The Kadomtsev–Petviashvili equation for dust ion-acoustic solitons in pair-ion plasmas

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Using the reductive perturbation method, we have derived the Kadomtsev–Petviashvili (KP) equation to study the nonlinear properties of electrostatic collisionless dust ion-acoustic solitons in pair-ion (p-i) plasmas. We have chosen the fluid model for the positive ions, the negative ions, and a fraction of static charged (both positively and negatively) dust particles. Numerical solutions of these dust ion-acoustic solitons are plotted and their characteristics are discussed. It is found that only the amplitudes of the electrostatic dust ion-acoustic solitons vary when the dust is introduced in the pair-ion plasma. It is also noticed that the amplitude and the width of these solitons both vary when the thermal energy of the positive or negative ions is varied. It is shown that potential hump structures are formed when the temperature of the negative ions is higher than that of the positive ions, and potential dip structures are observed when the temperature of the positive ions supersedes that of the negative ions. As the pair-ion plasma mimics the electron–positron plasma, thus our results might be helpful in understanding the nonlinear dust ion acoustic solitary waves in super dense astronomical bodies.

Keywords: charged and static dust particles, pair–ion plasma, soliton

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1. Introduction

The production of positrons in a laboratory[1–4] has made it possible to experimentally study the electron–positron (e–p) plasmas. The e–p plasmas are pair-ion (p-i) plasmas which are naturally encountered in pulsar magnetosphere, early universe, active galactic nuclei, and many other astronomical bodies.[5–11] Although it is possible to generate e–p plasmas in laboratories by injecting a low energy electron beam into positron plasmas,[12] it is extremely difficult to study the different plasma modes. The reason for this aggravating trouble is that the annihilation time in the e–p plasmas is less than the plasma period, and many orders of the plasma period are required in order to study the different modes in the e–p plasmas.

The problem of the small annihilation time in the e–p plasmas is overcome by using p-i fullerene plasmas. Pair-ion fullerene ($C_{60}^{\pm}$) plasmas help us to understand the collective modes in e–p plasmas. These collective modes are crucial from a plasma diagnostic point of view. Oohara and Hatakeyama[13,14] have developed an experimental method to generate the p-i fullerene ($C_{60}^{\pm}$) plasma. Many authors[15–21] have studied plasma collective modes in pure p-i plasma and p-i plasma with traces of electrons and/or dust particles.[22–26] The presence of charged dust particles in plasma not only modifies the existing plasma modes but also introduces many new eigen modes such as dust acoustic (DA), dust ion-acoustic (DIA), and dust lattice (DL) modes. The waves in dusty plasmas are extensively studied in Ref. [27]. The presence of dust in p-i plasma will mimic electron–positron–ion (e–p–i) plasma, which is also found in astronomical bodies. Therefore, the study of dust in p-i plasma will help us to understand the plasma diagnostics in e–p–i plasma.

The simplest way to study the solitary wave in plasma is to reduce the basic model equations into a well known nonlinear partial differential equation. The method is called the reductive perturbation method. Washimi and Taniuti[28] adopted this method to derive the Korteweg–de Vries (KdV) equation. Later on, this method was taken to study various plasma models. The Kadomtsev–Petviashvili (KP) equation is key to exploring a number of nonlinear problems both in physics and mathematics.[29,30] The KdV equation deals with soliton solutions, while the KP equation examines the stability of these solitons.[31–33] It also pictures small effects in the direction perpendicular to the propagation of these solitons in the plasmas.

In this paper, we derive the KP equation for the propagation of nonlinear dust ion acoustic (DIA) waves in unmagnetized p-i plasma with dust particles. In our model, we study the dynamics of a pair of ions where the dust particles are assumed to be charged and static. It is also worth mentioning here that to the best of our knowledge, the study of a nonlinear dust ion acoustic wave in p-i plasma has not yet been reported.

The paper is organized in as follows. In the next section, the mathematical model is presented, and normalizing factors are defined. A nonlinear soliton solution in the form of the KP equation for the p-i dusty plasma is derived in Section 3, while the numerical results are presented in Section 4. Section 5 is devoted to the conclusion.

