Thermodynamic properties of Reissner–Nordström–de Sitter quintessence black holes*

Wei Yi-Huan(魏益焕)[a)†] and Ren Jun(任 军)[b)]

[a)] Department of Physics, Bohai University, Jinzhou 121000, China
[b)] School of Science, Hebei University of Technology, Tianjin 300130, China

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The mass, electric charge, and temperature of black holes in the Reissner–Nordström–de Sitter quintessence (RN–dSQ) spacetime are obtained. The heat capacities of the RN–dSQ black holes for certain electric charges and masses are analyzed. The electrostatic energy and dark energy in the RN–dSQ black holes are also calculated.

Keywords: Reissner–Nordström–de Sitter quintessence (RN–dSQ) black hole, heat capacity, electrostatic energy, dark energy

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1. Introduction

Static, spherically symmetric spacetime filled with matter, an electrostatic field, and quintessence can have black-hole horizons (BHHs) and a cosmological horizons (CH).[1,2] The properties of black holes surrounded by quintessence have been generally studied.[3–14] Recently however, an investigation into the absorption of a massless scalar wave from a Schwarzschild black hole surrounded by quintessence has been carried out.[15] The circular orbits of the photons in the Schwarzschild black hole surrounded by quintessence were also studied.[16] All these studies involve the properties of dark energy.

The quintessence energy (a candidate for dark energy) considered in this paper has the negative pressure $p_{\rm Q} = -(2/3)\rho_{\rm Q}$, with $p_{\rm Q}$ being the energy density. The dark energy should permeate in the whole space and can also exist in a black hole. Therefore, in the title of this paper, the phrase 'surrounded by quintessence' has been replaced by simply 'quintessence'. The authors of Ref. [17] showed the spherical spacetime solutions with parameter n. A more general form of the polynomial solutions of the spherically symmetric spacetime is given in Refs. [1] and [18], where some of the thermodynamic properties of a black hole surrounded by quintessence have been discussed. As is well known, the de Sitter solution has played a significant role in the study of the thermodynamics of black holes.[19–21] One can consider the spacetime, including the quintessence energy (the dark energy) or the vacuum energy, coming from the cosmological constant (the dark energy). So far, the properties of dark energy have not been known very well. In fact, the range of the dark energy equation of state has not been precisely determined. The dark energy may exist in a single energy form or be the mixture of several different energy forms. Thus, it would be helpful to have a further understanding of the properties of dark energy to study the thermodynamic properties of spacetime filled with quintessence energy and vacuum energy.

In this paper, we study the thermal property and the energy of the Reissner–Nordström–de Sitter quintessence (RN–dSQ) spacetime. In Section 2, we calculate the mass, electric charge, and temperature of the RN–dSQ black hole, give the general expression for the heat capacity of the black hole, and discuss the behaviors of the heat capacity for a fixed electric charge and mass. In Section 3, we calculate the electrostatic energy and the dark energy in the RN–dSQ black hole.

2. Thermal properties of RN–dSQ spacetime

For the spherically symmetric spacetime described by the metric

$$\mathrm{d}s^2 = f\,\mathrm{d}t^2 - f^{-1}\,\mathrm{d}r^2 - r^2\,\mathrm{d}\Omega^2, \tag{1}$$

the energy–momentum tensor of the spacetime is given by

$$T_t^t = T_r^r = -\frac{1}{2r^2}\left(f + rf' - 1\right), \tag{2}$$

$$T_\theta^\theta = T_\varphi^\varphi = -\frac{1}{4r}\left(2f' + rf''\right), \tag{3}$$

where $f' = \mathrm{d}f/\mathrm{d}r$ and $f'' = \mathrm{d}^2 f/\mathrm{d}r^2$. In this paper, we consider the RN–dSQ spacetime described by the following metric function:

$$f = 1 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{r}{b} - \frac{r^2}{l^2}, \tag{4}$$

where m and Q are the mass and the electric charge of the spacetime, respectively. When $b \to \infty$, equation (4) is reduced to the Reissner–Norström–de Sitter spacetime solution.

