Propagation of an electromagnetic soliton in an anisotropic biquadratic ferromagnetic medium*

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Information storage technology based on anisotropic ferromagnets with sufficiently high magneto-optical effects has received much attention in recent years. Magneto-optical recording combines the merits of magnetic and optical techniques. We investigate the magneto-optical effects on a biquadratic ferromagnet and show that the dynamics of the system are governed by a perturbed nonlinear Schrödinger equation. The evolutions of amplitude and velocity of the soliton are found to be time independent, thereby admitting the lossless propagation of the electromagnetic soliton in the medium, which may have potential applications in soliton based optical communication systems. We also exploit the role of perturbation, which has a significant impact on the propagation of an electromagnetic soliton.

Keywords: solitons, classical spin models, Maxwell equations, nonlinear dynamics

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1. Introduction

The interest in soliton excitations has its own right, and the perturbed version has been substantially developed in recent years.[1–6] The result of this theory is clearly utilized in all branches of science, especially the soliton propagation pertaining to the electromagnetic (EM) wave propagation in optical and ferromagnetic media. In optical studies, the solitons can be exploited for signal transmission in optical fibers which carry all the information as optical or signal pulses. Stable optical pulses are generated due to the compensation between the nonlinear Kerr effect and the inherent dispersion of the medium, and such a compensation induces a stable nonlinear pulse called the optical soliton.[7] This optical soliton can travel a very long distance without changing its shape and velocity, which was first demonstrated by Hasewaga and Tappert[8] theoretically and later proved experimentally by Mollenauer *et al*.[9] In another context, the propagation of an intense electromagnetic wave through ferromagnetic media is associated with a wide variety of interesting phenomena, such as harmonic generation,[10,11] self-focusing,[12] domain wall propagation,[13–15] and EM soliton propagation.[16,17] The electromagnetic wave propagation in a ferromagnet is of relevance in a number of applications including magneto-optical recording, ultrafast high density data storage, and magnetic memory elements. The nonlinear effects arise due to the interaction between the electromagnetic wave and the ferromagnetic media, the existence and the dynamics of EM solitons attract great attention due to their possible applications. The optical pulses generated in the magnetic media are now used to read and write data in magnetic storage devices via the interaction between the magnetic field component of the electromagnetic field and the magnetization of the magnetic device. The electromagnetic wave propagates in the ferromagnetic medium modulated in the form of a soliton and also causes the spin wave to be excited in the solitonic nature. Many different physical effects bound to the fundamental spin–spin exchange interaction play a vital role in the generation of an EM soliton, including dispersion,[18] nonlinearities,[19] higher order magnetic interactions,[20–26] and so on. Theoretical analyses have been performed mainly in the framework of one-dimensional approximation, in which the solitons are solutions of nonlinear evolution equations, namely, nonlinear Schrödinger (NLS),[27] sine-Gordon,[28] and KdV[29] equations. The effect of dissipation and nonlinearity on the propagation of an electromagnetic wave in a saturated ferrite has been studied in the presence of an external magnetic field, and it is found to be governed by the Burgers equation.[30] In the perspective of the lossless propagation of EM signals, the ferromagnetic medium has also become a useful medium for propagation. This lossless propagation of an electromagnetic wave is mainly due to the nonlinear magnetic interactions present in the medium and the interactions with external magnetic fields, which has been experimentally observed in yttrium iron garnet.[31] McCall *et al*.[32] explained that for a lossless optical pulse in a two-level reso-

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nant medium, it was found that the energy difference between the two levels of the medium coincides with the optical wavelength and then coherent absorption takes place, and hence the medium becomes optically transparent to that particular wavelength. Since the major part of the existing investigations dedicated to the EM soliton have been performed in ferromagnetic medium saturated to a high level of magnetization, hence it is tempting to incorporate the presence of several higher order magnetic interactions into the model in addition to magnetic anisotropies and look for the possible existence of the EM soliton. It is found that the EM soliton undergoes damping, the velocity of the soliton decreases as time progresses, and the structural stability is maintained in the antiferromagnetic medium throughout the propagation.[33] However, when an anisotropy is added to the effective field as a small correction, the soliton loses its stability and decelerates due to damping. In this context, we report the breather-like evolution of an EM wave propagating in a weak antiferromagnetic medium with a Dzyaloshinskii–Moriya interaction.[34]

In this paper, we demonstrate the existence of localized EM soliton modes in an anisotropic ferromagnetic medium with biquadratic interaction. We employ a multi-scale perturbation method to study the problem, and most of the explicit parameters appearing in the equations are redefined to obtain a perturbed nonlinear Schrödinger equation. Furthermore, we use the the multiple scale perturbation technique to solve the resultant NLS equation and explore the existence of the EM soliton.

This paper is organized as follows. The governing equations are introduced in Section 2. In Section 3, a reductive perturbation technique is employed yielding a perturbed nonlinear Schrödinger equation. In Section 4, we construct the spin components of the EM soliton. The results are summarized in Section 5.

