Transfer function modeling and analysis of the open-loop Buck converter using the fractional calculus*

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Based on the fact that the real inductor and the real capacitor are fractional order in nature and the fractional calculus, the transfer function modeling and analysis of the open-loop Buck converter in a continuous conduction mode (CCM) operation are carried out in this paper. The fractional order small signal model and the corresponding equivalent circuit of the open-loop Buck converter in a CCM operation are presented. The transfer functions from the input voltage to the output voltage, from the input voltage to the inductor current, from the duty cycle to the output voltage, from the duty cycle to the inductor current, and the output impedance of the open-loop Buck converter in CCM operation are derived, and their bode diagrams and step responses are calculated, respectively. It is found that all the derived fractional order transfer functions of the system are influenced by the fractional orders of the inductor and the capacitor. Finally, the realization of the fractional order inductor and the fractional order capacitor is designed, and the corresponding PSIM circuit simulation results of the open-loop Buck converter in CCM operation are given to confirm the correctness of the derivations and the theoretical analysis.

Keywords: Buck converter, small signal equivalent circuit model, fractional calculus, transfer function

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1. Introduction

As a basic topology of the DC–DC converter, the Buck converter, whose characteristics are very different from the Boost converter since it serves to convert a direct-current (DC) input voltage to a lower DC output voltage, has been widely used in engineering applications, such as computer engineering and aerospace engineering.[1] As is well known, the modeling and analysis of the Buck converter is an important step for designing this converter to satisfy the real requirements, i.e., the model's precision has a vital influence on the performance of the final design. Therefore many researchers have made an effort to establish an appropriate model and explore the corresponding analysis method. Up to now, a few good models have been proposed to describe the Buck converter, and a few good analysis methods have been explored to investigate the dynamic behavior.[1–8] For example, under the assumption that the switching frequency is much higher than its characteristic frequency, the averaged model, which is used to derive the small signal model, can be obtained by averaging the circuit variable within each switching cycle to describe the dynamic behavior of the Buck converter in the low frequency region.[1–4] Additionally, since the dynamic behavior of the Buck converter in the high frequency region can not be analyzed using its averaged model, the discrete model, which can be used to analyze the whole dynamic behavior of the Buck converter, has been proposed. The main idea of this model is that it solves the differential equations in each operational state of the Buck converter and then samples and collects the circuit variables at a certain instant. The discrete model is usually used to identify the nonlinear phenomenon or to simulate the Buck converter precisely.[5–8] All the above research results are good guidance to allow engineers to design a Buck converter for use in engineering applications. Unfortunately, all the above research results are obtained by assuming that both the real inductor and the real capacitor are integer order in nature and are described by their integer order models.[1–8]

In recent years, with the development in the integer calculus and its applications,[9–11] the fractional calculus and its applications have also attracted much attention, and many important and valuable works have been obtained.[12–24] In particular, the studies on the mathematical models of the real inductor and the real capacitor show that both of them are fractional order in nature and should be modeled by using the fractional calculus.[15–22] For example, Westerlund pointed out that the real inductor is fractional order in nature and established its fractional order model to describe electricity characteristics.[15] Westerlund *et al.* also measured the order of the fractional order capacitor under different dielectrics in experiments.[16] Jonscher pointed out that the integer order capacitor cannot exist in nature because its impedance form would violate the causality.[17] Petráš adopted the fractional

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order inductor and the fractional order capacitor to design a circuit to realize the fractional Chua's circuit successfully, and the circuit experimental results were in good agreement with the simulation results.[18] Particularly, Jesus *et al.* developed fractional order capacitors of different orders by choosing different fractal structures.[21] Tenreiro Machado and Galhano pointed out that the different order of fractional order inductor can be produced based on the skin effect.[22] Therefore, the real inductor and the real capacitor should be modeled by using the fractional calculus to describe their real electric characteristics. Here, a question naturally arises: why can the results from the integer order model agree with the circuit simulations or experiments? The main reason is that the fractional orders of the inductor and the capacitor in the market are very close to one, and accordingly one can use the integer order model to approximately describe their dynamic behavior. This approximate will bring the wrong results if the fractional orders of the inductor and the capacitor are a little away from one, such as 0.59 and 0.42 of the capacitor in Ref. [21] and the different order of the inductor in Ref. [22]. Thus, the open-loop Buck converter should be modeled by using fractional calculus. In this paper, based on fractional calculus, the transfer functions of the open-loop Buck converter, which are the basis for analyzing and designing the converter in engineering applications, are established and analyzed. The averaged model can be obtained by using the state-space averaging[2] or the circuit averaging technique.[4] Since the state-space averaging is tedious due to matrix algebra manipulations, the circuit averaging technique, whose advantages are its simplicity and clearer insight into the converter behavior, is applied to derive the transfer functions of the open-loop Buck converter operating in the continuous conduction mode (CCM) operation here.

The rest of the paper is organized as follows. In Section 2, the circuit operation and the fractional order small signal model of the open-loop Buck converter are presented. Then the transfer functions from the input voltage to the output voltage, from the input voltage to the inductor current, from the duty cycle to the output voltage, from the duty cycle to the inductor current, and the output impedance of the open-loop Buck converter in CCM operation are established and investigated respectively. In Section 6, the effectiveness of the established transfer functions and the corresponding theoretical analysis are confirmed by the PSIM circuit simulations. Finally, some concluding remarks are given in Section 7.