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†Corresponding author. E-mail: weiyihuan@263.net

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When $l \to \infty$, it yields the black hole solution surrounded by quintessence.[1] If we write the four roots of $f = 0$ as r_{c+}, r_{c-}, r_+, and r_-, then the mass, the electric charge, and the parameters b and l of the spacetime are expressed as

$$m = \frac{(r_+ + r_-)r_{c+}r_{c-} + r_+r_-(r_{c+} + r_{c-})}{2[r_+r_- + (r_+ + r_-)(r_{c+} + r_{c-}) + r_{c+}r_{c-}]}, \quad (5)$$

$$Q^2 = \frac{r_+r_-r_{c+}r_{c-}}{r_+r_- + (r_+ + r_-)(r_{c+} + r_{c-}) + r_{c+}r_{c-}}, \quad (6)$$

$$b = \frac{r_+r_- + (r_+ + r_-)(r_{c+} + r_{c-}) + r_{c+}r_{c-}}{r_{c+} + r_{c-} + r_+ + r_-}, \quad (7)$$

$$-l^2 = r_+r_- + (r_+ + r_-)(r_{c+} + r_{c-}) + r_{c+}r_{c-}. \quad (8)$$

Equation (8) indicates that three positive roots exist at most among the four roots r_-, r_+, r_{c+}, and r_{c-}. Consider the case where $0 < r_- \le r_+ \le r_{c+}$ and $r_{c-} < 0$, that is, the RN–dSQ spacetime contains the black hole with the inner and outer horizons of radii r_- and r_+, and one CH of radius r_{c+}. In terms of r_+, r_-, b, and l, the mass and the electric charge of the RN–dSQ black hole are given by

$$m = \frac{1}{2}(r_+ + r_-)\left(1 - \frac{r_+^2 + r_-^2}{l^2}\right) - \frac{r_+^2 + r_+r_- + r_-^2}{2b}, \quad (9)$$

$$Q^2 = r_+r_-\left(1 - \frac{r_+ + r_-}{b} - \frac{r_+^2 + r_+r_- + r_-^2}{l^2}\right). \quad (10)$$

From Eqs. (9) and (10), the charge–mass ratio of the RN–dSQ black hole is obtained as

$$\chi_e = \frac{Q}{m} = \frac{2\chi^{1/2}\left(1 - \dfrac{1+\chi}{\tilde{b}} - \dfrac{1+\chi+\chi^2}{\tilde{l}^2}\right)^{1/2}}{(1+\chi)\left(1 - \dfrac{1+\chi^2}{\tilde{l}^2}\right) - \dfrac{1+\chi+\chi^2}{\tilde{b}}}, \quad (11)$$

where $\chi = r_-/r_+$ with $\chi \le 1$, $\tilde{b} = b/r_+$, and $\tilde{l} = l/r_+$ have been used. Figure 1 indicates that the maximum of χ_e should appear in the region $(\tilde{b} = 3, \tilde{l} = \infty)$ or $(\tilde{b} = \infty, \tilde{l} = \sqrt{6})$. From Fig. 2, it is seen that the maximum of χ_e is $\chi_{emax} \simeq 1.155$.

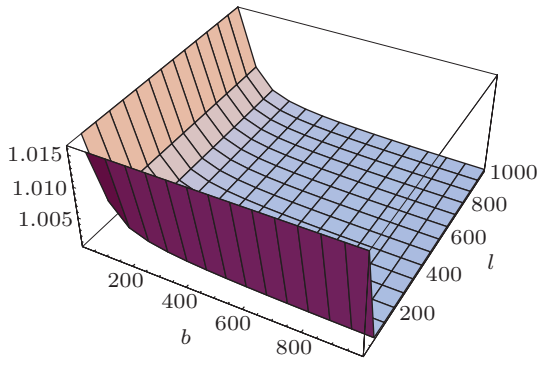


Fig. 1. (color online) χ_e for $\chi = 1$. r_+ is taken as the unit of the b and l axis.