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2. Mathematical model

We consider the two-dimensional (i.e., $\nabla = (\partial_x, \partial_y, 0)$) propagation of electrostatic dust ion acoustic (DIA) solitons in unmagnetized p-i plasmas with dust particles. We assume that the static dust particles are charged, and the positive and the negative ions are dynamic. The set of dynamic equations for the hot ions include the momentum equation

$$\frac{\partial \boldsymbol{v}_\pm}{\partial t} + (\boldsymbol{v}_\pm \cdot \nabla)\boldsymbol{v}_\pm = \pm\frac{e}{m_\pm}\boldsymbol{E} - \frac{1}{m_\pm n_\pm}\nabla p_\pm, \tag{1}$$

the continuity equation

$$\frac{\partial n_\pm}{\partial t} + \nabla \cdot (n_\pm \boldsymbol{v}_\pm) = 0, \tag{2}$$

and the pressure evolution equation

$$\frac{\partial p_\pm}{\partial t} + (\boldsymbol{v}_\pm \cdot \nabla)\, p_\pm + \gamma p_\pm (\nabla \cdot \boldsymbol{v}_\pm) = 0. \tag{3}$$

The system is closed by the Poisson equation, which can be written as

$$\nabla \cdot \boldsymbol{E} = 4\pi e(n_+ + sZ_{\rm d}n_{\rm d0} - n_-). \tag{4}$$

In equilibrium, we have

$$n_{+0} + sZ_{\rm d}n_{\rm d0} - n_{-0} = 0. \tag{5}$$

Here subscript d denotes the dust particles. The dust particles can be positively or negatively charged. The symbol $s = \pm$ is used for positively and negatively charged dust particles. The electric field intensity is defined as $\boldsymbol{E} = -\nabla\phi$, where ϕ is the electrostatic potential. The perturbed velocities, the perturbed densities, and the perturbed pressures of the pair of ions are denoted by $v_\pm$, $n_\pm$, and $p_\pm$, respectively. The equilibrium densities for the positive ions, the negative ions, and the static dust particles are given by $n_{+0,}$ $n_{-0,}$ and $n_{\rm d0,}$ respectively. The $Z_{\rm d}$ is the charge number for the dust particles. Since we are considering the pair-ion fullerene plasma, $m_+ = m_- = m$.

The normalized set of the above dynamic equations for the unmagnetized p-i plasma with static dust particles in two dimensions can be written as

$$\frac{\partial v_{+x}}{\partial t} + \left(v_{+x}\frac{\partial}{\partial x} + v_{+y}\frac{\partial}{\partial y}\right)v_{+x} = -\frac{\partial \Phi}{\partial x} - \frac{\mu}{n_+}\frac{\partial p_+}{\partial x}, \tag{6}$$

$$\frac{\partial v_{+y}}{\partial t} + \left(v_{+x}\frac{\partial}{\partial x} + v_{+y}\frac{\partial}{\partial y}\right)v_{+y} = -\frac{\partial \Phi}{\partial y} - \frac{\mu}{n_+}\frac{\partial p_+}{\partial y}, \tag{7}$$

$$\frac{\partial v_{-x}}{\partial t} + \left(v_{-x}\frac{\partial}{\partial x} + v_{-y}\frac{\partial}{\partial y}\right)v_{-x} = \frac{\partial \Phi}{\partial x} - \frac{1}{n_-}\frac{\partial p_-}{\partial x}, \tag{8}$$

$$\frac{\partial v_{-y}}{\partial t} + \left(v_{-x}\frac{\partial}{\partial x} + v_{-y}\frac{\partial}{\partial y}\right)v_{-y} = \frac{\partial \Phi}{\partial y} - \frac{1}{n_-}\frac{\partial p_-}{\partial y}, \tag{9}$$

$$\frac{\partial n_+}{\partial t} + \frac{\partial}{\partial x}(n_+ v_{+x}) + \frac{\partial}{\partial y}(n_+ v_{+y}) = 0, \tag{10}$$

