A Dugdale–Barenblatt model for a strip with a semi-infinite crack embedded in decagonal quasicrystals*

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The present study is to determine the solution of a strip with a semi-infinite crack embedded in decagonal quasicrystals, which transforms a physically and mathematically daunting problem. Then cohesive forces are incorporated into a plastic strip in the elastic body for nonlinear deformation. By superposing the two linear elastic fields, one is evaluated with internal loadings and the other with cohesive forces, the problem is treated in Dugdale–Barenblatt manner. A simple but yet rigorous version of the complex analysis theory is employed here, which involves a conformal mapping technique. The analytical approach leads to the establishment of a few equations, which allows the exact calculation of the size of cohesive force zone and the most important physical quantity in crack theory: stress intensity factor. The analytical results of the present study may be used as the basis of fracture theory of decagonal quasicrystals.

Keywords: quasicrystals, conformal mapping, Dugdale–Barenblatt model, stress intensity factor

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1. Introduction

Quasicrystals are a form of solids different from crystals and glassy solids, which possess quasi-periodic long-range translational symmetry and non-crystallographic rotational symmetry announced by Shechtman who was awarded the 2011 Nobel Prize.[1] The unusual properties of quasicrystals originate from their special atomic constitutions. Due to the quasiperiodicity, quasicrystals each have a special type of elastic degree of freedom, termed the phason degree of freedom apart from phonon degree which exists also for each crystal, and they have been studied extensively and widely in the physics of condensed matter.[2–19] By use analytical methods and simple models, the authors of Refs. [20] and [21] also studied the plastic deformation of decagonal quasicrystals. The exploration of mathematical methods of describing and analysing the structure quantitatively is a challenge to all researchers. On the basis of the physical framework and by extending the methodology of mathematical physics and classical elasticity, many methods have been proposed to handle the plane elasticity crack problems of quasicrystals described by symmetry groups, physical property tensors, elasticity, and dislocations. Gong *et al.*[22] have studied the physical property tensors of quasicrystals by using the group-theoretical method. Li and Liu[23] have also studied the physical property tensors of one-dimensional quasicrystals. A paper devoted to development of a mathematical theory of elasticity of quasicrystals and its applications by Fan,[24] and Li and Fan[25] also studied a decagonal quasicrystal with a Griffith crack and obtained some significant results. Recent studies on the elasto-/hydrodynamics of quasicrystals have made preliminary but significant progress. Wang *et al.*[14,15,26] provided some useful insight into the interface problems of quasicrystals. Gao[27] obtained the appropriate edge conditions for semi-infinite strips with mixed edge-data of two-dimensional quasicrystals. Liu et al.[28] described the general solutions of plane elasticity of one-dimensional quasicrystals.

Caillard *et al.*,[29] Bohsung and Trebin,[30] Wollgarten *et al.*,[31] and Rosenfeld *et al.*[32] have studied the plastic deformations of various quasicrystals in microscopic mechanism. For a similar subject, Zhu and Fan[33] and Wang *et al.*[34] have used the cracked icosahedral Al–Pd–Mn quasicrystal and cracked decagonal Al–Ni–Co quasicrystal to simulate dynamic behaviours of quasicrystals. As a kind of new material, quasicrystals are deformable under applied forces, thermal loads and certain internal effects. The theoretical physicists have proposed various descriptions of their elasticity values. The majority of researchers agree that the Landau density wave theory is the physical basis of the elasticity of quasicrystal. Essentially, the description suggests that there are two displacement fields $\boldsymbol{u}$ and $\boldsymbol{w}$ in a quasicrystal: the former is similar to that in crystals, named the phonon field; the latter is a displacement field, termed the phase field. The total displacement field in a quasicrystal is expressed as[18,19]

$$\overline{\boldsymbol{u}} = \boldsymbol{u}^{\parallel} \oplus \boldsymbol{u}^{\perp} = \boldsymbol{u} \oplus \boldsymbol{w}, \tag{1}$$

where $\oplus$ denotes the direct sum; $\boldsymbol{u}$ is in the physical space, or the parallel space $E_{\parallel}^{3}$; $\boldsymbol{w}$ is in the complement space, or the perpendicular space $E_{\perp}^{3}$, which is an internal space.