The temperatures of the three horizons of the RN–dSQ spacetime are obtained as

$$T_i = \frac{1}{4\pi}f'(r_i) = \frac{1}{4\pi}\left(\frac{2m}{r_i^2} - \frac{2Q^2}{r_i^3} - \frac{1}{b} - \frac{2r_i}{l^2}\right), \quad (12)$$

with $i = +, -, c+$. In terms of r_+, r_-, b, and l, the temperatures of the inner and outer BHHs of the RN–dSQ black hole are given by

$$T_\pm = \pm\frac{r_+ - r_-}{4\pi r_\pm^2}\left[1 - \frac{(r_+ + r_-) + r_\pm}{b} - \frac{(r_+ + r_-)^2 + 2r_\pm^2}{l^2}\right]. \quad (13)$$

It can be proved that $T_+ \ge 0$ and $T_- \le 0$ for $r_- \le r_+ \le r_{c+}$. In terms of $\chi = r_-/r_+$, $\tilde{b} = b/r_+$, and $\tilde{l} = l/r_+$, the temperature of the outer BHH is given by

$$T_+ = \frac{1-\chi}{4\pi r_+}\left(1 - \frac{2+\chi}{\tilde{b}} - \frac{3+2\chi+\chi^2}{\tilde{l}^2}\right). \quad (14)$$

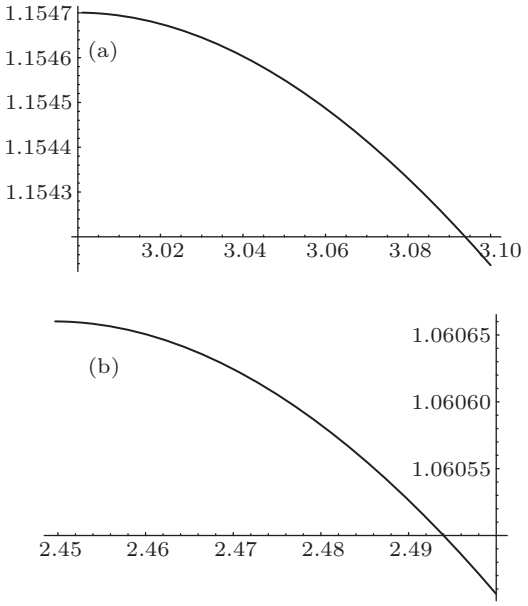


Fig. 2. (a) The relation between χ_e and $\tilde{b}$ with $\chi = 1$ and $\tilde{l} = \infty$. (b) The relation between χ_e and $\tilde{l}$ with $\chi = 1$ and $\tilde{b} = \infty$.

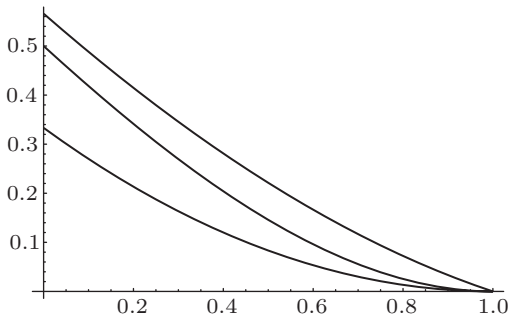


Fig. 3. The temperature of the outer BHH for the three cases: $\tilde{l} = \infty$, $\tilde{b} = 3$; $\tilde{b} = \tilde{l} = (3 + \sqrt{33})/2$; and $\tilde{b} = \infty$, $\tilde{l} = \sqrt{6}$ (from bottom to top). The horizontal and longitudinal axes represent T_+ with units $1/(4\pi r_+)$ and χ, respectively.