2. Model and dynamical equations

Let us consider a one-dimensional classical Heisenberg ferromagnetic spin system having a biquadratic exchange interaction in addition to the fundamental bilinear anisotropic Zeeman energy. Its Hamiltonian reads

$$H = -\sum_n \left[J(\boldsymbol{S}_n \cdot \boldsymbol{S}_{n+1}) + K(\boldsymbol{S}_n \cdot \boldsymbol{S}_{n+1})^2 - A(S_n^x) + \frac{2\beta}{\mu}(\boldsymbol{S}_n \cdot \boldsymbol{B}) \right], \tag{1}$$

where $\boldsymbol{S}_n$ denotes the classical spin vector at lattice site n, and $|\boldsymbol{S}_n|^2 = \text{const}$. The parameters J and K are the bilinear and the biquadratic exchange integrals between the nearest neighbors. The strength of the crystal field anisotropy along the easy axis of magnetization is characterized by parameter A, and the external magnetic field is the magnetic field component of the propagating EM wave along the x direction with $\beta = g\mu_B$, where g is the gyromagnetic ratio, μ_B represents the Bohr magneton, and μ is the magnetic permeability of the medium. In the general sense, the equation of motion for Eq. (1) can be derived using the spin Poisson bracket. In such a case, the spin $\boldsymbol{S}_n(t)$ may be replaced by $\boldsymbol{S}(x,t)$, where $x = na$ is continuous with a being the lattice parameter. We use the series expansion $\boldsymbol{S}_{n+1} = \boldsymbol{S} + a\boldsymbol{S}_x + (a^2/2!)\boldsymbol{S}_{xx} + \cdots$ in the discrete equation for the magnetic lattice and an appropriate rescaling of t, the equation of motion can be written to order a^4 in the form

$$\frac{\partial \boldsymbol{S}}{\partial t} = \boldsymbol{S} \wedge \left\{ J\boldsymbol{S}_{xx} + \gamma_1 \boldsymbol{S}_{xxxx} + \gamma_2 \left[(\boldsymbol{S} \cdot \boldsymbol{S}_{xx})\boldsymbol{S}_{xx} + \frac{2}{3}(\boldsymbol{S} \cdot \boldsymbol{S}_{xxx})\boldsymbol{S}_x \right] - 2AS^x \hat{\boldsymbol{n}} + \left(\frac{2\beta}{\mu} \right) \boldsymbol{B} \right\}, \tag{2}$$

where $\gamma_1 = a^2/12$, $\gamma_2 = Ka^2/(1+2K)$, and $\hat{\boldsymbol{n}} = (1,0,0)$. We consider the propagation of electromagnetic waves in a magnetic medium in the presence of an external magnetic field. The governing Maxwell's equations are[35]

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \qquad \nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t}, \tag{3}$$

where $\boldsymbol{E}$, $\boldsymbol{D}$, $\boldsymbol{H}$, and $\boldsymbol{B}$ are the electric field, the electric induction, the magnetic field, and the magnetic induction, respectively. The constitutive equation for $\boldsymbol{E}$ and $\boldsymbol{D}$ is

$$\boldsymbol{D} = \varepsilon \boldsymbol{E}, \tag{4}$$

where ε is the scalar permeability of the magnetic medium. Whereas the constitutive equation for $\boldsymbol{H}$ and $\boldsymbol{B}$ is

$$\boldsymbol{B} = \mu(\boldsymbol{H} + \boldsymbol{S}), \tag{5}$$

where μ is the magnetic permeability of the medium, and $\boldsymbol{S}$ is the magnetization density in the magnetic medium of propagation. Eliminating $\boldsymbol{E}$, $\boldsymbol{D}$, and $\boldsymbol{H}$ from Eqs. (3)–(5), we have

$$\frac{\partial^2 \boldsymbol{B}}{\partial x^2} - \mu\varepsilon \frac{\partial^2 \boldsymbol{B}}{\partial t^2} = \mu \left(\frac{\partial^2 \boldsymbol{S}}{\partial x^2} - \hat{\boldsymbol{n}} \frac{\partial^2 S^x}{\partial x^2} \right). \tag{6}$$

The above set of coupled equations (Eqs. (2)–(6)) completely describe the propagation of an EM wave in an anisotropic ferromagnetic medium with a biquadratic interaction.

3. The reductive perturbation method

After deriving the equations of motion, the task now lies in solving them to understand the underlying spin excitations. We find Eqs. (2)–(6) as a set of highly nontrivial nonlinear coupled partial differential equations, which are not amenable to exact analysis in general. In this section, we attempt to solve the coupled Landau–Lifshitz equations for magnetization and the Maxwell's equations for an electromagnetic field within

the framework of he reductive perturbation method along the lines of Taniuti, Yajima, and Leblond.[36,37] This technique adopts the small but finite nonlinear modulation of the slowly varying envelopes of the EM plane waves in the antiferromagnetic medium. In order to carry out this perturbation, the magnetization of the medium and the magnetic induction of the EM field have to be expanded nonuniformly in the anisotropic weak antiferromagnetic medium. Since the easy axis of magnetization of the anisotropic medium lies parallel to the direction of propagation (x direction), we assume that at the lowest order of expansion, the magnetization of the medium and the magnetic induction lie parallel to the propagation axis and turn around to the y–z plane at higher orders. Therefore writing $\boldsymbol{S}=(S^x,S^y,S^z)$ and $\boldsymbol{B}=(B^x,B^y,B^z)$ and expressing the Fourier components of $\boldsymbol{S}$ and $\boldsymbol{B}$ in powers of a small parameter ε, we have