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Dugdale–Barenblatt models are widely used to describe local fracture physical processes.[35,36] They originated from the work by Dugdale[37] and Barrenblatt[38] where tractions on zones near the crack tip was used to model cohesive forces of plastic yielding. Hence, we will extend the concept to a strip with a semi-infinite crack embedded in decagonal quasicrystals in the paper. The solutions of stress intensity factors of an infinite height and a finite height strip with a semi-infinite crack embedded in decagonal quasicrystals are obtained. The approach originates from the work by Muskhelishvili,[39] which involves the transcendental function conformal mapping technique. Of course, these solutions are inadequate for fracture theory of decagonal quasicrystals, and further studies of this kind of decagonal quasicrystal problem should be aided by the new results.

2. Solutions in terms of mathematical elastic framework

As described above, under given loading conditions, there are two induced displacement fields: $\boldsymbol{u}(\boldsymbol{r},t)$ for phonon field, and $\boldsymbol{w}(\boldsymbol{r},t)$ for phason field, which in turn yield the elastic strain tensors ε_{ij} and w_{ij} as

$$\varepsilon_{ij}=\frac{1}{2}\Big(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\Big),\quad w_{ij}=\frac{\partial w_i}{\partial x_j},\quad (w_{ij}\neq w_{ji}). \tag{2}$$

In linear elasticity, the stress tensors are related to the strain tensors

$$\sigma_{ij}=C_{ijkl}\varepsilon_{kl}+R_{ijkl}w_{kl},\quad H_{ij}=R_{klij}\varepsilon_{kl}+K_{ijkl}w_{kl}, \tag{3}$$

where u_i and w_i are phonon and phason displacements, σ_{ij} and ε_{ij} phonon stresses and strains, H_{ij} and w_{ij} phason stresses and strains, and C_{ijkl}, K_{ijkl}, and R_{ijkl} are the phonon, phason, phonon–phason coupling elastic constants, respectively. For a two-dimensional decagonal quasicrystal, we assume that the atom arrangement is periodic along the z direction, and quasi-periodic along the plane. Under a given loading, the in-plane mode of fracture of two-dimensional quasicrystal, i.e., in plane strain or plane stress state, one can introduce a quadruple harmonic equation about the stress potential $G(x,y)$ such that

$$\nabla^2\nabla^2\nabla^2\nabla^2 G=0, \tag{4}$$

where $\nabla^2=\partial^2/\partial x^2+\partial^2/\partial y^2$, and according to Ref. [24], the general solution can be written in terms of four analytic functions as

$$G=2\mathrm{Re}\Big[g_1(z)+\bar{z}g_2(z)+\frac{1}{2}\bar{z}^2g_3(z)+\frac{1}{6}\bar{z}^3g_4(z)\Big], \tag{5}$$

where $g_i(z)$ $(i=1,2,3,4)$ are analytic functions of a single complex variable $z=x+\mathrm{i}y$. In terms of these four analytic functions, the phonon stresses σ_{ij}, phason stresses H_{ij}, the phonon displacements u_i, and phason displacements w_i can be expressed, respectively, as follows:

$$\begin{cases}\sigma_{xx}=-32c_1\mathrm{Re}(\Omega(z)-2g_4'''(z)),\\ \sigma_{yy}=32c_1\mathrm{Re}(\Omega(z)+2g_4'''(z)),\\ \sigma_{xy}=\sigma_{yx}=32c_1\mathrm{Im}(\Omega(z)),\end{cases} \tag{6a}$$

$$\begin{cases}H_{xx}=32R_1\mathrm{Re}(\Theta'(z)-\Omega(z))-32R_2\mathrm{Im}(\Theta'(z)-\Omega(z)),\\ H_{xy}=-32R_1\mathrm{Im}(\Theta'(z)+\Omega(z))-32R_2\mathrm{Re}(\Theta'(z)+\Omega(z)),\\ H_{yx}=-32R_1\mathrm{Im}(\Theta'(z)-\Omega(z))-32R_2\mathrm{Re}(\Theta'(z)-\Omega(z)),\\ H_{yy}=-32R_1\mathrm{Re}(\Theta'(z)+\Omega(z))+32R_2\mathrm{Im}(\Theta'(z)+\Omega(z)),\end{cases} \tag{6b}$$

