Band structure characteristics of T-square fractal phononic crystals*

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The T-square fractal two-dimensional phononic crystal model is presented in this article. A comprehensive study is performed for the Bragg scattering and locally resonant fractal phononic crystal. We find that the band structures of the fractal and non-fractal phononic crystals at the same filling ratio are quite different through using the finite element method. The fractal design has an important impact on the band structures of the two-dimensional phononic crystals.

Keywords: fractal design, phononic crystal, band gap, filling ratio

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1. Introduction

When an electromagnetic wave propagates in periodic dielectric structures, a photonic band gap may be exhibited.[1] The study on photonic band gap structures is an important realm of photonic crystals.[2] The propagation of elastic/acoustic waves in artificial periodic materials, which is referred to as the phononic crystal analogy to the photonic crystal, has received much attention in recent years.[3–5] The existence of band gaps in the phononic crystals, which means that a certain frequency of the wave is forbidden, has many potential engineering applications such as elastic/acoustic filters, vibration/noise insulation, and transducers.

Phononic crystals are always an artificial periodic material with its unit cell comprising of an inclusion embedded in the matrix material. The geometry of the inclusion in most band gap studies are principally regular-shape geometries such as circle, polygons, and so on. Recently, some new behaviors have been discovered in studying the impact of fractal-shaped inclusion on the phononic crystals band structures. Norris *et al.* used the finite difference time-domain analysis to determine the impact of periodic fractal-shaped inclusions on the frequency response of two-dimensional phononic or acoustic band gap crystals.[6] Kuo and Piazza introduced the T-square fractal geometry design for a microscale phononic band gap structure in air/aluminum nitride.[7] However, in Ref. [7] the fractal-like phononic crystal is comprised of T-square shape pores, and the study is focused on the influences of the pore shapes on the band structures. Different from prior demonstrations, the impact of T-square fractal inclusions on the band structures is the subject in this paper. The unit cell of a T-square fractal-like phononic crystal consists of a center square scatter with four side square scatters repeating at its corners. It is different from the conventional design. The Bragg scattering and locally resonant mechanism are two different mechanisms resulting in the band gap. The gaps are the results of destructive interference of the wave reflections in the periodic structures, the so-called Bragg scattering mechanism.[8] The locally resonant mechanism is due to the localized resonances associated with scattering units.[9–13] In order to study the impact of T-square fractal design on the band structures of the two-dimensional phononic crystal, we will analyze both the Bragg scattering and locally resonant binary counterparts.

2. Unit cell with fractal pattern

We propose a type of two-dimensional phononic crystal with its unit cell consisting of a fractal inclusion. The iterating procedure of forming the T-square fractal unit cell is described as follows. Let us start with a square, and then repeat and scale down a copy to half the length and width of the basic square. The repeated squares are moved from the center to the corner of the original square. There are four levels of fractal unit cell of the phononic crystal shown in Fig. 1.

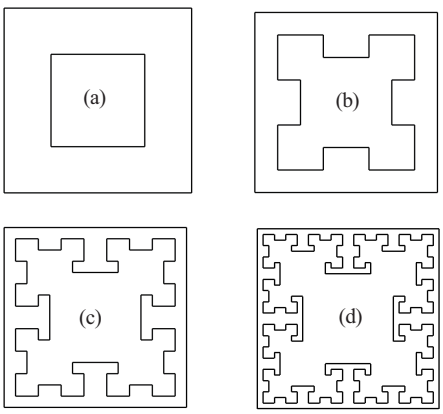


Fig. 1. T-square fractal with side square extension up to four levels. (a) First level, (b) second level, (c) third level, (d) fourth level.

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3. Band structures of the T-square fractal phononic crystal

Here, we consider the unit cells of the two-dimensional phononic crystal consisting of a center aluminum (for Bragg scattering mechanism) or rubber (for locally resonant mechanism) inclusion with four side aluminum or rubber square scatters repeating at its corners. The matrix material is epoxy. The COMSOL Multiphysics finite element solver is used to study T-square fractal-like phononic crystals with a binary structure.