$$\frac{\partial n_-}{\partial t} + \frac{\partial}{\partial x}(n_- v_{-x}) + \frac{\partial}{\partial y}(n_- v_{-y}) = 0, \tag{11}$$

$$\frac{\partial p_+}{\partial t} + \left(v_{+x}\frac{\partial}{\partial x} + v_{+y}\frac{\partial}{\partial y}\right)p_+ + \gamma p_+\left(\frac{\partial v_{+x}}{\partial x} + \frac{\partial v_{+y}}{\partial y}\right) = 0, \tag{12}$$

$$\frac{\partial p_-}{\partial t} + \left(v_{-x}\frac{\partial}{\partial x} + v_{-y}\frac{\partial}{\partial y}\right)p_- + \gamma p_-\left(\frac{\partial v_{-x}}{\partial x} + \frac{\partial v_{-y}}{\partial y}\right) = 0, \tag{13}$$

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = n_- - (1 - s\delta)n_+ - s\delta. \tag{14}$$

The normalization of the perturbed velocity of the pair of ions is done by the ion acoustic speed of the negative ions, $c_{s-} = (T_-/m)^{1/2}$, where T_- is the temperature of the negative ions in units of energy. The perturbed densities and the perturbed pressures for the pair of ions are normalized with the corresponding equilibrium densities and equilibrium pressures, respectively. The normalized electrostatic potential is defined as $\Phi = e\phi/T_+$. The space and time variables are normalized by the negative ion's Debye length $\lambda_{D-} = \left(T_-/4\pi n_{-0}e^2\right)^{1/2}$ and the plasma frequency of negative ions $\omega_{\rm p-} = \left(4\pi n_{-0}e^2/m\right)^{1/2}$, respectively. The temperature ratio of the positive ions to the negative ions is defined as $\mu = T_+/T_-$. We have also defined the density ratio of the static charged dust particles to the negative ions as $\delta = Z_{\rm d}n_{\rm d0}/n_{-0}$.

3. Soliton solution

In order to derive the KP equation, the well known reductive perturbation method[28] is used. The perturbed quantities can be expanded in the powers of ε as $(j = \pm)$

$$\begin{aligned}
n_j &= 1 + \varepsilon n_j^{(1)} + \varepsilon^2 n_j^{(2)} + \varepsilon^3 n_j^{(3)} + \cdots,\\
v_{jx} &= \varepsilon v_{jx}^{(1)} + \varepsilon^2 v_{jx}^{(2)} + \varepsilon^3 v_{jx}^{(3)} + \cdots,\\
v_{jy} &= \varepsilon^{\frac{3}{2}} v_{jy}^{(1)} + \varepsilon^{\frac{5}{2}} v_{jy}^{(2)} + \varepsilon^{\frac{7}{2}} v_{jy}^{(3)} + \cdots,\\
p_j &= 1 + \varepsilon p_j^{(1)} + \varepsilon^2 p_j^{(2)} + \varepsilon^3 p_j^{(3)} + \cdots,\\
\Phi &= \varepsilon \Phi^{(1)} + \varepsilon^2 \Phi^{(2)} + \varepsilon^3 \Phi^{(3)} + \cdots.
\end{aligned} \tag{15}$$

Here all the perturbed quantities are functions of x, y, and t, while ε is a small $(0 < \varepsilon \leq 1)$ expansion parameter characterizing the strength of the nonlinearity. The stretched variables are introduced in the standard fashion as follows:

$$X = \varepsilon^{1/2}(x - \lambda t), \quad Y = \varepsilon y, \quad \tau = \varepsilon^{3/2} t, \tag{16}$$

where λ is the phase velocity. Using Eqs. (15) and (16) in Eqs. (6)–(14) and collecting the lowest order terms $\left(\sim \varepsilon^{3/2}\right)$ of the momentum, the continuity and the pressure equations of the pair-ion fullerene plasma, we have