2. Circuit operation and its fractional order small signal model

The circuit schematic diagram of the open-loop Buck converter is shown in Fig. 1(a). It is obvious that it includes the fractional order inductor L_α and fractional order capacitor C_β. Subscript α denotes the order of the fractional order inductor, and subscript β denotes the order of the fractional order capacitor. According to Ref. [18] and the Laplace transform of the fractional calculus,[23] it is found that the fractional order inductor should be described by the fractional calculus, and the expressions about the relationship between its voltage v_L and current i_L in time and complex frequency domains are

$$\begin{cases} v_L = L_\alpha \dfrac{\mathrm{d}^\alpha i_L}{\mathrm{d}t^\alpha}, \\ v_L(s) = L_\alpha s^\alpha i_L(s). \end{cases} \tag{1}$$

The fractional order capacitor should be described by the fractional calculus, the expressions concerning the relationship between its voltage v_0 and its current i_C in time and complex frequency domains are

$$\begin{cases} i_C = C_\beta \dfrac{\mathrm{d}^\beta v_0}{\mathrm{d}t^\beta}, \\ i_C(s) = C_\beta s^\beta v_0(s). \end{cases} \tag{2}$$

The circuit of the open-loop Buck converter operates as follows. When there is a high level of P_w, which is the period signal to drive switch S, switch S is closed, and the diode does not conduct, this mode will go on within the time interval (0, dT), where d is the duty cycle, and T is the switching period. Then when there is a low level of P_w, switch S is open, and the diode is conducting, and this mode will go on within the time interval (dT, T). Since the CCM operation of the open-loop Buck converter is considered here, accordingly only the above two modes appear and the typical time domain waveforms are shown in Fig. 1(b).

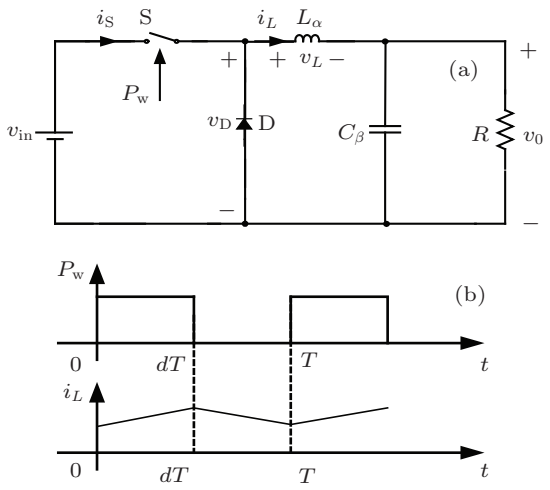


Fig. 1. Open-loop Buck converter and its typical waveform in CCM: (a) the circuit, (b) inductor current.

According to the circuit averaging technique,[4] switch S can be modeled as a current-dependent current source and

diode D can be modeled as a voltage-dependent voltage source in the CCM operation, i.e.,

$$\begin{cases} \langle i_{\rm S}\rangle = d\langle i_L\rangle, \\ \langle v_{\rm D}\rangle = d\langle v_{\rm in}\rangle, \end{cases} \tag{3}$$

where $\langle i_L\rangle$, $\langle i_{\rm S}\rangle$, $\langle v_0\rangle$, $\langle v_{\rm D}\rangle$, and $\langle v_{\rm in}\rangle$ are the averaged values of $i_{\rm L}$, $i_{\rm S}$, v_0, $v_{\rm D}$, and $v_{\rm in}$, respectively. Thus, the circuit averaged model of the Buck converter can be obtained and shown in Fig. 2.

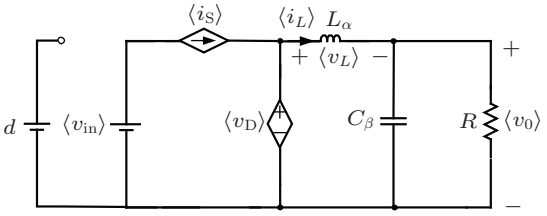


Fig. 2. Circuit averaged model of the open-loop Buck converter.

Assume that I_L, V_0, $V_{\rm in}$, and D are the DC values of $\langle i_L\rangle$, $\langle v_0\rangle$, $\langle v_{\rm in}\rangle$, and d, respectively, $\hat{i}_L$, $\hat{v}_0$, $\hat{v}_{\rm in}$, and $\hat{d}$ are the small AC variations of $\langle i_{\rm L}\rangle$, $\langle v_0\rangle$, $\langle v_{\rm in}\rangle$, and d, respectively. Then $\langle i_L\rangle$, $\langle v_0\rangle$, $\langle v_{\rm in}\rangle$, and d can be represented by the corresponding DC values plus the superimposed small AC variations with the assumption that the AC variations are small in magnitude compared to the steady state values, i.e.,

$$\begin{cases} \langle i_L\rangle = I_L + \hat{i}_L, \\ \langle v_0\rangle = V_0 + \hat{v}_0, \\ \langle v_{\rm in}\rangle = V_{\rm in} + \hat{v}_{\rm in}, \\ d = D + \hat{d}, \end{cases} \tag{4}$$

with

$$\begin{cases} \hat{i}_L \ll I_L, \\ \hat{v}_0 \ll V_0, \\ \hat{v}_{\rm in} \ll V_{\rm in}, \\ \hat{d} \ll D. \end{cases}$$