$$\begin{cases}u_x+\mathrm{i}u_y=32(4c_1c_2-c_3-c_1c_4)g_4''(z)-32(c_1c_4-c_3)(\overline{g_3'''(z)}+z\overline{g_4'''(z)}),\\ w_x+\mathrm{i}w_y=\dfrac{32(R_1-\mathrm{i}R_2)}{K_1-K_2}\overline{\Theta(z)},\end{cases} \tag{6c}$$

where

$$\Theta(z)=g_2^{(\mathrm{IV})}(z)+\bar{z}g_3^{(\mathrm{IV})}(z)+\bar{z}^2g_4^{(\mathrm{IV})}(z)/2,$$
$$\Omega(z)=g_3^{(\mathrm{IV})}(z)+\bar{z}g_4^{(\mathrm{IV})}(z),\quad L=C_{12},\quad M=C_{66},$$
$$c=M(K_1+K_2)-2(R_1^2+R_2^2),\quad c_1=\frac{c}{K_1-K_2}+M,$$
$$c_2=\frac{(L+M)(K_1+K_2)+c}{4c(L+M)},\quad c_3=\frac{(R_1^2+R_2^2)}{c},$$
$$c_4=\frac{(K_1+K_2)}{c}.$$

We define new symbols for convenience as follows:

$$g_2^{(\mathrm{IV})}(z)=h_2(z),\quad g_3'''(z)=h_3(z),\quad g_4''(z)=h_4(z). \tag{7}$$

2.1. Solution of an infinite height strip of decagonal quasicrystals

2.1.1. Solution of elastic deformation of an infinite height strip of decagonal quasicrystals

Consider an infinite height strip with a semi-infinite crack of decagonal quasicrystals shown in Fig. 1. The crack is along the axis x_3, a portion of which is subjected to a uniform pressure p or a shear force τ.

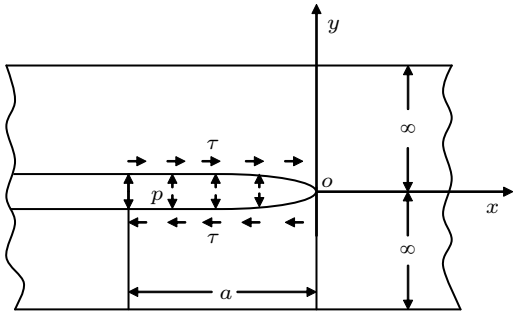


Fig. 1. An infinite height strip with a semi-infinite crack of decagonal quasicrystal.

The constructed conformal mapping corresponding to the transformation between the region Ω in the physical plane and the interior of the unit circle γ in ζ plane reads

$$z=\omega(\zeta)=m(1+\frac{2\zeta}{1-\zeta})^2, \tag{8}$$

where $m=a$ or $m=R$ (see below) is a constant. Under mapping (8), the crack tip $z=0$ is mapped to $\zeta=-1$, while $z=(-m,0^{\pm})$ are mapped to $\zeta=\pm\mathrm{i}$.

As shown in Fig. 1, the following boundary conditions should be satisfied:

$$\begin{cases} y=0,-a<x<0:\sigma_{yy}=-p,H_{yy}=0,\sigma_{xy}=-\tau,H_{xy}=0,\\ y=0,x<-a:\sigma_{yy}=0,H_{yy}=0,\sigma_{xy}=0,H_{xy}=0,\\ (x^2+y^2)^{1/2}\to\infty:\sigma_{ij}=0,H_{ij}=0. \end{cases} \tag{9}$$