$$v_{+x}^{(1)} = \frac{\lambda}{(\lambda^2 - \gamma\mu)}\Phi^{(1)}, \tag{17}$$

$$v_{-x}^{(1)} = -\frac{\lambda}{(\lambda^2 - \gamma)}\Phi^{(1)}, \tag{18}$$

$$n_+^{(1)} = \frac{1}{(\lambda^2 - \gamma\mu)}\Phi^{(1)}, \tag{19}$$

$$n_-^{(1)} = -\frac{1}{(\lambda^2-\gamma)}\Phi^{(1)}, \tag{20}$$

$$p_+^{(1)} = \frac{\gamma}{(\lambda^2-\gamma\mu)}\Phi^{(1)}, \tag{21}$$

$$p_-^{(1)} = -\frac{\gamma}{(\lambda^2-\gamma)}\Phi^{(1)}. \tag{22}$$

The lowest order terms of the y-component of momentum equations are

$$\partial_X v_{+y}^{(1)} = \frac{1}{\lambda}\partial_Y \Phi^{(1)} + \frac{\mu}{\lambda}\partial_Y p_+^{(1)}, \tag{23}$$

$$\partial_X v_{-y}^{(1)} = -\frac{1}{\lambda}\partial_Y \Phi^{(1)} + \frac{1}{\lambda}\partial_Y p_-^{(1)}. \tag{24}$$

The lowest order terms of Poisson's equation yield

$$n_-^{(1)} - (1-s\delta)n_+^{(1)} = 0. \tag{25}$$

The dispersion relation of the electrostatic DIA waves in the p-i plasma is obtained from the lowest order terms of Poisson's equation (25) together with Eqs. (19) and (20), which is given by

$$\lambda = \sqrt{\frac{\gamma(1-s\delta+\mu)}{2-s\delta}}. \tag{26}$$

Now, the next higher order terms ($\sim \varepsilon^{5/2}$) of the positive and the negative ion dynamic equations are described as follows:

$$-\lambda\partial_X v_{+x}^{(2)} + \partial_X \Phi^{(2)} + \mu\partial_X p_+^{(2)} = f_1, \tag{27}$$

$$-\lambda\partial_X v_{-x}^{(2)} - \partial_X \Phi^{(2)} + \partial_X p_-^{(2)} = f_2, \tag{28}$$

$$-\lambda\partial_X n_+^{(2)} + \partial_X v_{+x}^{(2)} = f_3, \tag{29}$$

$$-\lambda\partial_X n_-^{(2)} + \partial_X v_{-x}^{(2)} = f_4, \tag{30}$$

$$-\lambda\partial_X p_+^{(2)} + \gamma\partial_X v_{+x}^{(2)} = f_5, \tag{31}$$

$$-\lambda\partial_X p_-^{(2)} + \gamma\partial_X v_{-x}^{(2)} = f_6. \tag{32}$$

The next higher order terms ($\sim \varepsilon^2$) of Poisson's equation are given as

$$n_-^{(2)} - (1-s\delta)n_+^{(2)} = f_7, \tag{33}$$

where

$$\begin{aligned}
f_1 &= -\partial_\tau v_{+x}^{(1)} - v_{+x}^{(1)}\partial_X v_{+x}^{(1)} + \mu n_+^{(1)}\partial_X p_+^{(1)},\\
f_2 &= -\partial_\tau v_{-x}^{(1)} - v_{-x}^{(1)}\partial_X v_{-x}^{(1)} + n_-^{(1)}\partial_X p_-^{(1)},\\
f_3 &= -\partial_\tau n_+^{(1)} - \partial_X\left(n_+^{(1)} v_{+x}^{(1)}\right) + \partial_Y v_{+y}^{(1)},\\
f_4 &= -\partial_\tau n_-^{(1)} - \partial_X\left(n_-^{(1)} v_{-x}^{(1)}\right) + \partial_Y v_{-y}^{(1)},\\
f_5 &= -\partial_\tau p_+^{(1)} - v_{+x}^{(1)}\partial_X p_+^{(1)} - \gamma\partial_Y v_{+y}^{(1)} - \gamma p_+^{(1)}\partial_X v_{+x}^{(1)},\\
f_6 &= -\partial_\tau p_-^{(1)} - v_{-x}^{(1)}\partial_X p_-^{(1)} - \gamma\partial_Y v_{-y}^{(1)} - \gamma p_-^{(1)}\partial_X v_{-x}^{(1)},\\
f_7 &= \partial_X^2 \Phi^{(1)}.
\end{aligned}$$