By taking Eq. (4) into Eq. (3) and omitting high order small signal terms, i.e., $\hat{i}_L\hat{d} \approx 0$ and $\hat{v}_0\hat{d} \approx 0$, the resulting switch and diode models become

$$\begin{cases} \langle i_{\rm S}\rangle = DI_L + \hat{d}I_L + D\hat{i}_L, \\ \langle v_{\rm D}\rangle = DV_{\rm in} + \hat{d}V_{\rm in} + D\hat{v}_{\rm in}. \end{cases} \tag{5}$$

Using Eqs. (4) and (5) in Fig. 2 and then detaching the DC part and the small variation part, we can obtain the DC equivalent circuit model and the small-signal equivalent circuit model in Figs. 3 and 4, respectively.

From Fig. 3, the DC value of the open-loop Buck converter can be easily calculated, i.e., $V_0 = V_{\rm in}D$ and $I_L = V_{\rm in}D/R$. It is obvious that there is no difference between the results of the DC value from the integer order model and the fractional order model.

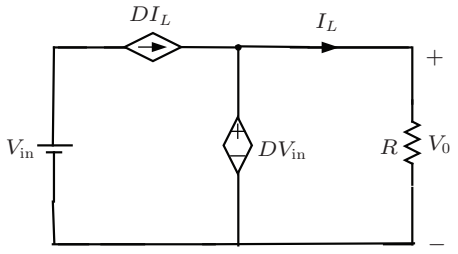


Fig. 3. The DC equivalent circuit model of the open-loop Buck converter.

From Fig. 4, the transfer functions from the input voltage to the output voltage, from the input voltage to the inductor current, from the duty cycle to the output voltage, from the duty cycle to the inductor current, and the output impedance of the open-loop Buck converter can be obtained, which are given in the following sections. The circuit parameters used in this paper are $v_{\rm in} = 24$ V, $C_\beta = 100$ μF, $L_\alpha = 3$ mH, $D = 0.6$, $R = 20$ Ω, $T = 40$ μs, $0 < \alpha < 1$, $0 < \beta < 1$.

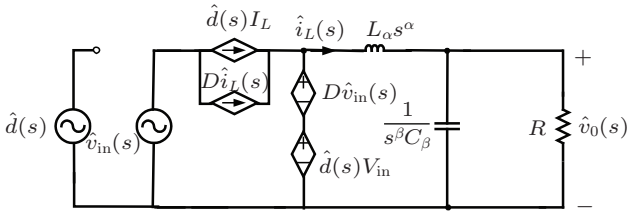


Fig. 4. The small-signal equivalent circuit model of the open-loop Buck converter.

3. Input voltage to output voltage transfer function and input voltage to inductor current transfer function

The transfer function from the input voltage to the output voltage of the open-loop Buck converter in CCM operation, or its open-loop audio susceptibility, is essential in determining the close-loop audio susceptibility in both current and voltage mode controls. The transfer function from the input voltage to the inductor current is important for determining the closed-loop audio-susceptibility in current model control. All the above transfer functions can be derived from the corresponding small-signal equivalent circuit model in Fig. 5, which is obtained by taking the perturbation of the duty cycle equal to zero in Fig. 4.

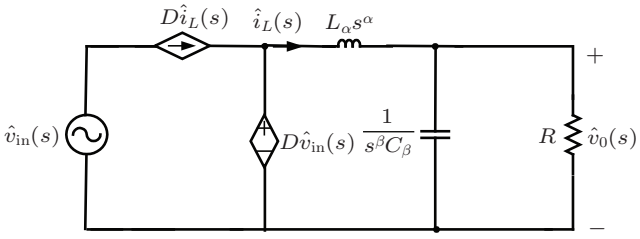


Fig. 5. The small-signal equivalent circuit model used to determine the input voltage to the output voltage transfer function and the input voltage to the inductor current transfer function.

Based on Kirchhoff's current and voltage laws, the following equations can be obtained from Fig. 5:

$$\begin{cases} L_\alpha s^\alpha \hat{i}_L(s) = D\hat{v}_{\rm in}(s) - \hat{v}_0(s), \\ C_\beta s^\beta \hat{v}_0(s) = \hat{i}_L(s) - \dfrac{\hat{v}_0(s)}{R}. \end{cases} \tag{6}$$

Since the transfer function from the input voltage to the output voltage expresses the effect of the input voltage changes on the output voltage when only the input voltage works, which implies that the perturbation of the duty cycle is equal to zero. Therefore,

$$G_{vv}(s) = \left.\frac{\hat{v}_0(s)}{\hat{v}_{\rm in}(s)}\right|_{\hat{d}(s)=0} = \frac{D}{L_\alpha C_\beta s^{\alpha+\beta} + L_\alpha s^\alpha / R + 1}. \tag{7}$$