In order not to repeat, we here only calculate the mode I loading problem. We begin by rewriting our unknown functions $h_3(z)=h_3(\zeta)$ and $h_4(z)=h_4(\zeta)$ in terms of the conformal mapping and they satisfy the following functional equations:

$$\begin{cases} h_4(\zeta)+\overline{h_3(0)}+\dfrac{1}{2\pi\mathrm{i}}\displaystyle\int_\gamma\frac{\omega(\sigma)}{\overline{\omega'(\sigma)}}\frac{\overline{h_4'(\sigma)}}{\sigma-\zeta}\mathrm{d}\sigma=\frac{1}{2\pi\mathrm{i}}\int_\gamma\frac{f_0}{\sigma-\zeta}\mathrm{d}\sigma,\\ h_3(\zeta)+\overline{h_4(0)}+\dfrac{1}{2\pi\mathrm{i}}\displaystyle\int_\gamma\frac{\overline{\omega(\sigma)}}{\omega'(\sigma)}\frac{h_4'(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma=\frac{1}{2\pi\mathrm{i}}\int_\gamma\frac{\overline{f_0}}{\sigma-\zeta}\mathrm{d}\sigma, \end{cases} \tag{10}$$

where σ represents the value of ζ at the boundary of γ in the mapping plane, and

$$f_0(z)=\frac{\mathrm{i}}{32c_1}\int(T_x+\mathrm{i}T_y)\mathrm{d}s=\begin{cases}-\dfrac{1}{32c_1}p(\omega(\sigma)),\ (-a<x<0),\\ -\dfrac{1}{32c_1}pa,\qquad\quad (x<-a),\end{cases}$$

$h_4(\zeta)$ and $h_3(\zeta)$ are single-valued analytic functions for $|\zeta|<1$.

By solving the preceding equations, one obtains

$$h_4(\zeta)=-\frac{1}{32c_1}\Big\{-\frac{pa}{\pi\mathrm{i}}\Big[\frac{2\zeta}{(1-\zeta)^2}\ln\frac{1+\mathrm{i}\zeta}{\zeta+i}-\ln\frac{\zeta+\mathrm{i}}{\zeta-\mathrm{i}}-\frac{2}{\mathrm{i}(1-\zeta)}\Big]\Big\}. \tag{11}$$

To determine the stress intensity factor, it is necessary to have the derivative $h_4'(\zeta)$. We obtain the derivative of Eq. (11) easily as follows:

$$h_4'(\zeta)=-\frac{pa}{16c_1\pi}\Big[\Big(\frac{1}{1+\zeta^2}+\frac{1}{(1-\zeta)^2}\Big)+\frac{1+\zeta}{\mathrm{i}(1-\zeta)^3}\ln\frac{1+\mathrm{i}\zeta}{\zeta+\mathrm{i}}+\frac{2\zeta}{(1-\zeta)^2(1+\zeta^2)}\Big]. \tag{12}$$

Meanwhile, we have the complex definition of stress intensity factor

$$K=K_{\mathrm{I}}-\mathrm{i}K_{\mathrm{II}}=\frac{\sqrt{2\pi}}{16c_1}\lim_{\zeta\to-1}\Big\{\sqrt{\omega(\zeta)-\omega(-1)}\frac{h_4'(\zeta)}{\omega'(\zeta)}\Big\}=\frac{\sqrt{\pi}}{16c_1}\frac{h_4'(-1)}{\sqrt{\omega''(-1)}}. \tag{13}$$

Substituting Eqs. (8) and (12) into Eq. (13), we obtain

$$K_{\mathrm{I}}=\frac{2\sqrt{2a\pi}p}{\pi}. \tag{14}$$

For the mode II loading problem, the calculation is similar to that for the mode I loading problem, and we obtain

$$K_{\mathrm{II}}=\frac{2\sqrt{2a\pi}\tau}{\pi}. \tag{15}$$

2.1.2. Solution of nonlinear deformation based on the Dugdale–Barenblatt manner of an infinite height strip of decagonal quasicrystals

We know that plastic deformation will occur in the vicinity of the fracture tip when the crack expands. The zone is placed at the fracture tip and subjected to so-called "cohesive forces" that impede the fracture growth. Therefore, we extend the Dugdale–Barenblatt model to this configuration of decagonal quasicrystal.