For $\Delta\mathscr{Q}_+ = T_+\Delta S_+$, with $\Delta\mathscr{Q}_+$ and ΔS_+ being small quantities, the heat capacity of the black hole is defined as

$$C_X^{(+)} = \lim_{\Delta T_+ \to 0}\left(\frac{\Delta\mathscr{Q}_+}{\Delta T_+}\right)_X = T_+\left(\frac{\partial S_+}{\partial T_+}\right)_X, \quad (15)$$

where the subscript X denotes any thermodynamic process. Substituting

$$T_+ = \frac{1}{4\pi}\left(\frac{2m}{r_+^2} - \frac{2Q^2}{r_+^3} - \frac{1}{b} - \frac{2r_+}{l^2}\right),$$

and $S_+ = \pi r_+^2$ into Eq. (15) yields

$$C_X^{(+)}=\frac{8\pi S_+T_+}{-\frac{4m}{r_+^2}+\frac{6Q^2}{r_+^3}-\frac{2r_+}{l^2}+\frac{2}{r_+}\left(\frac{\partial m}{\partial r_+}\right)_X-\frac{4Q}{r_+^2}\left(\frac{\partial Q}{\partial r_+}\right)_X+\frac{r_+}{b^2}\left(\frac{\partial b}{\partial r_+}\right)_X+\frac{4r_+^2}{l^3}\left(\frac{\partial l}{\partial r_+}\right)_X}. \tag{16}$$

For the three fixed parameters, Q, b, and l, inserting $(\partial m/\partial r_+)_{Q,b,l}=2\pi r_+T_+$ into Eq. (16) yields

$$\begin{aligned}C_{\rm Q}^{(+)}&=T_+\left(\frac{\partial S_+}{\partial T_+}\right)_{Q,b,l}\\&=\frac{mr_+-Q^2-\frac{r_+^3}{2b}-\frac{r_+^4}{l^2}}{2Q^2-mr_+-\frac{r_+^3}{2b}-\frac{2r_+^4}{l^2}}2S_+.\end{aligned} \tag{17}$$

Substituting Eqs. (9) and (10) into Eq. (17) yields

$$C_{\rm Q}^{(+)}=\frac{(1-\chi)\left(\frac{2+\chi}{\tilde b}+\frac{3+2\chi+\chi^2}{\tilde l^2}-1\right)}{1-3\chi+\frac{3\chi(1+\chi)}{\tilde b}+\frac{3(1+\chi+\chi^2+\chi^3)}{\tilde l^2}}2S_+. \tag{18}$$

Figures 4 and 5 show that the heat capacity $C_{\rm Q}^{(+)}$ is positive for the black hole with a higher charge–mass ratio χ.

For fixed m, b, and l, substituting

$$\left(\frac{\partial Q}{\partial r_+}\right)_{m,b,l}=-\frac{2S_+T_+}{Q}$$

into Eq. (16) yields

$$C_m^{(+)}=T_+\left(\frac{\partial S_+}{\partial T_+}\right)_{m,b,l}=\frac{mr_+-Q^2-\frac{r_+^3}{2b}-\frac{r_+^4}{l^2}}{Q^2-\frac{r_+^3}{b}-\frac{3r_+^4}{l^2}}2S_+. \tag{19}$$

In terms of χ, $\tilde b$, and $\tilde l$, $C_m^{(+)}$ is expressed as

$$C_m^{(+)}=\frac{(1-\chi)\left(1-\frac{2+\chi}{\tilde b}-\frac{3+2\chi+\chi^2}{\tilde l^2}\right)}{2\left(\chi-\frac{1+\chi+\chi^2}{\tilde b}-\frac{3+\chi+\chi^2+\chi^3}{\tilde l^2}\right)}. \tag{20}$$

For the small black hole ($\tilde l\gg1$, $\tilde b\gg1$), there can always be $C_m^{(+)}>0$ only if χ is not too small. For the medium black hole (say $\tilde l\sim10$, $\tilde b\sim10$), then $C_m^{(+)}>0$ if $\chi\geq0.15$. For the large black hole ($\tilde l\to\infty$ and $\tilde b\sim3$, $\tilde b\to\infty$ and $\tilde l\sim\sqrt{6}$, or $\tilde b=\tilde l\sim4.37$), χ should be large enough to ensure $C_m^{(+)}>0$.