The transfer function from the input voltage to the inductor current expresses the effect of the input voltage changes on the inductor current when only the input voltage works, which implies that the perturbation of the duty cycle is equal to zero. Thus,

$$G_{iv}(s) = \left.\frac{\hat{i}_L(s)}{\hat{v}_{\rm in}(s)}\right|_{\hat{d}(s)=0} = \frac{DC_\beta(s^\beta + 1/(C_\beta R))}{L_\alpha C_\beta s^{\alpha+\beta} + L_\alpha s^\alpha / R + 1}. \tag{8}$$

Obviously, compared with the transfer functions derived from the integer calculus,[2] these two transfer functions even cover the fractional orders of the inductor and the capacitor, and these two parameters will have an important influence on the dynamic behavior of these two transfer functions in both the frequency-domain and time-domain responses. For example, based on the definition and calculation of the bode diagram and step response for the fractional order system,[24] when α, $\beta = 0.8$ and α, $\beta = 1$, the theoretical calculation of the bode diagram and the corresponding step response about the above two transfer functions are shown in Figs. 6 and 7, respectively.

From Figs. 6 and 7, it is easy to observe that the results for α, $\beta = 0.8$ are very different from those for α, $\beta = 1$. For example, based on the step response of $G_{vv}(s)$, which is shown in Fig. 6(b), the rise time, the peak time, the settling time, and the percent overshoot in the case of α, $\beta = 0.8$ are much smaller than those in the case of α, $\beta = 1$. In other words, the dynamic performance of the Buck converter with the fractional order inductor and the fractional order capacitor are better than that with the integer order inductor and the integer order capacitor. This important point may be good guidance for designing the Buck converter for use in engineering applications in the future.

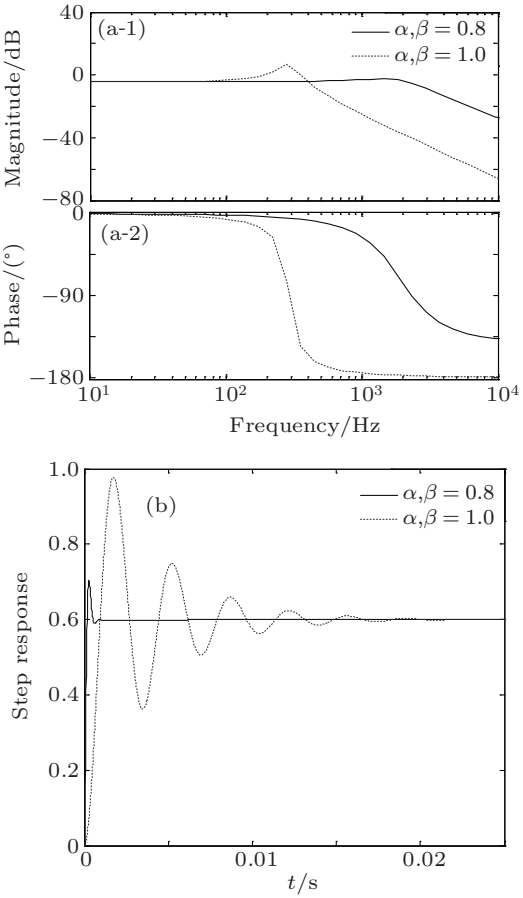


Fig. 6. (a) Bode diagram and (b) step response of $G_{vv}(s)$.

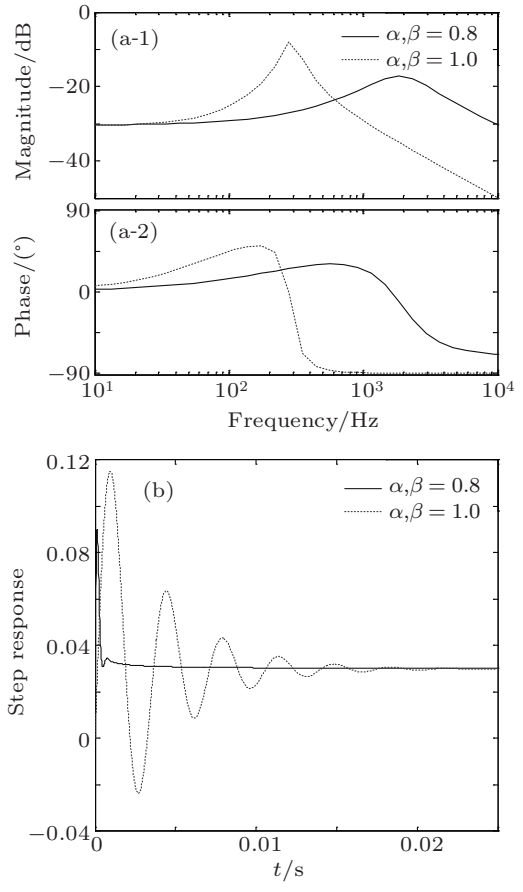


Fig. 7. (a) Bode diagram and (b) step response of $G_{iv}(s)$.