As shown in Fig. 2, over the portion $y=0$ and $-a<x<0$ of the crack, there is an equilibrium pressure p. Near the crack tip, there is a so-called cohesive force with length R whose value is unknown temporarily, this zone is subjected to traction $\sigma_{yy}=\sigma_Y$, where σ_Y represents the yield limit of the quasicrystalline material. The boundary conditions can be expressed as follows:

$$\begin{cases} y=0,-a<x<0:\sigma_{yy}=-p,H_{yy}=0,\sigma_{xy}=0,H_{xy}=0,\\ y=0,-R<x<0:\sigma_{yy}=\sigma_Y,H_{yy}=0,\sigma_{xy}=0,H_{xy}=0,\\ y=0,x<-a:\sigma_{yy}=0,H_{yy}=0,\sigma_{xy}=0,H_{xy}=0,\\ (x^2+y^2)^{1/2}\to\infty:\sigma_{ij}=0,H_{ij}=0. \end{cases} \tag{16}$$

In light of the superposition principle, this problem may be considered as a superposition of the following two boundary value sub-problems:

$$\begin{cases} y=0,-a<x<0:\sigma_{yy}=-p,H_{yy}=0,\sigma_{xy}=0,H_{xy}=0,\\ y=0,x<-a:\sigma_{yy}=0,H_{yy}=0,\sigma_{xy}=0,H_{xy}=0,\\ (x^2+y^2)^{1/2}\to\infty:\sigma_{ij}=0,H_{ij}=0, \end{cases} \tag{17}$$

and

$$\begin{cases} y=0,-R<x<0:\sigma_{yy}=\sigma_Y,H_{yy}=0,\sigma_{xy}=0,H_{xy}=0,\\ y=0,x<-R:\sigma_{yy}=0,H_{yy}=0,\sigma_{xy}=0,H_{xy}=0,\\ (x^2+y^2)^{1/2}\to\infty:\sigma_{ij}=0,H_{ij}=0. \end{cases} \tag{18}$$

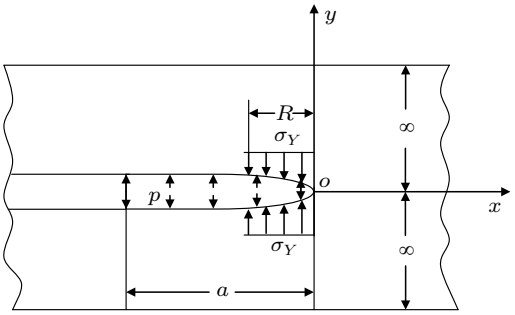


Fig. 2. Dugdale–Barenblatt model for this configuration of decagonal quasicrystal with a semi-infinite crack.

We have already obtained the stress intensity factor for problem (17)

$$K_{\mathrm{I}}^{p}=\frac{2\sqrt{2a\pi}p}{\pi}. \tag{19}$$

Let $m=R$ in mapping (8), we can also obtain the stress intensity factor for problem (18) (the calculation process is similar to that in the preceding case)

$$K_{\mathrm{I}}^{\sigma_Y}=-\frac{2\sqrt{2R\pi}\sigma_Y}{\pi}. \tag{20}$$

Because the stresses at the crack tip must be finite, i.e., there is no stress singularity, the total stress intensity factor must be equal to zero, i.e.,

$$K_{\mathrm{I}}^{p}+K_{\mathrm{I}}^{\sigma_Y}=0, \tag{21}$$

and this determines the size of cohesive force zone R

$$R=a\Big(\frac{p}{\sigma_Y}\Big)^2. \tag{22}$$

2.2. Solution of an asymmetrical crack in a strip of decagonal quasicrystal

As shown in Fig. 3, the description is identical to that in the foregoing section, and the difference between them lies in the height of the strip, which is finite here. This problem can also be considered as a superposition of the following two boundary value problems:

$$\begin{cases} \sigma_{xy}=\sigma_{yy}=0,H_{xy}=H_{yy}=0, & (y=H_2,-H_1,-\infty<x<+\infty),\\ \sigma_{xx}=\sigma_{xy}=0,H_{xx}=H_{xy}=0, & (-H_1<y<H_2,x=\pm\infty),\\ \sigma_{yy}=\sigma_{xy}=0,H_{yy}=H_{xy}=0, & (-\infty<x<-a,y=0),\\ \sigma_{yy}=-p,\sigma_{xy}=0,H_{yy}=H_{xy}=0, & (-a<x<0,y=0), \end{cases} \tag{23}$$