$$\begin{aligned}
S^x &= S_0+\varepsilon S_1^x+\varepsilon^2 S_2^x+\cdots,\\
S^\alpha &= \sqrt{\varepsilon}\left(S_1^\alpha+\varepsilon S_2^\alpha+\cdots\right),\\
B^x &= B_0+\varepsilon B_1^x+\varepsilon^2 B_2^x+\cdots,\\
B^\alpha &= \sqrt{\varepsilon}\left(B_1^\alpha+\varepsilon B_2^\alpha+\cdots\right),
\end{aligned}\tag{7}$$

where $\alpha=y,\,z$. Thus the magnetization and the magnetic field intensity have been expanded around uniform values S_0 and B_0 respectively along the direction of propagation. We now introduce the stretching variables $\xi=\varepsilon(z-vt)$ and $\tau=\varepsilon t$, which characterize the shape of the pulse propagating at speed v and the time variable accounting for the evolution of the propagating pulse, respectively. As the medium is ferromagnetic in nature, it is reasonable to treat the dielectric constant of the medium as small and hence rescale ε as $\varepsilon\to\varepsilon^2\varepsilon$. We assume that the bilinear and the biquadratic exchange interactions are stronger than the anisotropic interaction and the Zeeman energy by rescaling $J\to\varepsilon^{-1}J$, $\gamma_1\to\varepsilon^{-3}\gamma_1$, $\gamma_2\to\varepsilon^{-4}\gamma_2$, and $\beta\to\varepsilon\beta$. We now substitute the expansions for $\boldsymbol{S}$ and $\boldsymbol{B}$ given in Eq. (7) in the component form of Eqs. (2)–(6). After collecting and solving the coefficients at different orders of ε, we get the following results. At the order $\varepsilon^{(0)}$:

$$B_0=0,\;\; B_1^y=\mu S_1^y,\;\; B_1^z=\mu S_1^z.\tag{8}$$

At the order ε^1:

$$B_1^x=0,\;\; B_2^y=\mu S_2^y,\;\; B_2^z=\mu S_2^z,\tag{9}$$

$$\begin{aligned}
&\frac{\partial S_1^x}{\partial\tau}-v\frac{\partial S_1^x}{\partial\xi}\\
&=J\left(S_1^y\frac{\partial^2 S_1^z}{\partial\xi^2}-S_1^z\frac{\partial^2 S_1^y}{\partial\xi^2}\right)+\gamma_1\left(S_1^y\frac{\partial^4 S_1^z}{\partial\xi^4}-S_1^z\frac{\partial^4 S_1^y}{\partial\xi^4}\right)\\
&\quad+\gamma_2\left[\left(S_0\frac{\partial^2 S_1^x}{\partial\xi^2}+S_1^y\frac{\partial^2 S_1^y}{\partial\xi^2}+S_1^z\frac{\partial^2 S_1^z}{\partial\xi^2}\right)\right.\\
&\quad\left.\times\left(S_1^y\frac{\partial^2 S_1^z}{\partial\xi^2}-S_1^z\frac{\partial^2 S_1^y}{\partial\xi^2}\right)\right]+\frac{2}{3}\gamma_2\left[\left(S_0\frac{\partial^3 S_1^x}{\partial\xi^3}\right.\right.\\
&\quad\left.\left.+S_1^y\frac{\partial^3 S_1^y}{\partial\xi^3}+S_1^z\frac{\partial^3 S_1^z}{\partial\xi^3}\right)\left(S_1^y\frac{\partial S_1^z}{\partial\xi}-S_1^z\frac{\partial S_1^y}{\partial\xi}\right)\right],\\
&\frac{\partial S_1^y}{\partial\tau}-v\frac{\partial S_1^y}{\partial\xi}\\
&=-JS_0\frac{\partial^2 S_1^z}{\partial\xi^2}-\gamma_1 S_0\frac{\partial^4 S_1^z}{\partial\xi^4}-\gamma_2 S_0\frac{\partial^2 S_1^z}{\partial\xi^2}\left(S_0\frac{\partial^2 S_1^x}{\partial^2\xi}+S_1^y\frac{\partial^2 S_1^y}{\partial\xi^2}\right.\\
&\quad\left.+S_1^z\frac{\partial^2 S_1^z}{\partial\xi^2}\right)-\frac{2}{3}\gamma_2 S_0\frac{\partial S_1^z}{\partial\xi}\left(S_0\frac{\partial^3 S_1^x}{\partial\xi^3}+S_1^y\frac{\partial^3 S_1^y}{\partial\xi^3}+S_1^z\frac{\partial^3 S_1^z}{\partial\xi^3}\right)\\
&\quad-2A(S_2^z S_0+S_1^z S_1^x)-\frac{2\beta}{\mu}(S_0B_1^z),\\
&\frac{\partial S_1^z}{\partial\tau}-v\frac{\partial S_1^z}{\partial\xi}\\
&=JS_0\frac{\partial^2 S_1^y}{\partial\xi^2}+\gamma_1 S_0\frac{\partial^4 S_1^y}{\partial\xi^4}+\gamma_2 S_0\frac{\partial^2 S_1^y}{\partial\xi^2}\left(S_0\frac{\partial^2 S_1^x}{\partial^2\xi}+S_1^y\frac{\partial^2 S_1^y}{\partial\xi^2}\right.\\
&\quad\left.+S_1^z\frac{\partial^2 S_1^z}{\partial\xi^2}\right)+\frac{2}{3}\gamma_2 S_0\frac{\partial S_1^y}{\partial\xi}\left(S_0\frac{\partial^3 S_1^x}{\partial\xi^3}+S_1^y\frac{\partial^3 S_1^y}{\partial\xi^3}+S_1^z\frac{\partial^3 S_1^z}{\partial\xi^3}\right)\\
&\quad+2A(S_2^y S_0+S_1^y S_1^x)+\frac{2\beta}{\mu}(S_0B_1^y).
\end{aligned}\tag{10}$$