On solving relations given in Eqs. (27)–(33), the following relation is obtained:

$$\frac{1-s\delta}{\lambda^2-\gamma\mu}\partial_X f_1 - \frac{1}{\lambda^2-\gamma}\partial_X f_2 + \frac{1-s\delta}{\lambda}\partial_X f_3 - \frac{1}{\lambda}\partial_X f_4 + \frac{\mu(1-s\delta)}{\lambda(\lambda^2-\gamma\mu)}\partial_X f_5 - \frac{1}{\lambda(\lambda^2-\gamma)}\partial_X f_6 - \partial_X^2 f_7 = 0, \tag{34}$$

where the relation given in Eq. (26) has also been used to obtain the above equation.

The KP equation in unmagnetized p-i plasma with static charged dust particles is obtained in terms of $\Phi^{(1)}$ by using the relations of f's along with Eqs. (23) and (24) in Eq. (34), i.e.,

$$\partial_X\left[\partial_\tau \Phi^{(1)} + A\Phi^{(1)}\partial_X \Phi^{(1)} + B\partial_X^3 \Phi^{(1)}\right] + C\partial_Y^2 \Phi^{(1)} = 0, \tag{35}$$

where the nonlinear, dispersive, and diffractive coefficients A, B, and C are respectively defined as follows:

$$A = \frac{A_2}{A_1}, \quad B = \frac{1}{A_1}, \quad C = \frac{A_3}{A_1}, \tag{36}$$

with

$$\begin{aligned}
A_1 &= \frac{2\lambda(1-s\delta)}{(\lambda^2-\gamma\mu)^2} + \frac{2\lambda}{(\lambda^2-\gamma)^2},\\
A_2 &= \frac{(1-s\delta)(3\lambda^2-2\gamma\mu+\mu\gamma^2)}{(\lambda^2-\gamma\mu)^3} - \frac{3\lambda^2-2\gamma+\gamma^2}{(\lambda^2-\gamma)^3},\\
A_3 &= \frac{(1-s\delta)(\lambda^2-2\gamma\mu)}{(\lambda^2-\gamma\mu)^2} - \frac{\lambda^2-2\gamma}{(\lambda^2-\gamma)^2}.
\end{aligned} \tag{37}$$

In order to find the analytical solution of the KP equation (35), we have used the well-known tanh method.[34,35] We have defined the transformed coordinate $\xi = X + Y - u\tau$ of the comoving frame with speed u and used the boundary conditions $\Phi^{(1)} \to 0$ and $\partial_\xi \Phi^{(1)}$, $\partial_\xi^2 \Phi^{(1)}$, $\partial_\xi^3 \Phi^{(1)} \to 0$ as $\xi \to \infty$ for the localized solution

$$\Phi^{(1)}(\xi) = \frac{12B}{A}\left[1 - \tanh^2(X+Y-u\tau)\right]. \tag{38}$$

The speed of the comoving frame $u = 4B + C$ depends on the weak dispersive and diffraction coefficients and has been obtained by using the boundary conditions, i.e., as $\xi \to \infty$, then $\Phi^{(1)}(\xi) \to 0$ and $\tanh^2(\xi) \to 1$. The solution of the nonlinear potential is given by $\Phi \simeq \varepsilon\Phi^{(1)}$.