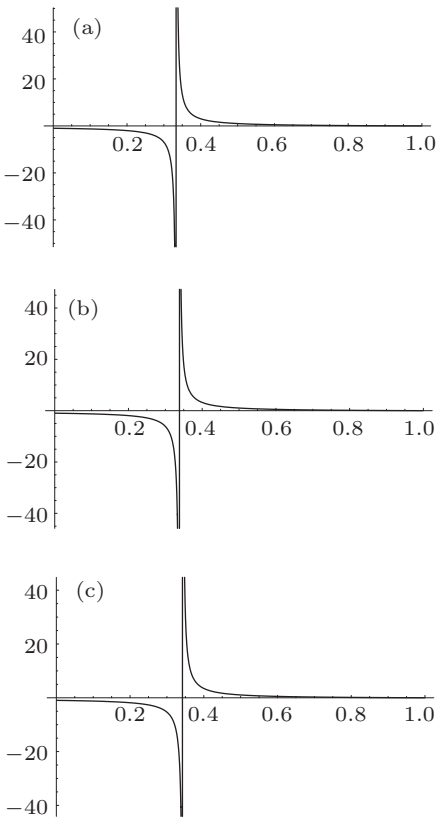


Fig. 4. The heat capacity $C_{\rm Q}^{(+)}$ for (a) $\tilde b=\tilde l=1000$, (b) $\tilde b=\tilde l=100$, and (c) $\tilde b=\tilde l=50$. The horizontal and longitudinal axes represent $C_{\rm Q}^{(+)}$ with units $2S_+$ and χ, respectively.

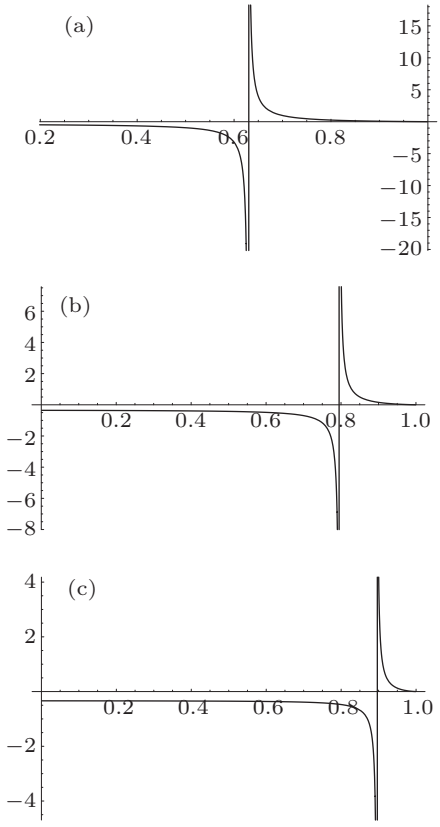


Fig. 5. The heat capacity $C_{\rm Q}^{(+)}$ for (a) $\tilde b=\tilde l=5$, (b) $\tilde b=\tilde l=4.5$, and (c) $\tilde b=\tilde l=4.38$.