Without loss of generality, we assume a new complex field u in order to identify Eq. (10) with a more standard nonlinear evolution equation

$$u=(S_1^y-\mathrm{i}S_1^z),\quad |u|^2=-2S_0S_1^x.\tag{11}$$

Using the above new complex field and after some lengthy calculations, we obtain

$$\begin{aligned}
&\mathrm{i}u_\tau+u_{\xi\xi}+2|u|^2u\\
&=\lambda\left\{u_{\xi\xi\xi\xi}+c\left[3(|u|^2)_{\xi\xi}u_{\xi\xi}+2(|u|^2)_{\xi\xi\xi}u_\xi\right.\right.\\
&\quad\left.\left.-6|u_\xi|^2u_{\xi\xi}-3u_\xi^2u_{\xi\xi}^*\right]\right\},
\end{aligned}\tag{12}$$

where $\lambda=-\gamma_1/J$ and $c=\gamma_2/3\gamma_1$. In Eq. (12), suffices τ and ξ represent the partial derivatives with respect to τ and ξ, respectively.

4. Perturbed soliton spin excitations

The generalized perturbed nonlinear Schrödinger (pNLS) equation takes the following form:

$$\mathrm{i}q_t+q_{xx}+2|q|^2q=p[q,q^*],\tag{13}$$

which occurs in several contexts in nonlinear physics.[38,39] The perturbation $p[q,q^*]$ in general is small and depends on function q and its complex conjugate q^*, and sometimes also depends on the derivatives of q. The unperturbed problem, i.e., $p[q,q^*]=0$, is well known to be solvable[40] by inverse scattering transform, and finite energy soliton solutions exist that are dynamically stable and robust with respect to collision.

The unperturbed NLS equation describes the evolution of two novel solitons, namely, temporal and spatial solitons, in the literature of modern optics.[41] Temporal solitons are termed as the propagation of light wave packets for high speed communication along the optical fiber reported by Hasegawa and Tappert.[42] Secondly, spatial solitons occupy the propagation of a stationary envelope of monochromatic light waves in the planar wave guide under the paraxial approximation. This fact along with the integrability afforded by excluding the perturbative term makes the unperturbed NLS equation one of the most important models in modern optics. In the present case, the perturbed NLS equation (12) obtained in the previous section describes the propagation of an electromagnetic soliton in a biquadratic ferromagnetic medium. The term proportional to λ can be treated as a weak perturbation, and the entire equation can be solved without any approximation by using the multiple scale perturbation method laid down by Kodama and Ablowitz.[43] The basic idea of this approach is summarized as follows. We consider a nonlinear partial differential equation of the form

$$Q(q,\ q_t,\ q_x,\ q_{xx},...;\lambda)=0,\quad 0<\lambda\ll 1, \tag{14}$$

where Q is a nonlinear function of q, q_t, $q_x,\ldots$, and λ is an external parameter. Let the unperturbed equation (for $\lambda=0$) be given by

$$Q(q_0,\ q_{0t},\ q_{0x},...)=0, \tag{15}$$

where q_0 is the exact solution to Eq. (15). In the present case, we will consider q_0 to be the soliton solution to Eq. (15). Now we introduce the so-called fast variables θ_i $(i=1,\ 2,...,\ m)$, a slow variable $T=\lambda t$, and parameters P_l $(l=1,\ 2,\ldots,N)$ depending on the slow variable. We then rewrite q_0 in terms of these newly defined variables as

$$q_0=\hat{q}_0(\theta_1,\ \theta_2,...,\theta_m,\ T;\ P_1,\ P_2,...,P_N). \tag{16}$$

It may be noted that in the case of one-soliton solution, we need only one fast variable. Now we assume an expansion for q in the form

$$q=\hat{q}_0+\lambda\hat{q}_1+\cdots\equiv\sum_{n=0}^{\infty}\lambda^n\hat{q}_n. \tag{17}$$

Using Eq. (17) in the perturbed equation (14) and collecting the coefficients of various powers of λ, we obtain a system of differential equations with variable coefficients. Solving this system of equations recursively and taking care of the secularity problem appropriately, we can in principle obtain the required perturbed solution. After obtaining such a perturbed solution, it is essential that we check the validity of the solution in all regions of the fast variable space and adequately modify the perturbative expressions. We make use of the above technique to solve the perturbed nonlinear Schrödinger equation (12). Considering Eq. (12) and treating λ as a small parameter, we find that as $\lambda\to 0$, it admits the following envelope soliton solution:

$$u_0=\eta\,\mathrm{sech}\,\eta(\theta-\theta_0)\exp\left[\mathrm{i}\zeta(\theta-\theta_0)+\mathrm{i}(\sigma-\sigma_0)\right], \tag{18}$$

where

$$\frac{\partial\theta}{\partial t}=-2\zeta,\quad \frac{\partial\theta}{\partial\xi}=1,$$
$$\frac{\partial\sigma}{\partial t}=\eta^2+\zeta^2,\quad \frac{\partial\sigma}{\partial\xi}=0. \tag{19}$$