The KdV equation yields a one-dimensional soliton solution. This one-dimensional soliton has a strikingly stable propagation. However, if we consider the propagation of soliton in higher dimensions like the one in the two-dimentional KP equation, the solution may become precarious.[36] It is, therefore, obligatory to address the stability of a two-dimensional KP soliton. To discuss the stability of the KP equation, the variation in perturbation scheme of y-axis defined in Eq. (15) is very important. If the perturbations in the perpendicular direction of propagation of soliton are too weak, then we will get the one-dimensional KdV soliton solution, whereas strong perturbations along the y axis can exclude the possibility of soliton solution. We use the method established on energy considerations[31,32] to find the stability condition of the KP

soliton. Using the transformed coordinate ξ, the KP equation (35) in the form of energy integral equation can be written as

$$\frac{1}{2}\left(\frac{\partial \Phi^{(1)}}{\partial \xi}\right)^2 + V(\Phi^{(1)}) = 0, \tag{39}$$

where the pseudo potential is defined as

$$V(\Phi^{(1)}) = -\frac{(u-C)}{2B}(\Phi^{(1)})^2 + \frac{A}{6B}(\Phi^{(1)})^3. \tag{40}$$

The necessary condition for a stable soliton solution is $\left|\frac{\mathrm{d}^2 V(\Phi^{(1)})}{\mathrm{d}(\Phi^{(1)})^2}\right|_{\Phi^{(1)}=0} < 0$. According to this condition, we obtain

$$\left|\frac{\mathrm{d}^2 V(\Phi_1)}{\mathrm{d}\Phi_1^2}\right|_{\Phi_1=0} = -\frac{(u-C)}{B}. \tag{41}$$

Equation (41) demands that $(u-C)/B$ should be positive for the existence of soliton solution. This condition holds only if $B>0$ and $(u-C)>0$ or $B<0$ and $(u-C)<0$. Now from the expression of dispersive coefficient B given in Eq. (36), the condition $B>0$ means that $(u-C)>0$ and $u>C$ must be satisfied. Thus, for a stable soliton solution, $u>C$ must always hold.

4. Numerical results

Now we address some numerical results of dust ion acoustic solitons in pair-ion plasma with traces of dust particles. Dust particles are neutrals in general, however, these dust particles become charged when positive or negative charged particles are attached to them. Several mechanisms, such as thermionic emission, field emission, radioactivity, and impact ionization, are responsible for charging (positively or negatively) the dust particles.[27] Therefore, we consider both the positively and negatively charged dust particles. The dust particles are taken as impurities (i.e., a few percent of positive or negative ions) in the pure p-i plasma. We have used Eq. (38) to plot $\Phi^{(1)}(\xi)$ for different values of ion temperature ratio μ and ion density ratio δ. In general, it is assumed that in p-i fullerene ($C_{60}^{\pm}$) plasma, $m_+ = m_-$, $Z_+ = Z_-$, $n_{+0} = n_{-0}$ (without dust particles), and $T_+ = T_-$. It is, however, observed experimently that T_+ is not exactly equal to T_-.[27] This deviation in the temperatures of the two ions may be due to the fact that the charging process of the positive ions is different from that of the negative ions.[33] The positive ions are produced in the process of electron impact ionization $C_{60} + \bar{e}(> 12\ \mathrm{eV}) \rightarrow C_{60}^{+} + 2\bar{e}$, whilst the low energy electrons are attached to acquire the negative ions $C_{60} + \bar{e}(< 12\ \mathrm{eV}) \rightarrow C_{60}^{-}$. Thus, it is appropriate to talk about plasma when the two ions are at slightly different temperatures. It is clear from Fig. 1 that a electrostatic potential hump structure is found when the dust is positively charged and the thermal energy of the negative ions is higher than that of the positive ions (i.e., $\mu = \frac{T_+}{T_-} = 0.5$). When the thermal energy of the positive ions is higher than that of the negative ions, then the potential dip structures are formed as shown in Fig. 2, where $\mu = 2$. Figures 3–7 are plotted to discuss the variations in the amplitude and the width of the dust ion acoustic soliton in the p-i plasma when different parameters are used. For example, Fig. 3 shows the increase in the amplitude of the potential hump structure when the positively charged dust particles are introduced in the p-i plasma whilst the negative ions are hotter than the positive ions. It can also be seen from Fig. 4 that the introduction of the negatively charged dust particles to pure p-i plasma decreases the amplitude of the potential dip structure whilst the positive ions are hotter than the negative ions. It is clear from Fig. 5 that the amplitude and the width of the electrostatic potential hump structures increase with the increase in the thermal energy of the negative ions, i.e., when μ changes from 0.5 to 0.4 (the increase in the thermal energy of the negative ions is the decrease of μ as $\mu = T_+/T_-$). It is also evident from Fig. 6 that the amplitude and the width of the electrostatic potential dip structures increase with the increase in the temperature of the positive ions, i.e., when μ changes from 2 to 2.5. As already mentioned, the electrostatic potential hump structures appear when the dust particles are positively charged and the thermal energy of the negative ions is higher than that of the positive ions. These hump structures do not flip over to potential dip structures if the positively charged dust particles are replaced by the negatively charged dust particles as long as the negative ions are hotter than the positive ions. The potential hump structures turn over to potential dip structures only if the positive ions become hotter than the negative ions. Figure 7 shows the decrease in the amplitude and the width of electrostatic potential hump structures when the negative dust particles supersede the positive dust particles whilst the negative ions are hotter than the positive ions.