3. The energies of RN–dSQ spacetime

From the Misner–Sharp (MS) energy formula[22]

$$E_{\rm MS}=\frac{1}{2}r(1-h^{ab}\partial_a r\partial_b r), \tag{21}$$

the energy of the spherical spacetime described by the metric (1) is obtained as

$$E = E_{\rm MS} = \frac{1}{2} r(1-f). \tag{22}$$

The horizons of the spacetime are determined by $f(r_{\rm H}) = 0$, with $r_{\rm H}$ being the radius of the horizon. From Eq. (22), the energy enclosed by any horizon is given by

$$E(r_{\rm H}) = \frac{1}{2} r_{\rm H}. \tag{23}$$

In the region $f(r) \geq 0$, the energy density of the spacetime is given by

$$\rho = \frac{1}{4\pi r^2}\frac{{\rm d}E}{{\rm d}r} = \frac{1}{8\pi r^2}(1-f) - \frac{1}{8\pi r} f'. \tag{24}$$

On the horizon of the spacetime, the energy density is

$$\rho_{\rm H} = \rho(r_{\rm H}) = \frac{1}{8\pi r_{\rm H}^2} - \frac{\kappa_{\rm H}}{4\pi r_{\rm H}}, \tag{25}$$

with $\kappa_{\rm H} = f'(r_{\rm H})/2$ being the surface gravity on the horizon. Since $\rho = T_0^0$, $p = -T_1^1$, and $T_0^0 = T_1^1$, the pressure of the spacetime described by the metric (1) is given by $p = -\rho$.

For the spacetime described by metric (1), the spatial volume of the system enclosed by the surface of radius r is expressed as

$$V = \int {\rm d}^3 x \sqrt{g} = \frac{4}{3}\pi r^3,$$

with g being the metric determinant.[23,24] For any system enclosed by the horizon of the spherical spacetime described by Eq. (1), the energy can always be written as[25–27]

$$T_{\rm H}\,{\rm d}S_{\rm H} - p_{\rm H}\,{\rm d}V_{\rm H} = {\rm d}E(r_{\rm H}), \tag{26}$$

where $T_{\rm H} = \kappa_{\rm H}/2\pi$, $S_{\rm H} = \pi r_{\rm H}^2$, $p_{\rm H} = -\rho_{\rm H}$, and $V_{\rm H} = (4/3)\pi r_{\rm H}^3$ are the temperature of the horizon, the entropy of the horizon, the pressure on the horizon, and the volume enclosed by it, respectively. Assuming that there are several horizons in the spacetime, then the volume enclosed by the two horizon surfaces of radii r_i and r_j with $r_j \geq r_i$ can be defined as $V_{ij} = V_j - V_i = (4/3)\pi(r_j^3 - r_i^3)$.[24] The first law of thermodynamics for the spacetime region enclosed by the two horizons is obtained as

$${\rm d}E_{ij} = T_j\,{\rm d}S_j - T_i\,{\rm d}S_i - (p_j\,{\rm d}V_j - p_i\,{\rm d}V_i), \tag{27}$$

where $E_{ij} = (r_j - r_i)/2$ is the energy of the spacetime region bounded by the two horizons. For the region $(f(r) \leq 0)$, equation (27) leads to the first law of thermodynamics for the black hole.[28] For the RN–dSQ black hole, equation (27) gives

$${\rm d}E_{\rm BH} = T_+\,{\rm d}S_+ - T_-\,{\rm d}S_- - (p_+\,{\rm d}V_+ - p_-\,{\rm d}V_-), \tag{28}$$

where $S_\pm = \pi r_\pm^2$ are the entropies of the outer and inner horizons, $V_\pm = (4/3)\pi r_\pm^3$ may be interpreted as the volumes of the regions enclosed by the two horizons,

$$E_{\rm BH} = E(r_+) - E(r_-) = \frac{1}{2}(r_+ - r_-) \tag{29}$$

is the energy of the black hole, and

$$p_\pm = \frac{-1}{8\pi r_\pm^3}\left(r_\mp + \frac{r_\pm^2 - r_+ r_- - r_\mp^2 + r_\pm^2}{b} + \frac{r_\pm^3 - r_+^2 r_- - r_-^2 r_+ - r_\mp^3 + 2r_\pm^3}{l^2}\right) \tag{30}$$

are the pressures on the two black-hole horizons.

