Generation of transient chaos from two-dimensional systems via a switching approach*

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A new approach of generating transient chaos from two-dimensional (2D) continuous autonomous systems within finite time is presented. Based on an absolute-value switching law, the phenomenon of transient chaos takes place by switching between three 2D systems. Basic dynamic behavior of the systems is investigated. Numerical examples illustrate the validity of the results.

Keywords: two-dimensional systems, chaos generation, switching control, transient chaos

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1. Introduction

It is well known that chaos can be utilized and controlled. Many researchers have been exploiting chaotic dynamics including the elimination and generation of chaos, engineering applications of chaos, the discovery of new chaos, and some important results have been obtained in recent years. For example, a family of switched unified chaotic systems are generated by switching between nonlinear functions of six subsystems.[1] The problem of feedback control of a delayed dynamical system is solved.[2] A novel four-dimensional autonomous hyperchaotic system is presented and investigated by means of a continuous spectrum.[3] Based on state feedback, a four-dimensional hyperchaotic system is constructed.[4] A new three-dimensional (3D) quadratic autonomous chaotic system which contains five system parameters and three quadratic cross-product terms is reported and a single four-wing chaotic attractor with wide parameter ranges is generated.[5] Two different sliding mode controllers are designed to stabilize a fractional order unified chaotic system.[6] A novel four-dimensional autonomous chaotic system is presented and can generate one-, two-, three-, and four-wing chaotic attractors with the variation of a single parameter.[7] A novel neural network based on a diagonal recurrent neural network and chaos is constructed.[8]

Chaos can be applied to many engineering fields such as communications, signal processing, nonlinear circuits design, etc. So much attention has been focused on generating chaos from analyzing and controlling chaos in the past. The problems of generation of chaos or hyperchaos have been studied.[9–12] The appearance of transient chaos in many fields has also been considered.[13–15]

In contrast to chaos, transient chaos can be applied mainly to optimize neural networks. A neural network model with transient chaos, or a transiently chaotic neural network (TCNN) as an approximation method for combinatorial optimization problems has been put forward.[16] The proposed neural network has richer and more flexible dynamics and can be expected to have a stronger ability in searching for globally optimal or near-optimal solutions in Ref. [16]. A class of simple chaotic Hopfield neural networks is presented and a bifurcation from transient chaos to chaos is discussed.[17]

For some three-dimensional or higher-dimensional continuous systems, chaos can be generated by switching between piecewise-linear controllers. In 2002, Lü *et al.*[18] designed a switching piecewise-linear controller, which can generate chaos from a three-dimensional linear autonomous system within a wide range of parameter values. In 2004, based on the presented method in Ref. [18], Zheng *et al.*[19] improved a piecewise-linear controller to generate two symmetrical chaotic attractors simultaneously via a switching controller. Liu *et al.*[20] developed a new switching method to generate chaos or chaos-like dynamics from two or more three-dimensional linear autonomous systems. By using a switching piecewise-linear controller, some new chaotic attractors with two or three merged basins of attraction are generated from a three-dimensional linear autonomous system.[21] Guo *et al.*[22] used a hysteretical switching method, by which novel chaotic funnel-shaped attractors are generated from a simple three-dimensional system. Tavazori and Haeri[23] presented a system with switching fractional multi-model structure which can generate chaos.

Multi-scroll chaotic attractors can also be generated via a switching approach. Lü *et al.*[24,25] presented a hysteresis series switching approach for generating multi-scroll or multi-directional chaotic attractors from a three-dimensional linear autonomous system. A saturated function series approach for generating multidirectional multi-scroll chaotic attractors from fractional differential systems via switching control was

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designed in Ref. [26]. A hysteresis series switching approach for generating multi-directional multi-scroll chaotic attractors from a three-dimensional linear autonomous fractional differential system was put forward in Ref. [27]. In 2008, Lü *et al.*[28] introduced a novel thresholding approach for creating multi-scroll chaotic attractors via switching control.