4. Duty cycle to output voltage transfer function and duty cycle to inductor current transfer function

The transfer function from the duty cycle to the output voltage of the open-loop Buck converter in CCM operation is essential in voltage mode control and vital for describing the voltage loop for current mode control. The transfer function from the duty cycle to the inductor current of the system is also important for describing the current loop for current mode control. These two transfer functions can be obtained from the small-signal equivalent circuit model under the condition that the perturbation of the input voltage is equal to zero in Fig. 4, which is shown in Fig. 8. Accordingly, based on the Kirchhoff's current and voltage laws, the following equations can be obtained:

$$\begin{cases} L_\alpha s^\alpha \hat{i}_L(s) = \hat{d}(s)V_{\text{in}} - \hat{v}_0(s), \\ C_\beta s^\beta \hat{v}_0(s) = \hat{i}_L(s) - \dfrac{\hat{v}_0(s)}{R}. \end{cases} \tag{9}$$

Thus, the transfer function from the duty cycle to the output voltage, which reflects the effect of the duty cycle change on the output voltage when the perturbation of input voltage is equal to zero, is

$$G_{vd}(s) = \left.\frac{\hat{v}_0(s)}{\hat{d}(s)}\right|_{\hat{v}_{\text{in}}(s)=0} = \frac{V_{\text{in}}}{L_\alpha C_\beta s^{\alpha+\beta} + L_\alpha s^\alpha / R + 1}. \tag{10}$$

The transfer function from the duty cycle to the inductor current, which reflects the effect of the duty cycle change on the inductor current when the perturbation of input voltage is equal to zero, is

$$G_{id}(s) = \left.\frac{\hat{i}_L(s)}{\hat{d}(s)}\right|_{\hat{v}_{\text{in}}(s)=0} = \frac{V_{\text{in}} C_\beta (s^\beta + 1/(C_\beta R))}{L_\alpha C_\beta s^{\alpha+\beta} + L_\alpha s^\alpha / R + 1}. \tag{11}$$

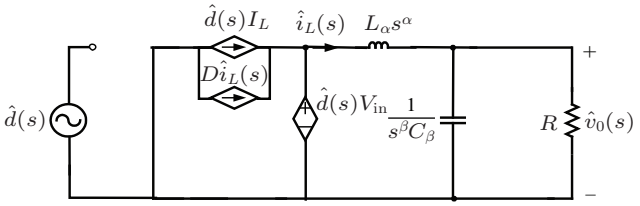


Fig. 8. The small-signal equivalent circuit model used to determine the duty cycle to the output voltage transfer function and the duty cycle to the inductor current transfer function.

Obviously, compared with the transfer functions from the duty cycle to the output voltage and the duty cycle to the inductor current of the open-loop Buck converter in CCM operation using the integer calculus, these two transfer functions also cover the fractional orders of the inductor and the capacitor.

Moreover, from the bode diagram and the step response of these two transfer functions, which are shown in Figs. 9 and 10, respectively, the same results as in Section 3 can be obtained.

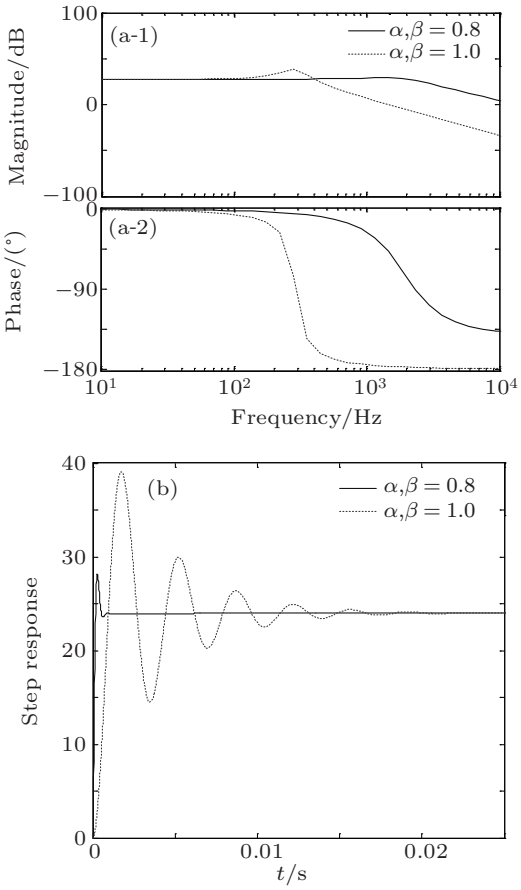


Fig. 9. (a) Bode diagram and (b) step response of $G_{vd}(s)$.