Now under the assumption of quasi stationarity, we assume that the parameters ζ, η, θ_0, and σ_0 in Eqs. (18) and (19) are functions of $T=\lambda t$. Specifically, the quasi-stationarity solution of Eq. (15) is assumed to take the form

$$u=\hat{u}(\theta,\ T;\ \lambda)\exp\left[i\zeta(\theta-\theta_0)+i(\sigma-\sigma_0)\right]. \tag{20}$$

The above assumption seems to be a conceptually simple generalization of the unperturbed form and is appropriate for explicit calculations. We assume the Poincare-type asymptotic expansion for $\hat{u}$ in Eq. (20)

$$\hat{u}(\theta,\ T,\ \lambda)=\sum_{m=0}^{\infty}\lambda^m\hat{u}_m(\theta,\ T). \tag{21}$$

We restrict ourselves in the present paper to calculations of $\mathrm{O}(\lambda)$ and therefore

$$\hat{u}(\theta,\ T;\ \lambda)=\hat{u}_0(\theta,\ T)+\lambda\hat{u}_1(\theta,\ T). \tag{22}$$

The leading order solution $\hat{u}_0$ taken from Eq. (18) is given by

$$\hat{u}_0=\eta\,\mathrm{sech}\,\eta(\theta-\theta_0). \tag{23}$$

Using Eqs. (20)–(22) in Eq. (12), at order λ, we have

$$-\eta^2\hat{u}_1+\hat{u}_{1\theta\theta}+2\hat{u}_0^2\hat{u}_1^*+4\hat{u}_0^2\hat{u}_1=F(\hat{u}_0), \tag{24}$$

where

$$\begin{aligned}F(\hat{u}_0)\ =\ &\left[\zeta_T(\theta-\theta_0)-\zeta\theta_{0T}-\sigma_{0T}\right]\hat{u}_0+\hat{u}_{0\theta\theta\theta}\\&-6\zeta^2\hat{u}_{0\theta\theta}+\zeta^4\hat{u}_0+c\big(4\hat{u}_{0\theta\theta\theta}\hat{u}_{0\theta}\hat{u}_0+9\hat{u}_{0\theta}^2\hat{u}_{0\theta\theta}\\&+6\hat{u}_{0\theta\theta}^2\hat{u}_0-9\zeta^2\hat{u}_{0\theta\theta}\hat{u}_0^2-9\zeta^2\hat{u}_{0\theta}^2\hat{u}_0-3\zeta^4\hat{u}_0^3\big)\\&+\mathrm{i}\Big[4\zeta\hat{u}_{0\theta\theta\theta}-\hat{u}_{0T}-4\zeta^2\hat{u}_{0\theta}+c\big(4\zeta\hat{u}_0^2\hat{u}_{0\theta\theta\theta}\\&+18\zeta\hat{u}_{0\theta\theta}\hat{u}_{0\theta}\hat{u}_0+10\zeta\hat{u}_{0\theta}^3-12\zeta^3\hat{u}_{0\theta}\hat{u}_0^2\big)\Big].\end{aligned}$$

Separating the complex equation (24) into a set of equations in real variables by putting $\hat{u}_1=\hat{\phi}_1+\mathrm{i}\,\hat{\psi}_1$, where $\hat{\phi}_1$ and $\hat{\psi}_1$ are real, we obtain

$$L_1\hat{\phi}_1=-\eta^2\hat{\phi}_1+\hat{\phi}_{1\theta\theta}+6\hat{u}_0^2\hat{\phi}_1=\Re F(\hat{u}_0),$$
$$L_2\hat{\psi}_1=-\eta^2\hat{\psi}_1+\hat{\psi}_{1\theta\theta}+2\hat{u}_0^2\hat{\psi}_1=\Im F(\hat{u}_0), \tag{25}$$

where L_1 and L_2 are the self-adjoint operators. Considering the homogeneous part of Eq. (25), we can check directly that

$\hat{u}_{0\theta}$ and $\hat{u}_0$ are solutions to it respectively, and hence the secularity conditions yield

$$\int_{-\infty}^{\infty} \hat{u}_{0\theta} \Re F \, \mathrm{d}\theta = 0, \quad \int_{-\infty}^{\infty} \hat{u}_0 \Im F \, \mathrm{d}\theta = 0. \tag{26}$$

Now making use of $\hat{u}_{0\theta}$, $\hat{u}_0$, $\Re F(\hat{u}_0)$, and $\Im F(\hat{u}_0)$ in the secularity condition (21) and evaluating the integrals, we get

$$\zeta_T = 0, \quad \eta_T = 0. \tag{27}$$

From Eq. (27), it is evident that the velocity and the amplitude of the soliton remain unaffected during the propagation. Thus the electromagnetic spin soliton moving with constant velocity and amplitude signifies that the lossless propagation of electromagnetic signals along a ferromagnetic medium is possible.