3.1. Accuracy of the method

In order to demonstrate the availability of the finite element method, band structures for a square lattice of rubber cylinders embedded in epoxy given in Ref. [10] are calculated, and the corresponding comparison with reference to the results given by lumped mass method is made. The radius of the cylinder is 8 mm, and the lattice constant is 20 mm. The elastic parameters, such as density ρ and Lame constants λ and μ, used in the calculation are: $\rho = 1.3 \times 10^3$ kg/m^3, $\lambda = 6 \times 10^5$ Pa, $\mu = 4 \times 10^4$ Pa for rubber, $\rho = 1.18 \times 10^3$ kg/m^3, $\lambda = 4.43 \times 10^9$ Pa, $\mu = 1.59 \times 10^9$ Pa for epoxy. Our calculated reduced frequency range of the dispersion relation of in-plane modes is from 0.00953 to 0.00958, which coincides with the data in Ref. [10] (from 0.00988 to 0.00996). Hence, it manifests the accuracy of the present method.

3.2. Impact of T-square fractal design on the Bragg scattering band structures

The dispersion relations of in-plane modes in different fractal level aluminum/epoxy composites, which are the typical Bragg scattering phononic crystals, are shown in Fig. 2. The unit cells are arranged in a simple square lattice. The lattice constant a equals 0.01 m. The elastic parameters employed in the calculations are $\rho = 2.73 \times 10^3$ kg/m^3, $\lambda = 6.82 \times 10^{10}$ Pa, $\mu = 2.87 \times 10^{10}$ Pa for aluminum, $\rho = 1.18 \times 10^3$ kg/m^3, $\lambda = 4.43 \times 10^9$ Pa, $\mu = 1.59 \times 10^9$ Pa for epoxy. The reduced frequency is calculated by $\omega a/2\pi c_{t,e}$, where $c_{t,e}$ is the transverse wave velocity of epoxy. Figure 2 depicts that the phononic crystal band frequency increases with the increasing level of the fractal. The first band gap exists between the third and fourth bands for all level fractal phononic crystals.

Then, for further studying the impact of fractal on band structures, we compare the fractal phononic crystal with a non-fractal one, both of which have the same filling ratio. Here, the value of filling ratio is the area ratio between the inclusion and unit cell. The values of filling ratio of different level fractal phononic crystals are listed in Table 1. Similar with the first level fractal, non-fractal phononic crystal is composed of a square inclusion immersed in the matrix material.

Figure 3 shows the variation of the first band gap edges and width of the fractal and non-fractal phononic crystal with the filling ratio. It can be seen from Fig. 3(a) that for both fractal and non-fractal cases, the frequency gets higher with increasing filling ratio. The gap bottom in fractal inclusion is higher than that in non-fractal case, while the gap top in fractal inclusion is lower than that of non-fractal case. The gap width of non-fractal case is wider than the fractal case, as shown in Fig. 3(b). When the filling ratio equals 0.4375, the gap width of fractal phononic crystal has a maximum. The presence of the side squares essentially shortens the distance of the neighbor inclusion in unit cells. The wavelength of the standing wave becomes shorter, and higher vibrational modes are suppressed. This phenomenon is more obvious when the fractal level is higher.

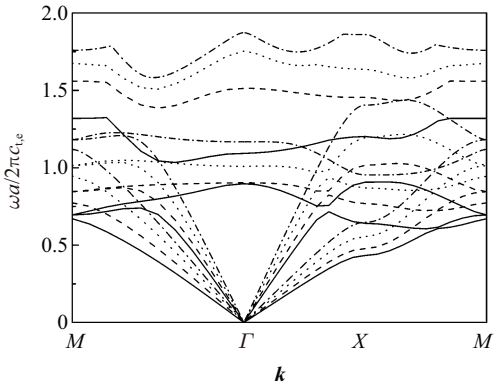


Fig. 2. Band structures of in-plane modes in two-dimensional phononic crystal composed of aluminum and epoxy, first level fractal (solid line), second level fractal (dashed line), third level fractal (dotted line), and fourth level fractal (dash-dot line).