In order to obtain an understanding for the energy distribution in the RN–dSQ spacetime, let us calculate the energies of the three ingredients in the black hole. The energy of the spacetime described by the metric function (4) is given by

$$E = E_{\rm MS} = m + E_{\rm e} + E_{\rm q} + E_{\rm vac}, \tag{31}$$

where

$$E_{\rm e} = -\frac{Q^2}{2r}, \quad E_{\rm q} = \frac{r^2}{2b}, \quad E_{\rm vac} = \frac{r^3}{2l^2} \tag{32}$$

are the energies of the electrostatic field, the quintessence field, and the vacuum, respectively. The energies in the regions enclosed by the outer and inner horizons of the RN–dSQ black hole are

$$E_{\rm e\pm} = -\frac{Q^2}{2r_\pm}, \quad E_{\rm q\pm} = \frac{r_\pm^2}{2b}, \quad E_{\rm vac\pm} = \frac{r_\pm^3}{2l^2}. \tag{33}$$

The three energies in the RN–dSQ black hole (the region enclosed by the two BHHs of the RN–dSQ black hole) are given by

$$E_{\rm eBH} = \frac{Q^2}{2}\left(\frac{1}{r_-} - \frac{1}{r_+}\right), \quad E_{\rm qBH} = \frac{r_+^2 - r_-^2}{2b}, \quad E_{\rm vacBH} = \frac{r_+^3 - r_-^3}{2l^2}. \tag{34}$$

Substituting Eq. (10) into $E_{\rm eBH}$ given by Eq. (34) yields

$$E_{\rm eBH} = \frac{r_+ - r_-}{2}\left(1 - \frac{r_+ + r_-}{b} - \frac{r_+^2 + 3r_+ r_- + 3r_-^2}{l^2}\right). \tag{35}$$

From $E_{\rm dBH} = E_{\rm BH} - E_{\rm eBH}$, the dark energy in the RN–dSQ black hole is obtained as

$$E_{\rm dBH} = \frac{r_+ - r_-}{2}\left(\frac{r_+ + r_-}{b} + \frac{r_+^2 + 3r_+ r_- + 3r_-^2}{l^2}\right). \tag{36}$$

For the RN–dSQ black hole with a small charge–mass ratio ($\chi \ll 1$), equation (35) is reduced to

$$E_{\rm eBH} \simeq \frac{r_+ - r_-}{2}\left(1 - \frac{1}{\tilde{b}} - \frac{1}{\tilde{l}^2}\right).$$

The dark energy in the RN–dSQ black hole is thus given by

$$E_{\rm dBH} = E_{\rm qBH} + E_{\rm eBH} \simeq \frac{r_+ - r_-}{2}\left(\frac{1}{\tilde{b}} + \frac{1}{\tilde{l}^2}\right).$$

For the small black hole ($\tilde{b} \gg 1$, $\tilde{l} \gg 1$) with $Q \neq 0$, we have $E_{\rm eBH} = (r_+ - r_-)/2$ and $E_{\rm dBH} \simeq 0$. For the large black hole, the quantity of the dark energy in the black hole can be comparable with the electrostatic energy. For $\tilde{b} \simeq 3$ and $\tilde{l} = \infty$, we have $E_{\rm dBH} \simeq E_{\rm eBH}/2$.

4. Conclusions

The RN–dSQ spacetime has two BHHs and one CH, which is the generalization of the Reissner–Nordström–de Sitter spacetime and Reissner–Nordström quintessence spacetime.[1] The small black holes with a large charge–mass ratio in the RN–dSQ spacetime can be more thermally stable in the process where only the mass or the electric charge changes. This implies that the small RN–dSQ black hole with a large charge–mass ratio should be thermally stable in a general process where both the electric charge and the mass change. The energy in the small RN–dSQ black hole is mainly the electrostatic energy. For the large RN–dSQ black hole, the quantity of the dark energy is large enough and can be comparable with the electrostatic energy.

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