4.1. Perturbed solitons

The perturbed soliton solutions can be constructed by solving Eq. (25) for $\hat{\phi}_1$ and $\hat{\psi}_1$ using ζ_T and η_T. The homogeneous part of the first equation in Eq. (25) admits two particular solutions $\hat{\phi}_{11}$ and $\hat{\phi}_{12}$, which are of the form

$$\hat{\phi}_{11} = \operatorname{sech}\eta(\theta-\theta_0)\tanh\eta(\theta-\theta_0), \tag{28}$$

$$\hat{\phi}_{12} = -\frac{1}{\eta}\left[\operatorname{sech}\eta(\theta-\theta_0) - \frac{3}{2}\eta(\theta-\theta_0)\hat{\phi}_{11} - \frac{1}{2}\tanh\eta(\theta-\theta_0)\sinh\eta(\theta-\theta_0)\right]. \tag{29}$$

Knowing the two particular solutions, we can obtain the general solution for $\hat{\phi}_1$ through the formula

$$\hat{\phi}_1 = \delta_1\hat{\phi}_{11} + \delta_2\hat{\phi}_{12} - \hat{\phi}_{11}\int_{-\infty}^{\theta}\hat{\phi}_{12}\Re F \, \mathrm{d}\theta' + \hat{\phi}_{12}\int_{-\infty}^{\theta}\hat{\phi}_{11}\Re F \, \mathrm{d}\theta', \tag{30}$$

where δ_1 and δ_2 are arbitrary constants. Using Eqs. (28) and (29) and $\Re F$ in Eq. (30) and after evaluating the integrals, we obtain the general solution for $\hat{\phi}_1$. However, the solution contains the secular term, which makes the solution unbounded unless

$$\delta_2 = 0. \tag{31}$$

Further, using the boundary conditions

$$\hat{\phi}_1(0)|_{\theta_0=0} = 0\,, \quad \hat{\phi}_{1\theta}(0)|_{\theta_0=0} = 0, \tag{32}$$

we obtain

$$\delta_1 = 0. \tag{33}$$

Using Eqs. (31) and (33), the explicit form of $\hat{\phi}_1$ is constructed as

$$\begin{aligned}\hat{\phi}_1 = {} & \left[\frac{19}{2}\eta^4 + \frac{63}{4}\eta^6 + \frac{11}{8}\zeta^4 + \frac{39}{4}\zeta^2\eta^2 - \frac{79}{4}c\eta^6 + \frac{1}{2}(\zeta\theta_{0T}+\sigma_{0T})\right]\Theta\tanh\eta\Theta\,\operatorname{sech}\eta\Theta\left[\frac{7}{3}\eta^3 - \frac{35}{2}\eta^5 + \frac{3}{4\eta}\zeta^4 + \frac{15}{2}\zeta^2\eta + \frac{1}{2\eta}(\zeta\theta_{0T}+\sigma_{0T}) + \frac{31}{4}c\eta^5 - \frac{21}{2}c\zeta^2\eta^3 - \frac{15}{4}c\zeta^4\eta\right]\operatorname{sech}\eta\Theta + \left(\frac{47}{6}\eta^3 - 14\eta^5 - \frac{3}{2}\zeta^2\eta + \frac{3}{4\eta}\zeta^4 + \frac{151}{20}c\eta^5 + \frac{27}{2}c\zeta^2\eta^3 + 6c\zeta^4\eta\right)\operatorname{sech}^3\eta\Theta - \left(\frac{13}{3}\eta^3 + \frac{7}{2}\eta^5 + \frac{11}{60}c\eta^5 + 9c\zeta^2\eta^3 + 3c\zeta^4\eta\right)\operatorname{sech}^5\eta\Theta, \end{aligned} \tag{34}$$

where $\Theta = \theta - \theta_0$. In a similar fashion, the solution for ψ_1 can be evaluated by solving the second equation in Eq. (25), and the homogeneous part admits two particular solutions $\hat{\psi}_{11}$ and $\hat{\psi}_{12}$

$$\hat{\psi}_{11} = \operatorname{sech}\eta(\theta-\theta_0), \tag{35}$$

$$\hat{\psi}_{12} = \frac{1}{2\eta}[\eta(\theta-\theta_0)\operatorname{sech}\eta(\theta-\theta_0) + \sinh\eta(\theta-\theta_0)]. \tag{36}$$

Knowing the two particular solutions, we can obtain the general solution for $\hat{\psi}_1$ through the formula

$$\hat{\psi}_1 = \delta_3\hat{\psi}_{11} + \delta_4\hat{\psi}_{12} - \hat{\psi}_{11}\int_{-\infty}^{\theta}\hat{\psi}_{12}\Im F \, \mathrm{d}\theta' + \hat{\psi}_{12}\int_{-\infty}^{\theta}\hat{\psi}_{11}\Im F \, \mathrm{d}\theta', \tag{37}$$

where δ_3 and δ_4 are arbitrary constants. By substituting Eqs. (35) and (36) in Eq. (37) and after evaluating the integrals, we obtain the general solution for $\hat{\psi}_1$. In order to remove the secularity terms which make the solution unbounded, we choose

$$\delta_4 = 0. \tag{38}$$

By using the boundary conditions

$$\hat{\psi}_1(0)|_{\theta_0=0} = 0, \quad \hat{\psi}_{1\theta}(0)|_{\theta_0=0} = 0, \tag{39}$$

