** 693
[20] de Alcantara Bonfim O F, Boechat B, Cordeiro C, Sá Barreto F C and Florencio J 2001 *J. Phys. Soc. Jpn.* **70** 829
[21] Jafari R, Kargarian M, Langari A and Siahatgar M 2008 *Phys. Rev. B* **78** 214414
[22] Kargarian M, Jafari R and Langari A 2009 *Phys. Rev. A* **79** 042319
[23] Zhang X P, Ren Z Z, Zheng Q and Zhi Q J 2009 *Chin. Phys. B* **18** 3210
[24] Viswanath V S and Müller G, 1994 *The Recursion Method-Applications to Many-Body Dynamics* (Berlin: Springer)
[25] Mori H 1965 *Prog. Theor. Phys.* **34** 399
[26] Lee M H 1982 *Phys. Rev. Lett.* **49** 1072
[27] Lee M H 1982 *Phys. Rev. B* **26** 2547
[28] Lee M H 1983 *J. Math. Phys.* **24** 2512
[29] Lee M H, Hong J and Florencio J 1987 *Phys. Scr.* **1987** 498
[30] Stolze J, Viswanath V S and Müller G 1992 *Z. Phys. B* **89** 45
[31] Xu Z B, Kong X M and Liu Z Q 2008 *Phys. Rev. B* **77** 184414
[32] Schotte U, Hoser A and Stüβer N 2000 *Solid State Commun.* **113** 523
[33] Zhao J Z, Wang X Q, Xiang T, Su Z B and Yu L 2003 *Phys. Rev. Lett.* **90** 207204
[34] Kavokin K V 2001 *Phys. Rev. B* **64** 075305
[35] Kavokin K V 2004 *Phys.Rev. B* **69** 075302