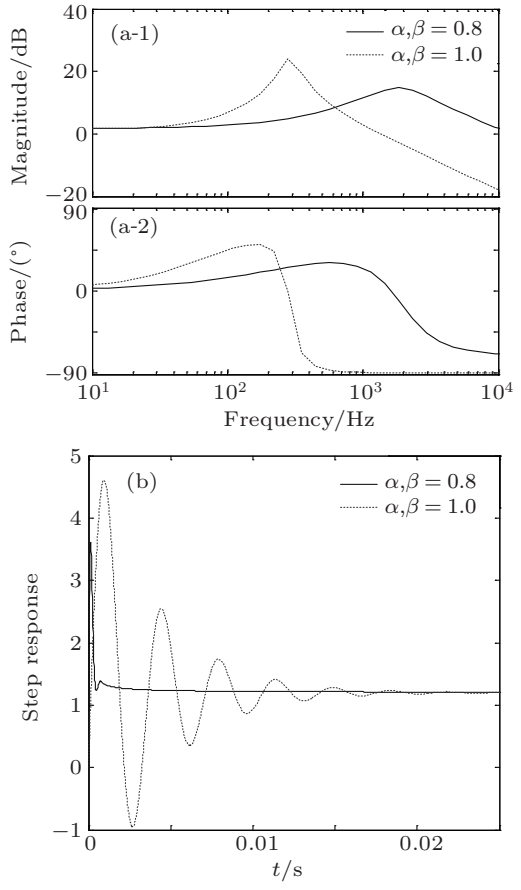


Fig. 10. (a) Bode diagram and (b) step response of $G_{id}(s)$.

5. Output impedance

The small signal equivalent circuit model for the output impedance is shown in Fig. 11, which is obtained taking both the perturbations of the duty cycle and the input voltage equal to zero and deriving the transfer functions from $\hat{i}(s)$ to $\hat{v}(s)$.

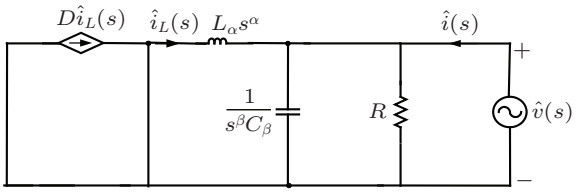


Fig. 11. The small-signal equivalent circuit model used to determine the output impedance.

From Fig. 11, based on Kirchhoff's current and voltage laws, the following equations can be obtained:

$$\begin{cases} L_\alpha s^\alpha \hat{i}_L(s) = -\hat{v}(s), \\ C_\beta s^\beta \hat{v}(s) = \hat{i}_L(s) + \hat{i}(s) - \dfrac{\hat{v}(s)}{R}. \end{cases} \tag{12}$$

Accordingly, the output impedance can be derived as

$$Z_{\text{out}}(s) = \frac{\hat{v}(s)}{\hat{i}(s)}\bigg|_{\substack{\hat{v}_{\text{in}}(s)=0 \\ \hat{d}(s)=0}} = \frac{L_\alpha s^\alpha}{L_\alpha C_\beta s^{\alpha+\beta} + L_\alpha s^\alpha / R + 1}. \tag{13}$$

Obviously, results like those in Sections 3 and 4 can also be obtained from the bode diagram and the step response of $Z_{\text{out}}(s)$, which are shown in Figs. 12(a) and 12(b), respectively.

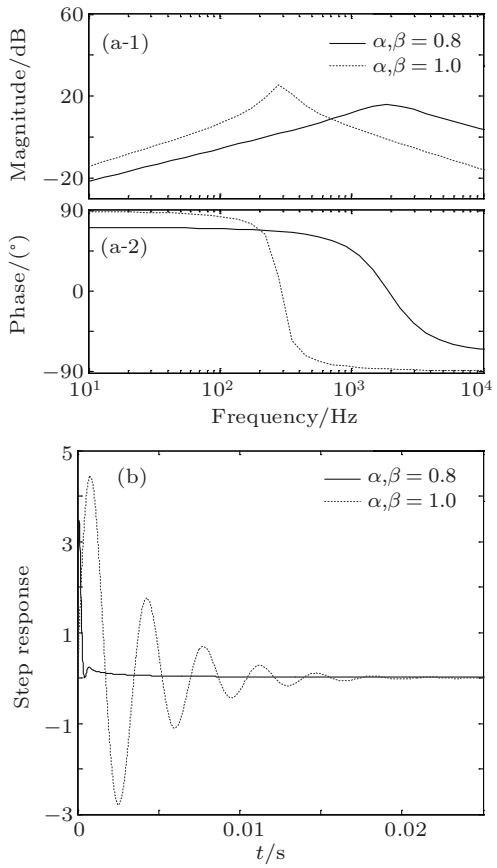


Fig. 12. (a) Bode diagram and (b) step response of $Z_{\text{out}}(s)$.

6. PSIM simulation

The power electronics simulator software PSIM, which is made to simulate power electronic converters and motor drives, has been widely used to simulate the power converter.[25,26] Here, this software is applied to simulate the Buck converter with the fractional order inductor and the fractional order capacitor to confirm the theoretical analysis in the previous sections. Note that the fractional order inductor and the fractional order capacitor could not be bought in the market until now. Also, the simulation elements of the fractional order inductor and the fractional order capacitor can not be found in the PSIM software. Fortunately, based on the approximate method to calculate the fractional calculus, such as the chain fractance[23] and Oustaloup's approximation,[27] the dynamic behavior of the fractional order inductor and the fractional order capacitor can be approximately described.