we get

$$\delta_3 = 0. \tag{40}$$

Using Eqs. (38) and (40), we write the final form of solution for $\hat{\psi}_1$ as

$$\begin{aligned}\hat{\psi}_1 = {} & \left(2\zeta\eta^3 - \frac{37c}{10}\zeta\eta^5 - \frac{1}{2}\eta\Theta_T\right)\Theta\operatorname{sech}\eta\Theta + \left(\frac{49}{6}c\zeta\eta^4 - 2\zeta\eta^2 + \frac{7}{9}c\zeta\eta^4 + 3c\zeta^3\eta^2\right)\tanh\eta\Theta\,\operatorname{sech}\eta\Theta - \frac{35}{9}c\zeta\eta^4\tanh\eta\Theta\,\operatorname{sech}^3\eta\Theta + \frac{c}{5}\zeta\eta^4\tanh\eta\Theta\,\operatorname{sech}^5\eta\Theta. \end{aligned} \tag{41}$$

After obtaining the explicit forms of $\hat{\phi}_1$ and $\hat{\psi}_1$, the first order perturbed soliton solution $\hat{u}_1$ can be constructed using the formula $\hat{u}_1 = \hat{\phi}_1 + \mathrm{i}\,\hat{\psi}_1$ and is given by

$$\begin{aligned}\hat{u}_1 = & \left[\frac{19}{2}\eta^4+\frac{63}{4}\eta^6+\frac{11}{8}\zeta^4+\frac{39}{4}\zeta^2\eta^2-\frac{79}{4}c\eta^6\right.\\ & \left.+\frac{1}{2}(\zeta\theta_{0T}+\sigma_{0T})\right]\Theta\tanh\eta\Theta\,\mathrm{sech}\eta\Theta-\left[\frac{7}{3}\eta^3-\frac{35}{2}\eta^5\right.\\ & +\frac{3}{4\eta}\zeta^4+\frac{15}{2}\zeta^2\eta+\frac{1}{2\eta}(\zeta\theta_{0T}+\sigma_{0T})+\frac{31}{4}c\eta^5\\ & \left.-\frac{21}{2}c\zeta^2\eta^3-\frac{15}{4}c\zeta^4\eta\right]\mathrm{sech}\eta\Theta+\left(\frac{47}{6}\eta^3-14\eta^5\right.\\ & -\frac{3}{2}\zeta^2\eta\frac{3}{4\eta}\zeta^4+\frac{151}{20}c\eta^5+\frac{27}{2}c\zeta^2\eta^3\\ & \left.+6c\zeta^4\eta\right)\mathrm{sech}^3\eta\Theta-\left(\frac{13}{3}\eta^3+\frac{7}{2}\eta^5+\frac{11}{60}c\eta^5\right.\\ & \left.+9c\zeta^2\eta^3+3c\zeta^4\eta\right)\mathrm{sech}^5\eta\Theta+\mathrm{i}\left[\left(2\zeta\eta^3-\frac{37c}{10}\zeta\eta^5\right.\right.\\ & \left.-\frac{1}{2}\eta\Theta_T\right)\Theta\mathrm{sech}\eta\Theta+\left(\frac{49}{6}c\zeta\eta^4-2\zeta\eta^2+\frac{7}{9}c\zeta\eta^4\right.\\ & \left.+3c\zeta^3\eta^2\right)\tanh\eta\Theta\,\mathrm{sech}\eta\Theta-\frac{35}{9}c\zeta\eta^4\tanh\eta\Theta\\ & \times\mathrm{sech}^3\eta\Theta+\frac{c}{5}\zeta\eta^4\tanh\eta\Theta\,\mathrm{sech}^5\eta\Theta\Big]. \qquad (42)\end{aligned}$$

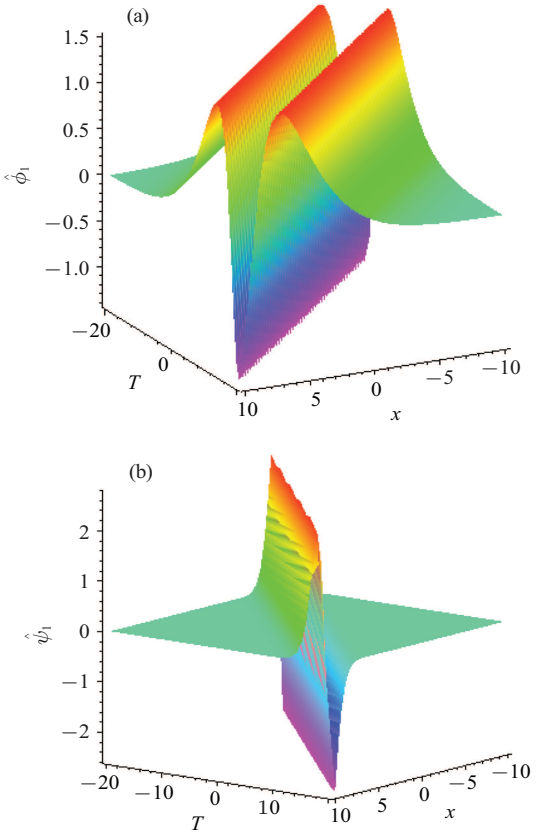


Fig. 1. (color online) Real (a) and imaginary (b) parts of the perturbed one soliton solution with $\eta = 1.5$, $\zeta = 0.5$, and $\lambda = 0.5$.