Table 1. Filling ratio of different level fractal phononic crystals.

	First level	Second level	Third level	Fourth level
Filling ratio	0.25	0.4375	0.5781	0.6836

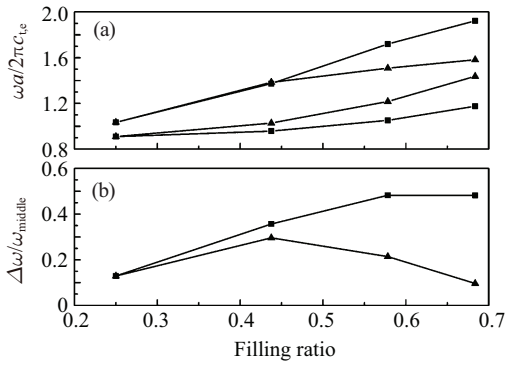


Fig. 3. (a) The upper and bottom edges of the first bandgap of the in-plane modes in the two-dimensional Bragg scattering fractal (triangle) and non-fractal (square) phononic crystal as a function of the filling ratio. (b) The corresponding normalized gap width of the fractal (triangle) and non-fractal (square) phononic crystal as a function of the filling ratio.

On the basis of the discussion above, we can see that the level of the fractal impacts the band structures in the form of the filling ratio. There are two ways of changing the filling ratio of the unit cell. One is by changing the level of the fractal, and the other is by varying the lattice constant. To take the third level fractal case as an example, we change the filling ratio through varying the lattice constant a. The lattice constant is set to be 0.0095, 0.01, 0.012, 0.014, and 0.016 m. Here, the fractal and non-fractal cases in the same filling fraction are also compared. The results displayed in Fig. 4 exhibit the same trend, as shown in Fig. 3. Thus, we can see that changing the level of fractal and varying the lattice constant have the same impact on the band structure to the Bragg scattering phononic crystal.

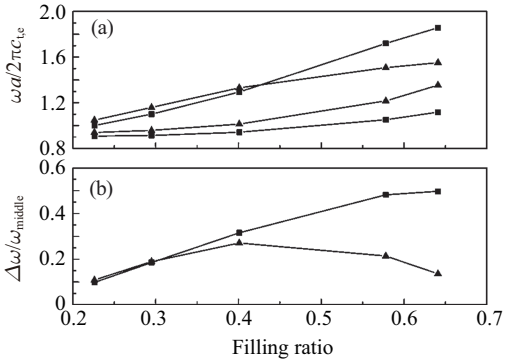


Fig. 4. (a) The upper and bottom edges of the first bandgap of the in-plane modes in the two-dimensional Bragg scattering fractal with the third level fractal (triangle) and non-fractal (square) phononic crystal as a function of the filling ratio. (b) Corresponding normalized gap width of the fractal (triangle) and none fractal (square) phononic crystal as a function of the filling ratio.

3.3. Impact of T-square fractal design on the locally resonant band structures

The locally resonant range of the frequency gap is usually two orders of magnitude lower than the one resulting from the Bragg scattering mechanism.[8] The physical origin for this is due to the localized resonances associated with inclusion in the unit cell. We study the impact of the fractal on the locally resonant phononic crystal composed of the square lattice of soft rubber in epoxy. The lattice constant a equals 0.01 m. The elastic constants employed in the calculations are $\rho = 1.3 \times 10^3$ kg/m^3, $\lambda = 6 \times 10^5$ Pa, $\mu = 4 \times 10^4$ Pa for rubber, $\rho = 1.18 \times 10^3$ kg/m^3, $\lambda = 4.43 \times 10^9$ Pa, $\mu = 1.59 \times 10^9$ Pa for epoxy. The dispersion relations of in-plane modes in different fractal levels are listed in Fig. 5. The flat branches cross the whole Brillouin zone is the remarkable feature of the band structures of the locally resonant phononic crystals. Figure 5 depicts that the phononic crystal band frequency decreases with the increasing level of the fractal. The first band gap exists between the third and fourth bands for all level fractal phononic crystals, the same as the Bragg scattering mechanism.