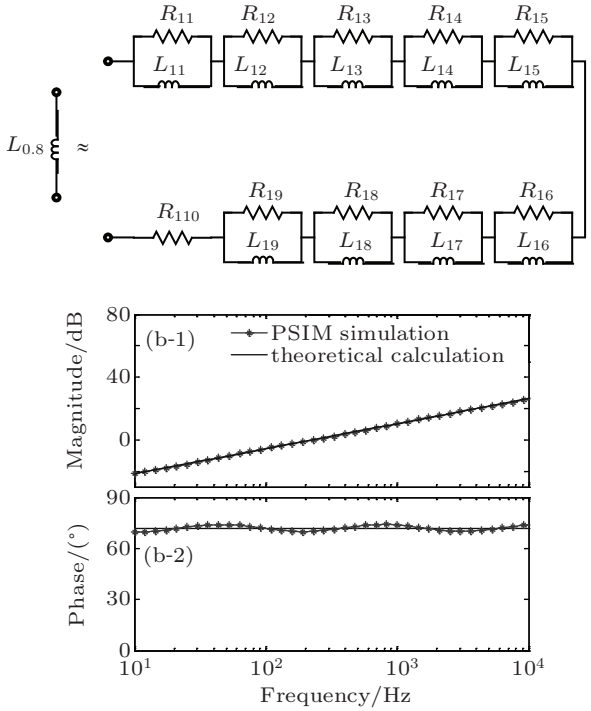


Fig. 13. (a) Fractional order inductor and its approximated model, (b) bode diagram of $L_{0.8}s^{0.8}$. Here, $R_{11} = 7.16$ kΩ, $R_{12} = 340.84$ Ω, $R_{13} = 34.25$ Ω, $R_{14} = 3.54$ Ω, $R_{15} = 367$ mΩ, $R_{16} = 38$ mΩ, $R_{17} = 4$ mΩ, $R_{18} = 0.4$ mΩ, $R_{19} = 42$ μΩ, $R_{110} = 5$ μΩ, $L_{11} = 95$ μH, $L_{12} = 77$ μH, $L_{13} = 131.6$ μH, $L_{14} = 231.6$ μH, $L_{15} = 408$ μH, $L_{16} = 719.4$ μH, $L_{17} = 1.268$ mH, $L_{18} = 2.235$ mH, $L_{19} = 3.934$ mH.

For example, when $L_\alpha = 3$ mH, $\alpha = 0.8$, the approximate model of the fractional order inductor can be obtained by using the chain fractance, which is shown in Fig. 13. The PSIM simulation results about the bode diagram of the approximated model are in good agreement with the corresponding theoretical calculation results. For $C_\beta = 100$ μF, $\alpha = 0.8$, the chain fractance is shown in Fig. 14, the PSIM simulation results concerning the bode diagram of the approximated model

are also in good agreement with the corresponding theoretical calculation. Therefore, the conclusion can be drawn that it is effective to use the corresponding approximated models of the fractional order inductor and the fractional order capacitor to describe their dynamic behavior. Consequently, the Buck converter with the fractional order inductor and the fractional order capacitor can be simulated with the PSIM software.

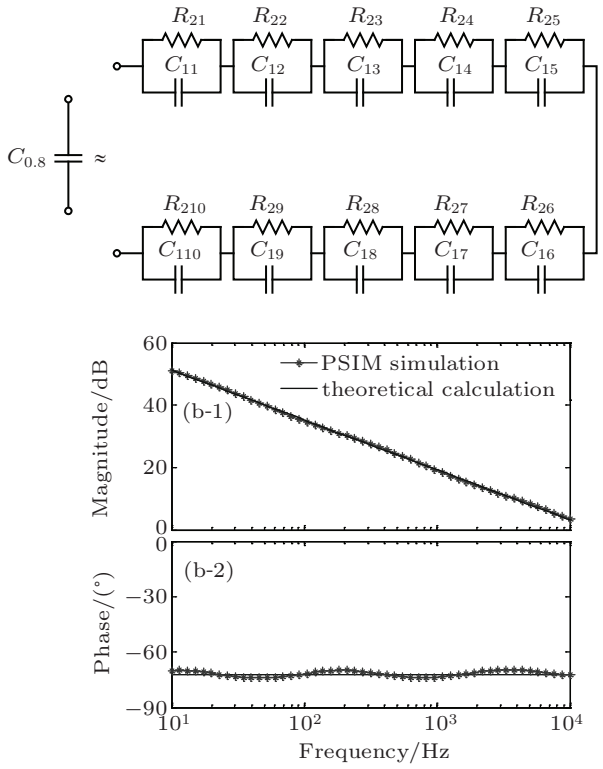


Fig. 14. (a) Fractional order capacitor and its approximated model, (b) bode diagram of $1/(C_{0.8}s^{0.8})$. Here, $R_{21} = 20$ mΩ, $R_{22} = 160$ mΩ, $R_{23} = 1.5$ Ω, $R_{24} = 14.6$ Ω, $R_{25} = 141$ Ω, $R_{26} = 1.36$ kΩ, $R_{27} = 13.131$ kΩ, $R_{28} = 126.742$ kΩ, $R_{29} = 1.222$ MΩ, $R_{210} = 102.85$ MΩ, $C_{11} = 6.5$ μF, $C_{12} = 13.98$ μF, $C_{13} = 24.5$ μF, $C_{14} = 43.2$ μF, $C_{15} = 76.2$ μF, $C_{16} = 134.2$ μF, $C_{17} = 236.6$ μF, $C_{18} = 417$ μF, $C_{19} = 736$ μF, $C_{110} = 560$ μF.