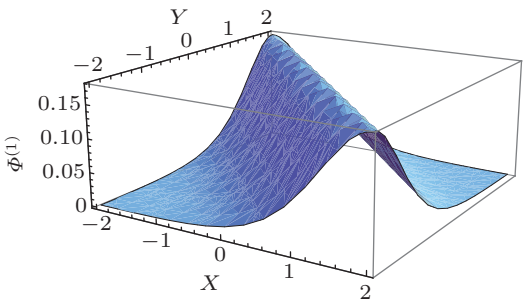


Fig. 1. (color online) Two-dimensional electrostatic potential hump structure in pair-ion dusty plasma shown for $s = +ve$, $\delta = 0.1$, $\mu = 0.5$, $u = 0.823$, $c = 0.61$.

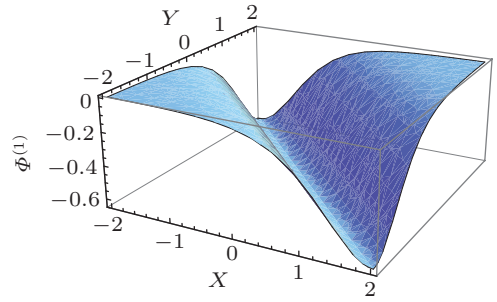


Fig. 2. (color online) Two-dimensional electrostatic potential dip structure in pair-ion dusty plasma shown for $s = -ve$, $\delta = 0.11$, $\mu = 2$, $u = 1.4$, $c = 0.86$.

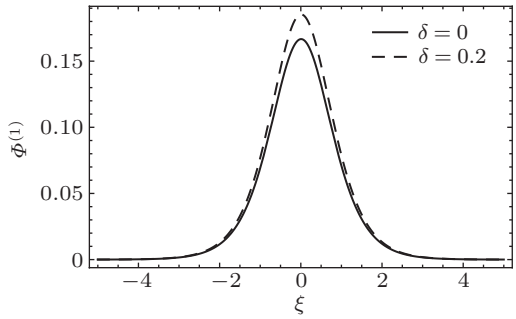


Fig. 3. Increase in the amplitude of electrostatic potential hump structure in the absence ($\delta = 0$, solid curve) as well as in the presence ($\delta = 0.2$, dotted curve) of positively charged dust particles in pair-ion plasma when negative ions are hotter than positive ions, i.e., $\mu = 0.5$.

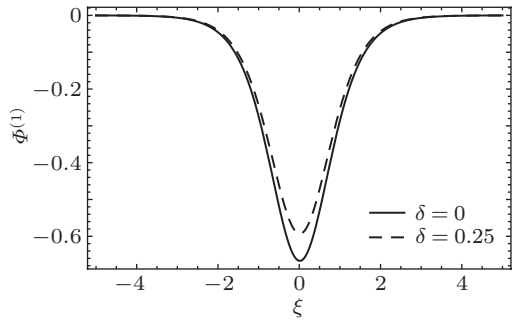


Fig. 4. Decrease in the amplitude of electrostatic potential dip structure in the absence ($\delta = 0$, solid curve) as well as in the presence ($\delta = 0.25$, dotted curve) of negatively charged dust particles in pair-ion plasma when positive ions are hotter than negative ions, i.e., $\mu = 2$.