The impact of fractal design on the locally resonant phononic crystal is quite different from that on the Bragg scattering mechanism. Results shown in Fig. 6 reflect the variation of the first band gap edges and width of the fractal and non-fractal locally resonant phononic crystals. It can be seen from Fig. 6(a) that the frequency gets lower with increasing filling ratio for both fractal and non-fractal cases. The gap edges which contain both the upper and bottom in the fractal case are higher than that of the non-fractal case. The gap width of the fractal case is wider than the non-fractal case, as shown in Fig. 6(b).

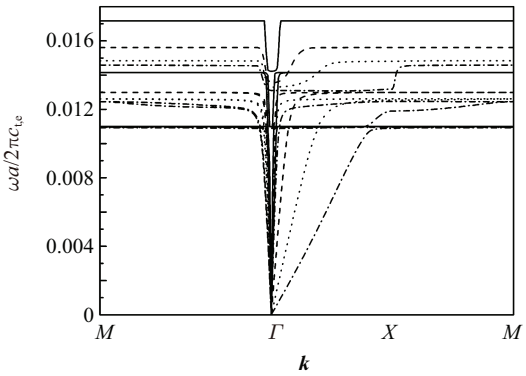


Fig. 5. Band structures of in-plane modes in two-dimensional phononic crystal composed of rubber and epoxy, first level fractal (solid line), second level fractal (dashed line), third level fractal (dotted line), and fourth level fractal (dash-dot line).

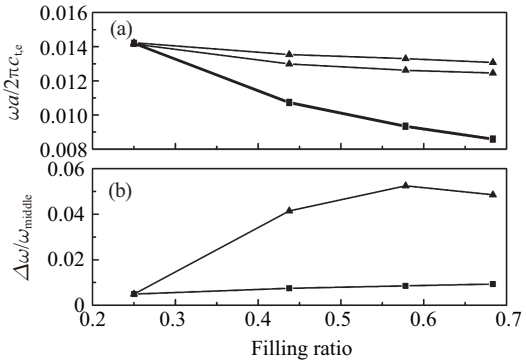


Fig. 6. (a) The upper and bottom edges of the first bandgap of the in-plane modes in the two-dimensional locally resonant fractal (triangle) and non-fractal (square) phononic crystal as a function of the filling ratio. (b) The corresponding normalized gap width of the fractal (triangle) and non-fractal (square) phononic crystal as a function of the filling ratio.

Similar to the case of Bragg scattering mechanism, taking the third level fractal case as example is still suitable for the locally resonant case. The lattice constant is also set to be 0.0095, 0.01, 0.012, 0.014, and 0.016 m. The results are listed in Fig. 7. We can see that the first band gaps of both fractal and non-fractal cases have the same variation tendency with the increasing filling ratio in locally resonant mechanism. The gap width of the fractal case increases with the increasing filling ratio. While the gap width of the none fractal almost keeps constant. Their variation tendency is different. It is noticeable

that changing the levels of fractal and varying the lattice constant almost make the same impact on the band structure of the locally resonant phononic crystal.

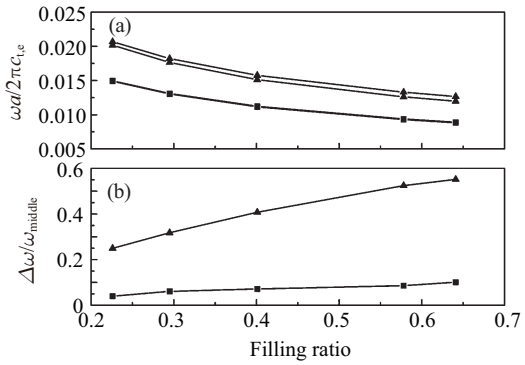


Fig. 7. (a) The upper and bottom edges of the first bandgap of the in-plane modes in the two-dimensional locally resonant fractal with the third level fractal (triangle) and non-fractal (square) phononic crystal as a function of the filling ratio. (b) The corresponding normalized gap width of the fractal (triangle) and non-fractal (square) phononic crystal as a function of the filling ratio.