Here, under $\alpha, \beta = 0.8$ and $\alpha, \beta = 1$, only the bode diagram of $G_{vv}(s)$ and the typical time domain waveforms of the output voltage from the PSIM simulation shown in Figs. 15 and 16 are given to confirm the effectiveness of the corresponding derived transfer functions in the previous sections. Note that based on the characteristic of the PSIM, the bode diagram of $G_{vv}(s)$ is directly obtained from the switching model of the Buck converter but not from its averaged model or small signal model. At 0.04 s, the input voltage is changed from 24 V to 25 V. Obviously, the results from the PSIM are in good agreement with the theoretical calculations.

From Figs. 15 and 16, it is also found that there is an important difference between the integer order model and the fractional order model, and accordingly the Buck converter with the fractional order inductor and the fractional order capacitor should be modeled using the fractional calculus to describe their real dynamic behavior.

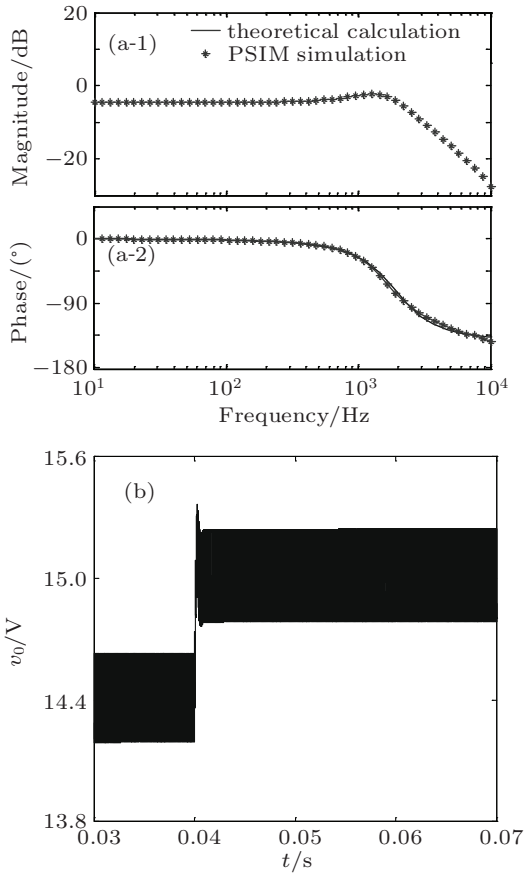


Fig. 15. (a) Bode diagram and (b) step response of $G_{vv}(s)$ with $\alpha, \beta = 0.8$.

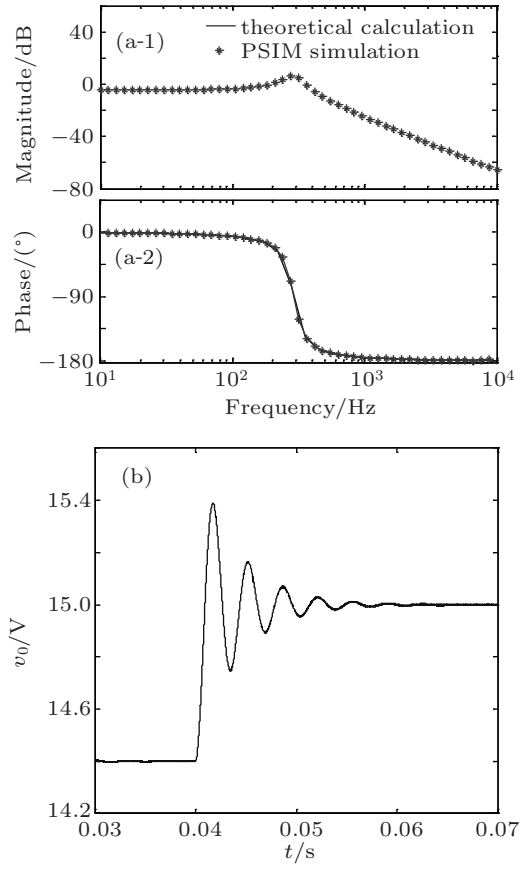


Fig. 16. (a) Bode diagram and (b) step response of $G_{vv}(s)$ with $\alpha, \beta = 1$.

7. Conclusion

Based on the established transfer functions of the open-loop Buck converter in a CCM operation, it is found that all the derived transfer functions cover the orders of the fractional order inductor and the fractional order capacitor. Furthermore, the results for the bode diagrams and the step responses of the derived transfer functions from the theoretical calculation and the PSIM circuit simulations show that both the orders of the fractional order inductor and the fractional order capacitor have an important influence on the dynamic behavior of the open-loop Buck converter in a CCM operation, such as the rise time, the peak time, the settling time, and the percent overshoot. Therefore, if we directly establish the integer order transfer functions instead of the fractional order transfer functions to describe the open-loop Buck converter in a CCM operation with the fractional order inductor and the fractional order capacitor, the obtained results will be wrong. In other words, the fractional order transfer functions of the open-loop Buck converter in a CCM operation with the fractional order inductor and the fractional order capacitor must be established to describe its real dynamic behavior.

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