Based on the chaos generation theory of multi-scroll chaotic attractors, chaotic circuits can be well designed and implemented. In 2010, Zhang and Yu[29] developed a novel method for generating grid multi-scroll chaotic attractors via switching piecewise-linear controller and designed chaotic circuits. A new switched fourscroll hyperchaotic system with two subsystems and a practical circuit were designed.[30] In 2011, a novel approach for generating various grid multi-wing butterfly chaotic attractors from the multi-PWL Lü system by constructing a heteroclinic loop and switching control was developed.[31]

It is well known that chaos cannot be generated from a single two-dimensional (2D) continuous system. However, transient chaos can be generated by switching several 2D continuous systems. In this paper, a transient chaotic attractor is generated by switching three 2D continuous autonomous systems. An absolute-value switching law is designed to guarantee that the trajectories of the systems enter a transient chaotic region within finite time. The parameters of the designed switching law or the systems are selected with a wide range. The new switching approach with adjustable parameters can generate a lot of new transient chaos. Basic dynamic behaviors of the 2D transient chaotic systems are analyzed in this paper.

A traditional switching approach of 3D continuous systems can generate chaos by changing the nonlinear function of systems, or constructing a switching surface, or utilizing piecewise-linear controllers. In contrast to the conventional switching approach, a novel switching approach of 2D continuous systems in this paper can generate transient chaos by constructing a closed planar region with particular shape. An absolute-value switching law is employed to generate transient chaos from three 2D continuous autonomous systems with particular structures. Finally, numerical examples verify the effectiveness of the presented approach.

2. Design of 2D systems

Consider the following switched system with three 2D subsystems:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}_{\sigma}(\boldsymbol{x}), \tag{1}$$

where $\boldsymbol{x} = [x_1\ x_2]^{\mathrm{T}} \in \boldsymbol{R}^2$ is a state vector, $\boldsymbol{f}_{\sigma}(\boldsymbol{x}) \in \boldsymbol{R}^2$ is a vector field, σ is a switching signal that activates a subsystem at some instant, and $\sigma = i \in S$ ($S = \{1,2,3\}$).

Three 2D subsystems are expressed as follows:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}_1(\boldsymbol{x}) = \boldsymbol{A}_1\boldsymbol{x}, \tag{2}$$

$$\dot{\boldsymbol{x}} = \boldsymbol{f}_2(\boldsymbol{x}) = \boldsymbol{A}_2\boldsymbol{x}, \tag{3}$$

$$\dot{\boldsymbol{x}} = \boldsymbol{f}_3(\boldsymbol{x}) = \boldsymbol{g}(\boldsymbol{x}), \tag{4}$$

where

$$\boldsymbol{A}_1 = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix},\ \boldsymbol{A}_2 = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix},\ \boldsymbol{g}(\boldsymbol{x}) = \begin{bmatrix} 0 \\ kx_1^2 \end{bmatrix} \text{ or } \begin{bmatrix} kx_2^2 \\ 0 \end{bmatrix}.$$

The parameters of the switched system (1) satisfy the following conditions:

1) $\lambda(\boldsymbol{A}_1) = \{\lambda_1, \lambda_2\}$, $(\exists \lambda_i, \mathrm{Re}(\lambda_i) > 0, i \in \{1,2\})$;

2) $\lambda(\boldsymbol{A}_2) = \{a + \mathrm{j}b, a - \mathrm{j}b\}$, $(a < 0, b > 0, \mathrm{j}^2 = -1)$;

3) $k > 0$.

The above conditions imply that $\boldsymbol{A}_1$ is not Hurwitz stable, and $\boldsymbol{A}_2$ is Hurwitz stable.

3. Design of an absolute-value switching law

An absolute-value switching law of system (1) is designed as follows:

$$\sigma = i = \begin{cases} 1, & (m_1|x_1| + n_1|x_2| < 1), \\ 2, & (m_2|x_1| + n_2|x_2| > 1), \\ 3, & \text{otherwise}, \end{cases} \tag{5}$$

where

$$m_i > 0, \quad n_i > 0, \quad (i = 1,2).$$

Let

$$P = \{(x_1, x_2) | m_1|x_1| + n_1|x_2| < 1\}, \tag{6}$$

$$Q = \{(x_1, x_2) | m_2|x_1| + n_2|x_2| > 1\}, \tag{7}$$

and P and Q satisfy the following condition:

$$P \subset Q^{\mathrm{C}}. \tag{8}$$

Here, Q^{C} denotes the complementary set of Q.