In order to understand the physical origin of the phenomenon above, we restudy the band structures of the third level fractal and non-fractal locally resonant phononic crystal in Fig. 8. Points F_1 and F_2 are respectively in the bottom and upper edges of the first band gap for the fractal case. Points NF_1 and NF_2 are respectively in the bottom and upper edges of the first band gap for non-fractal case. The resonant modes of the four points are listed in Fig. 9. The direction and length of the arrows shown in the figure represent the direction and amplitude of the displacement vectors, respectively. For the vibration modes at points F_1 and NF_1 (on the bottom edges of the first band gap) in Figs. 9(a) and 9(c), the vibration concentrates in the inclusion. For the vibration modes at points F_2 and NF_2 (on the upper edges of the first band gap) in Figs. 9(b) and 9(d), the vibration concentrates in both the inclusion and matrix material. For the locally resonant phononic crystal, the vibration of inclusion and matrix material can be simplified into mass-spring oscillates.[10] As for the vibration modes illustrated in Fig. 9, the time harmonic forces from the oscillators to the hosting structure are not zero, then the

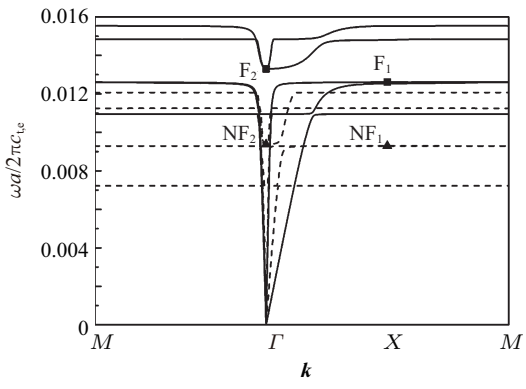


Fig. 8. Band structures of the two-dimensional third-level fractal phononic crystal (solid line) and corresponding non-fractal case (dashed line).

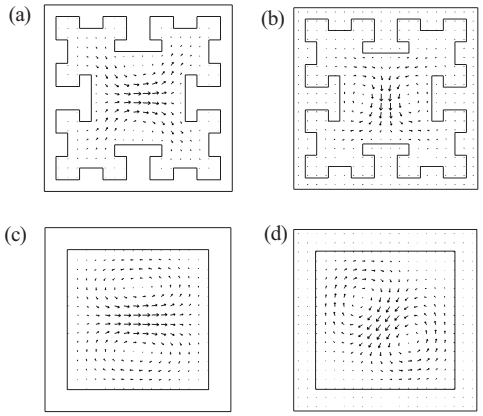


Fig. 9. Vibration modes of (a) point F_1, (b) point F_2 for the third level fractal phononic crystal, (c) point NF_1, (d) point NF_2 for corresponding two-dimensional non-fractal phononic crystal.

band gap appears. For the locally resonant two-dimensional binary phononic crystal, the localized resonant properties of single inclusion are the main influencing factors for the band structure. Here, for the fractal phononic crystal, the area of the inclusion participating in resonance is much less than that of non-fractal case. This is the main reason why the fractal photonic crystal has higher band gap frequencies compared with non-fractal case.

4. Conclusion

We have studied the fractal design impact on the band structures of the Bragg scattering and locally resonant two-dimensional phononic crystal. Numerical results show that the band structure based on the two generating mechanisms are both influenced by the fractal design through comparing with non-fractal structure. We also study the physical mechanism of band structure by analyzing the vibration modes of points located in the band gap. With the increase in the fractal level, the gap width of the Bragg scattering fractal case is narrower than that of the non-fractal case due to the presence of the side squares, which shortens the distance of the neighbor inclusion in unit cells and suppresses the higher vibrational modes. As the fractal level increased, for the locally resonant fractal case, the gap width is wider than that of the non-fractal. This is due to the area of the inclusion participating in resonance being much smaller than that of the non-fractal case. The band structure analysis of phononic crystal with different level fractals can be proved helpful in the future design of phononic band gap based devices.

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