and

$$\begin{cases} \sigma_{xy}=\sigma_{yy}=0,H_{xy}=H_{yy}=0, & (y=H_2,-H_1,-\infty<x<+\infty),\\ \sigma_{xx}=\sigma_{xy}=0,H_{xx}=H_{xy}=0, & (-H_1<y<H_2,x=\pm\infty),\\ \sigma_{yy}=\sigma_{xy}=0,H_{yy}=H_{xy}=0, & (-\infty<x<-R,y=0),\\ \sigma_{yy}=\sigma_Y,\sigma_{xy}=0,H_{yy}=H_{xy}=0, & (-R<x<0,y=0), \end{cases} \tag{24}$$

where a, H_1, and H_2 are the constants with the dimension of length. The size of cohesive force zone R is also unknown temporarily.

For this complicated configuration, with the help of conformal mapping

$$z=\omega(\zeta)=\frac{H_1}{\pi}\ln(1-\zeta)+\frac{H_2}{\pi}\ln\Big(1+\frac{H_1}{H_2}\zeta\Big), \tag{25}$$

the cracked material shown in Fig. 3 can be transformed into the upper-half plane of the ζ plane. Under mapping (25), points A, O, and B in the z plane are transformed into points $\overline{\sigma}_{-a}$, 0, and σ_{-a}, respectively, in the ζ plane. Points $z=(-R,0^+)$ and

$z=(-R,0^-)$ are mapped to $\overline{\sigma}_{-R}$ and σ_{-R}.

By solving Eq. (10) based on boundary conditions (23) and (24), we can obtain the following result:

$$h_4'(\zeta)=\frac{1}{32c_1}\frac{q(t)}{2\pi^2\mathrm{i}}\left\{\frac{H_1}{1-\zeta}\ln\left(\frac{1-\sigma}{\sigma-\zeta}\right)+\frac{H_1}{1+H_1\zeta/H_2}\cdot\ln\left(\frac{1+H_1\sigma/H_2}{\sigma-\zeta}\right)\right\}_{x_1}^{\overline{x_1}}, \tag{26}$$

where $q(t)=p$ and $x_1=\overline{\sigma}_{-a}$ for problem (23), and $q(t)=-\sigma_Y$ and $x_1=\overline{\sigma}_{-R}$ for problem (24).

We will solve the size of cohesive force zone as we did in the preceding section. Substituting Eqs. (25) and (26) into the corresponding complex definition of stress intensity factor

$$K=K_{\mathrm{I}}-\mathrm{i}K_{\mathrm{II}}=\frac{\sqrt{2\pi}}{16c_1}\lim_{\zeta\to 0}\left\{\sqrt{\omega(\zeta)-\omega(0)}\frac{h_4'(\zeta)}{\omega'(\zeta)}\right\}=\frac{\sqrt{\pi}}{16c_1}\frac{h_4'(0)}{\sqrt{\omega''(0)}}, \tag{27}$$

we have

$$K_{\mathrm{I}}^{p}=\frac{\sqrt{H_1}p}{\pi}\left(\frac{H_1+H_2}{H_2}\right)^{-1/2}\left[\ln\left(\frac{1+H_1\sigma_{-a}/H_2}{1-\sigma_{-a}}\right)-\ln\left(\frac{1+H_1\overline{\sigma}_{-a}/H_2}{1-\overline{\sigma}_{-a}}\right)\right], \tag{28}$$

and

$$K_{\mathrm{I}}^{\sigma_Y}=-\frac{\sqrt{H_1}\sigma_Y}{\pi}\left(\frac{H_1+H_2}{H_2}\right)^{-1/2}\left[\ln\left(\frac{1+H_1\sigma_{-R}/H_2}{1-\sigma_{-R}}\right)-\ln\left(\frac{1+H_1\overline{\sigma}_{-R}/H_2}{1-\overline{\sigma}_{-R}}\right)\right]. \tag{29}$$

Since the transform of Eq. (25) is very complicated, the simple and exact inversion is difficult to obtain. However, if the lengths a, H_1, and H_2 are given, the specific values of σ_{-a} and σ_{-R} are easy to obtain.