The expression (5) is equivalent to the following expression:

$$\sigma = i = \begin{cases} 1, & (x \in P), \\ 2, & (x \in Q), \\ 3, & (x \in Q^{\mathrm{C}} - P), \end{cases} \tag{9}$$

where

$$R^2 = Q \oplus Q^{\mathrm{C}}, \tag{10}$$

and $Q \oplus Q^{\mathrm{C}}$ denotes the direct sum of Q and Q^{C}.

4. Analysis of basic dynamical behaviors

The equilibrium (0,0) is considered first. The equilibrium falls in region P. When the trajectories of system (1) enter region P, subsystem (2) is active. Because subsystem (2) is unstable, the trajectories will go though region P and enter region $Q^{\mathrm{C}} - P$, and subsystem (4) is active. We make some discussion below according to the equation of subsystem (4).

Case 1 When

$$\boldsymbol{g}(\boldsymbol{x}) = [\,0\ kx_1^2\,]^{\mathrm{T}}, \quad \dot{x}_1 = 0, \quad \dot{x}_2 = kx_1^2 > 0,$$

the trajectory of subsystem (4) will be vertical and upward.

Case 2 When

$$g(x) = [\, kx_2^2 \;\; 0 \,]^{\mathrm{T}}, \quad \dot{x}_1 = kx_2^2 > 0, \quad \dot{x}_2 = 0,$$

the trajectory subsystem (4) will be horizontal and forward.

When the trajectories enter region Q, subsystem (3) is active. Because subsystem (3) is stable and the trajectories are spiral in the clockwise direction, the trajectories will go back to $Q^{\mathrm{C}} - P$, and subsystem (4) is active again. The trajectories will be vertical and upward or will be horizontal and forward again, and enter region Q again. Subsystem (3) is active again. The trajectories are spiral in the clockwise direction, and will return to region $Q^{\mathrm{C}} - P$ again. That is, system (1) exhibits folding and stretching dynamics in regions Q and $Q^{\mathrm{C}} - P$ repeatedly, leading to complex dynamics. Therefore, transient chaos can be generated from 2D continuous autonomous systems via switching control under some certain conditions within a finite time.

Remark 1 Transient chaos cannot be generated from system (1) by the above absolute-value switching law (5) under some particular parameters.

5. Numerical examples

Example 1 Consider the switched system (1) in the form

$$\boldsymbol{A}_1 = \begin{bmatrix} 3 & -10 \\ 10 & 2 \end{bmatrix}, \; \boldsymbol{A}_2 = \begin{bmatrix} -0.2 & 10 \\ -10 & -0.1 \end{bmatrix}, \; \boldsymbol{g}(\boldsymbol{x}) = \begin{bmatrix} 0 \\ 20x_1^2 \end{bmatrix}.$$

The parameters are defined as

$$a_1 = 3, \; b_1 = -10, \; c_1 = 10, \; d_1 = 2,$$
$$\lambda(\boldsymbol{A}_1) = \{2.5 + 9.9875\mathrm{j}, \; 2.5 - 9.9875\mathrm{j}\},$$
$$a_2 = -0.2, \; b_2 = 10, \; c_2 = -10, \; d_2 = -0.1,$$
$$\lambda(\boldsymbol{A}_2) = \{-0.15 + 9.9999\mathrm{j}, \; -0.15 - 9.9999\mathrm{j}\},$$
$$k = 20 > 0.$$

Absolute-value switching law is given by

$$\sigma = i = \begin{cases} 1, & (10|x_1| + 0.5|x_2| < 1), \\ 2, & (5|x_1| + 0.5|x_2| > 1), \\ 3, & \text{otherwise}, \end{cases} \tag{11}$$

where $m_1 = 10$, $n_1 = 0.5$, $m_2 = 5$, $n_2 = 0.5$, and $m_i > n_i > 0$ $(i = 1, 2)$.

Let

$$P = \{(x_1, x_2) | 10|x_1| + 0.5|x_2| < 1\},$$
$$Q = \{(x_1, x_2) | 5|x_1| + 0.5|x_2| > 1\},$$

and P and Q satisfy the following condition:

$$P \subset Q^{\mathrm{C}}.$$

The expression (11) is equivalent to the following expression:

$$\sigma = i = \begin{cases} 1, & (x \in P), \\ 2, & (x \in Q), \\ 3, & (x \in Q^{\mathrm{C}} - P). \end{cases} \tag{12}$$

Figure 1 shows the distribution of switching regions.