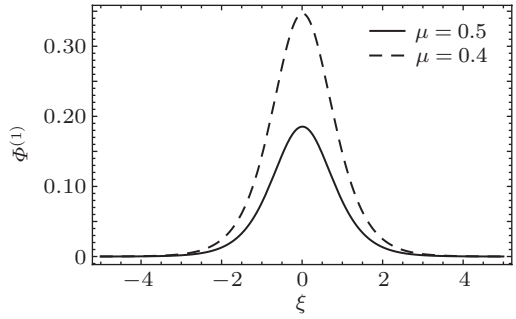


Fig. 5. Increases in amplitude and width with the increase in negative-ion temperature (i.e., decrease in μ) in pair-ion dusty plasma shown for $\delta = 0.2$, $\mu = 0.5$ (solid curve) and $\mu = 0.4$ (dotted curve).

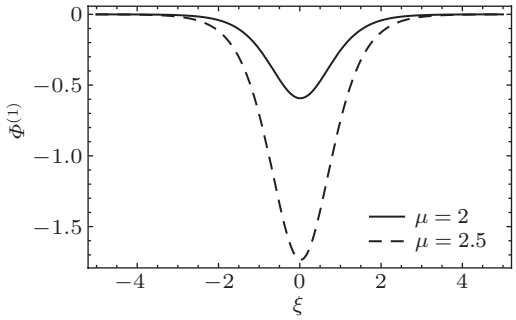


Fig. 6. Increases in amplitude and width with the increase in positive-ion temperature (i.e., increase in μ) in pair-ion dusty plasma shown for $\delta = 0.25$, $\mu = 2$ (solid curve) and $\mu = 2.5$ (dotted curve).

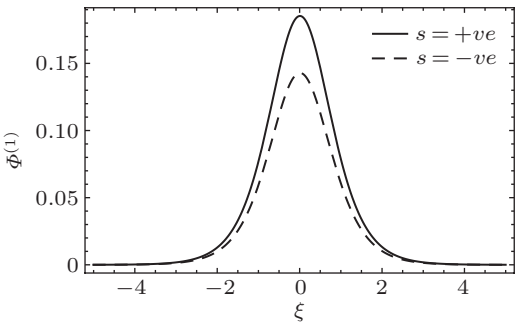


Fig. 7. Decreases in the amplitude and the width of electrostatic potential hump structure when the dust particles are positively (solid curve) or negatively (dotted curve) charged for $\mu = 0.5$ and $\delta = 0.2$.

5. Summary

To summarize, we have analyzed the low amplitude nonlinear dust ion acoustic electrostatic solitons in unmagnetized p-i plasma. The dust particles are assumed to be static while the ions are dynamic. The Kadomtsev–Petviashvili equation has been derived for the nonlinear propagation of electrostatic solitary waves in the pair-ion (p-i) plasma using the reductive perturbation technique. To elaborate the effects of the static charged dust particles on solitary waves in the p-i plasma, the numerical results are plotted in the absence and the presence of the static charged dust particles. We discuss how the temperatures of the positive and the negative ions affect the results. It is found that electrostatic potential hump structures are formed as long as the thermal energy of the negative ions is higher than that of the positive ions. It is also observed that the potential hump structures flip over (i.e., hump structures become dip structures) to potential dip structures only if the thermal energy of the positive ions exceeds that of the negative ions. It is also noticed that the flipping of the electrostatic nonlinear potential structures is only controlled by the thermal energy of the negative ions and not by the type of charge on the static dust particles. The potential hump structures do not flip over to potential dip structures when the positive dust particles are replaced by the negative dust particles, whilst the thermal energy of the negative ions is higher than that of the positive ions.

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