In particular, if $H_1=H_2=H$, we can obtain

$$\begin{cases}\overline{\sigma}_{-a}=\sqrt{1-\mathrm{e}^{-\pi a/H}},\\ \overline{\sigma}_{-R}=\sqrt{1-\mathrm{e}^{-\pi R/H}},\end{cases} \tag{30}$$

and substituting Eq. (30) into Eqs. (28) and (29), one obtain

$$K_{\mathrm{I}}^{p}=\frac{\sqrt{2H}p}{\pi}\ln\frac{1+\sqrt{1-\mathrm{e}^{-\pi a/H}}}{1-\sqrt{1-\mathrm{e}^{-\pi a/H}}}, \tag{31}$$

and

$$K_{\mathrm{I}}^{\sigma_Y}=-\frac{\sqrt{2H}\sigma_Y}{\pi}\ln\frac{1+\sqrt{1-\mathrm{e}^{-\pi R/H}}}{1-\sqrt{1-\mathrm{e}^{-\pi R/H}}}. \tag{32}$$

Because the stresses at the crack tip must be finite, i.e., there is no stress singularity, the total stress intensity factor results in the following equality:

$$\ln\frac{1+\sqrt{1-\mathrm{e}^{-\pi a/H}}}{1-\sqrt{1-\mathrm{e}^{-\pi a/H}}}=\frac{\sigma_Y}{p}\ln\frac{1+\sqrt{1-\mathrm{e}^{-\pi R/H}}}{1-\sqrt{1-\mathrm{e}^{-\pi R/H}}}. \tag{33}$$

In a small-scale yielding state of decagonal quasicrystal, i.e., $R\ll\alpha$ and $R/H\to 0$, we can obtain

$$R=\frac{Hp^2}{4\pi\sigma_Y^2}\left(\ln\frac{1+\sqrt{1-\mathrm{e}^{-\pi a/H}}}{1-\sqrt{1-\mathrm{e}^{-\pi a/H}}}\right)^2=\frac{\pi}{8}\left(\frac{K_{\mathrm{I}}^{p}}{\sigma_Y}\right)^2. \tag{34}$$

Finally, these special results (31), (32), and (34) are consistent with the results obtained in Ref. [40]. If $a/H\to 0$, expressions (31), (32), and (34) can be converted into expressions (19), (20), and (22), respectively.

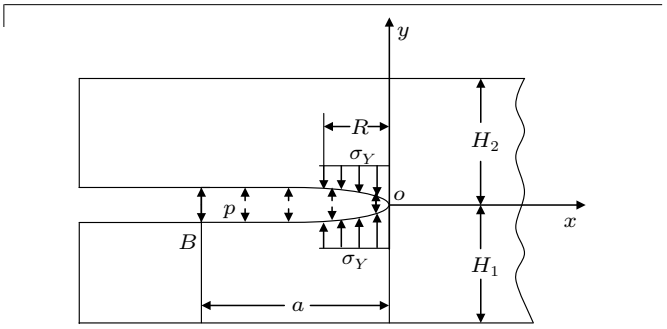


Fig. 3. An asymmetrical crack in a strip of decagonal quasicrystal.

3. Summary

The problem of a strip with a semi-infinite crack embedded in decagonal quasicrystals is a mathematically daunting problem. In the present study on decagonal quasicrystals, systematical and direct methods of mathematical mechanics and complex variable functions are developed to solve the equations under homologous boundary value conditions. This is new development of the theory of Muskhelishvili, which is widely used in the statics problem of the elastic medium. With the aid of the conformal mapping technique, some exact analytical solutions including the infinite height and finite height strip with a semi-infinite crack, a portion of which is subjected to mode-I loading or mode-II loading, are obtained. Particularly, the results in Ref. [40] belong to the special cases of the results in this paper, meanwhile, when the height $H_1=H_2=H\to\infty$, the results accord with those in the case of infinite height strip. For nonlinear deformation of decagonal quasicrystal, we obtain the size of cohesive force zone according to the Dugdale–Barenblatt concept. These solutions can be reduced to the results of classical elasticity when the phason field and phonon–phason coupling vanish. Finally, in a small-scale yielding state of decagonal quasicrystals, we recommend the results (22) and (34) as parameters of the size

of cohesive force zone for the fracture deformation theory of the corresponding configuration of decagonal quasicrystal.

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