Take the initial condition as

$$\boldsymbol{x}_0 = [\, 0.01 \;\; 0.01 \,]^{\mathrm{T}},$$

transient chaos can be generated under the absolute-value switching law (11) (see Figs. 2–4).

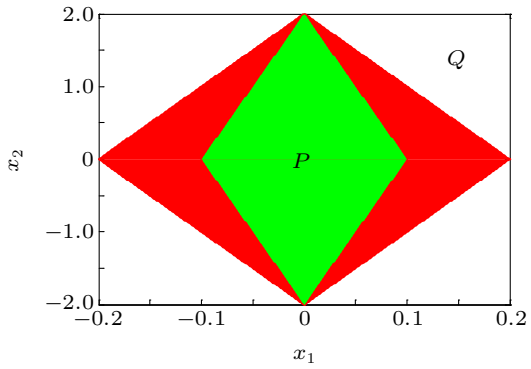


Fig. 1. (color online) Distribution of switching regions.

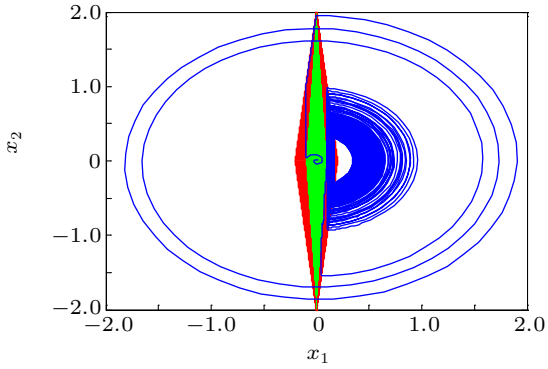


Fig. 2. (color online) A new single-scroll chaotic attractor.

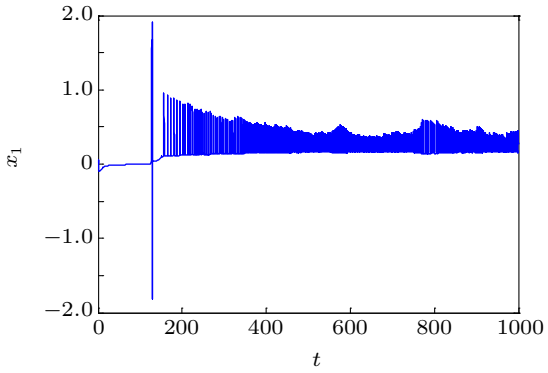


Fig. 3. (color online) Response curve of state x_1.

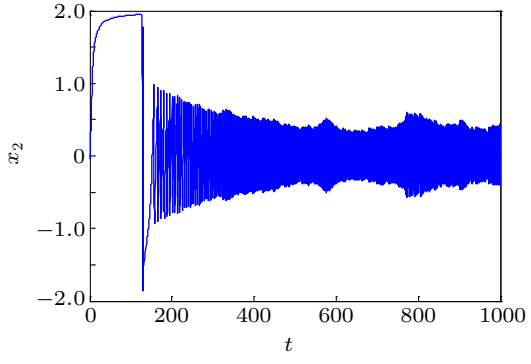


Fig. 4. (color online) Response curve of state x_2.

Example 2 Consider the switched system (1) in the form

$$\boldsymbol{A}_1 = \begin{bmatrix} 3 & -10 \\ 10 & 2 \end{bmatrix}, \ \boldsymbol{A}_2 = \begin{bmatrix} -0.2 & 10 \\ -10 & -0.1 \end{bmatrix}, \ \boldsymbol{g}(\boldsymbol{x}) = \begin{bmatrix} 20x_2^2 \\ 0 \end{bmatrix}.$$

The parameters are defined as

$$a_1 = 3,\ b_1 = -10,\ c_1 = 10,\ d_1 = 2,$$
$$\lambda(\boldsymbol{A}_1) = \{2.5 + 9.9875\mathrm{j},\ 2.5 - 9.9875\mathrm{j}\},$$
$$a_2 = -0.2,\ b_2 = 10,\ c_2 = -10,\ d_2 = -0.1,$$
$$\lambda(\boldsymbol{A}_2) = \{-0.15 + 9.9999\mathrm{j},\ -0.15 - 9.9999\mathrm{j}\},$$
$$k = 20 > 0.$$

The absolute-value switching law is given by

$$\sigma = i = \begin{cases} 1, & (0.5|x_1| + 10|x_2| < 1), \\ 2, & (0.5|x_1| + 5|x_2| > 1), \\ 3, & \text{otherwise}, \end{cases} \tag{13}$$

where $m_1 = 0.5$, $n_1 = 10$, $m_2 = 0.5$, $n_2 = 5$, and $n_i > m_i > 0$ $(i = 1, 2)$.