We have plotted the real and the imaginary parts of the perturbed soliton with the parameters $\eta = 1.5$, $\zeta = 0.5$, and $\lambda = 0.5$. From the figures, it is inferred that the nonlinear spin excitations in the biquadratic ferromagnet are excited in the form of a soliton. Figure 1(a) depicts the profile of a soliton with a wing-like structure for the real part, the kink-like structure for the imaginary part of the perturbed soliton is shown in Fig. 1(b), where it is acted upon by the electromagnetic wave. Using the complex variable $u = S_1^y - iS_1^z$ with $|u|^2 = 2S^0S_1^x$, the component of the spin density $\boldsymbol{S}$ can be constructed. The one soliton solution takes the form

$$\begin{aligned}S_1^x &= -\frac{1}{2S_0}\left(\hat{u}_0^2+2\lambda\hat{u}_0\hat{\phi}_1\right),\\ S_1^y &= (\hat{u}_0+\lambda\hat{\phi}_1)\cos\alpha-\lambda\hat{\psi}_1\sin\alpha,\\ S_1^z &= -(\hat{u}_0+\lambda\hat{\phi}_1)\sin\alpha-\lambda\hat{\psi}_1\cos\alpha, \qquad (43)\end{aligned}$$

where $\alpha = \xi(\theta-\theta_0)+(\sigma-\sigma_0)$ and $\hat{u}_0 = \eta\mathrm{sech}\eta(\theta-\theta_0)$. Figure 2 shows the lossless propagation of x, y, and z components of the magnetization of the medium under the influence

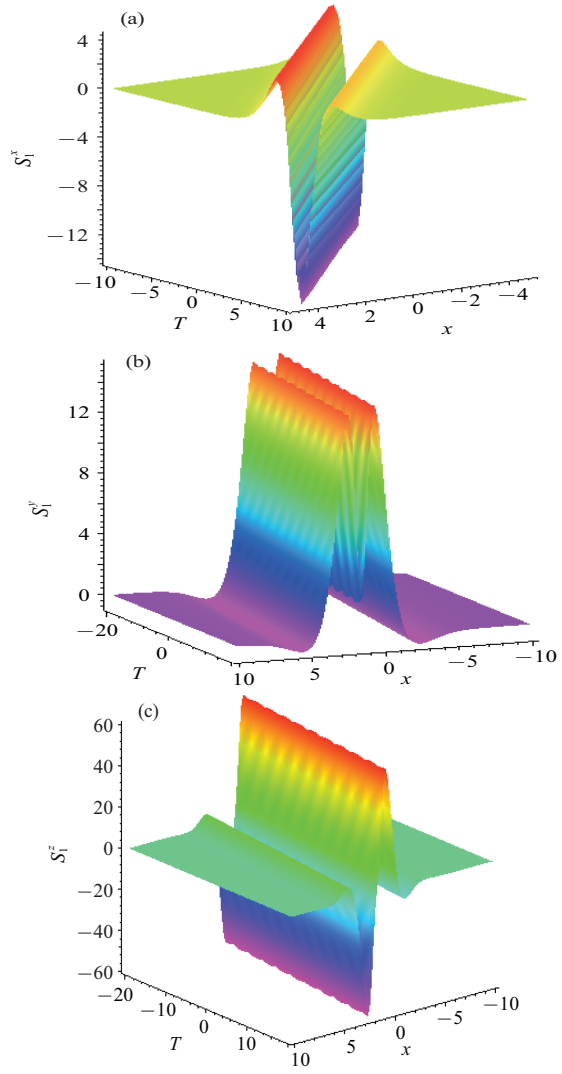


Fig. 2. (color online) Evolution of spin components (a) S_1^x, (b) S_1^y, and (c) S_1^z of the spin density function $\boldsymbol{S}$ with parameters $\eta = 1.2$, $\zeta = 0.5$, and $\lambda = 0.5$.

of electromagnetic wave in the form of soliton. This soliton travels without loss of energy and keeps its shape intact during its evolution along the spin lattice. Since the soliton is not bell-like, its structure is distorted at the tail (Figs. 2(a) and 2(c)) and the central localized region (Fig. 2(b)), it ensures that the participation of spins in the lattice are more due to the perturbation. From Eq. (43), we find that it is also possible to construct the magnetic field components of the EM wave through the relationships $B_1^y = \mu S_1^y$ and $B_1^z = \mu S_1^z$. Thus the EM-spin soliton and the EM-soliton travel without change in shape, indicating the possibility of lossless communication.

5. Conclusion

We have investigated the nature of nonlinear spin excitations in an anisotropic biquadratic ferromagnetic system when acted upon by an EM field. Using the reductive perturbation method, both the space and the time variables are stretched, and the magnetization of the medium and the magnetic field component of the EM field are expanded suitably and admit the higher order nonlinear Schrödinger equation that governs the nonlinear spin excitations in the medium. The nonlinear spin excitations under the influence of an EM wave is understood through a multi-scale perturbation analysis. The results of the perturbation analysis show that the soliton travels with constant velocity and amplitude along the spin lattice. The perturbation analysis also confirms the existence of a perturbed soliton. In addition, we present some simple localized (soliton) excitations of the components of the spin density function.

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