Let

$$P = \{(x_1, x_2) | 0.5|x_1| + 10|x_2| < 1\},$$
$$Q = \{(x_1, x_2) | 0.5|x_1| + 5|x_2| > 1\},$$

and P and Q satisfy the following condition:

$$P \subset Q^{\mathrm{C}}.$$

The expression (13) is equivalent to the following expression:

$$\sigma = i = \begin{cases} 1, & (x \in P), \\ 2, & (x \in Q), \\ 3, & (x \in Q^{\mathrm{C}} - P). \end{cases} \tag{14}$$

Figure 5 shows the distribution of the switching regions.

Take the initial condition as

$$\boldsymbol{x}_0 = [\,0.01 \ \ 0.01\,]^{\mathrm{T}},$$

transient chaos can be generated under the absolute-value switching law (13) (see Figs. 6–8).

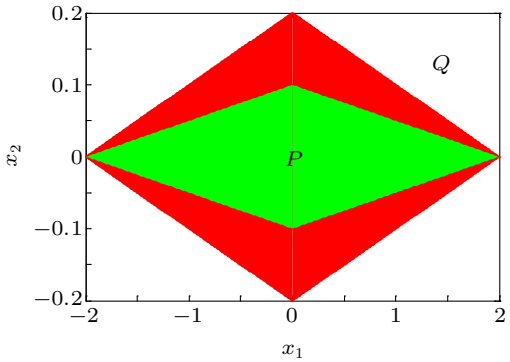


Fig. 5. (color online) Distribution of switching regions.

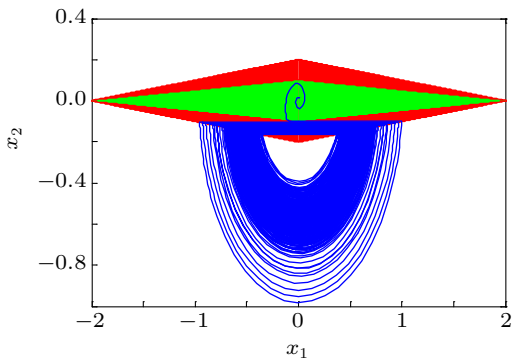


Fig. 6. (color online) A new single-scroll chaotic attractor.

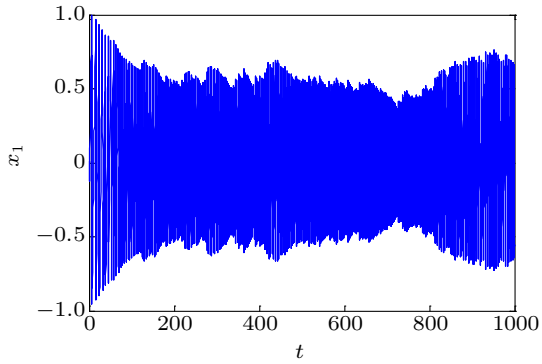


Fig. 7. (color online) Response curve of state x_1.

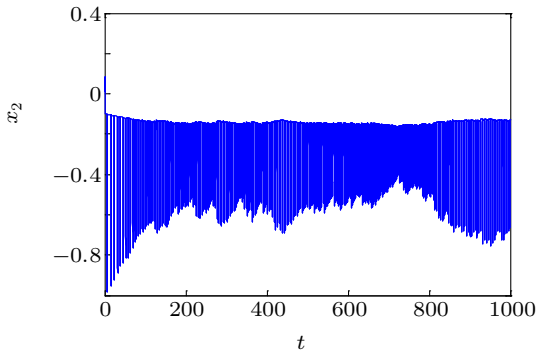


Fig. 8. (color online) Response curve of state x_2.

6. Conclusions

A new switching approach of generating transient chaos from 2D systems is firstly put forward in this paper. Three 2D systems and the absolute-value switching law are designed. The switching law and 2D systems are within a wide range of parameters. Finally, numerical examples illustrate the feasibility and validity of the presented novel approach.

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