Nonclassicality and decoherence of the coherent superposition operation of photon subtraction and addition on the squeezed state*

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The statistical properties of m-coherent superposition operation $(\mu a+\nu a^{\dagger})^m$ on the single-mode squeezed vacuum state (M-SSVS) and its decoherence in a thermal environment are studied. Converting the M-SSVS to a squeezed Hermite polynomial excitation state, we obtain a compact expression for the normalization factor of M-SSVS, which is the Legendre polynomial of the squeezing parameter. We also derive the explicit expression of the Wigner function (WF) of the M-SSVS, and find the negative region of the WF in phase space. The decoherence effect on this state is then discussed by deriving the time evolution of the WF. Using the negativity of the WF, the loss of nonclassicality is then discussed.

Keywords: nonclassicality, decoherence, coherent superposition of photon subtraction and addition, Wigner function

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1. Introduction

Despite the fact that Gaussian states have been created in quantum optics laboratories, they are not of high-enough quality for many aspects of quantum information processing,[1–6] and quantum entanglement is playing a more important role in the quantum information of spreading and processing.[7] Many ideas have been suggested to construct optimized entangled states. In the construction of entangled states, the application of a non-Gaussian operation on a Gaussian state[8,9] can be considered as the most commonly used method. One of the simplest methods in this operation may be through either subtracting photons or adding photons from a light beam using simple optical processes, such as beam splitting, frequency-down conversion, and homodyne detection.[10–12] Subtracting or adding photon states has recently received increasing attention from both experimentalists and theoreticians.[13–19] One of the reasons for this is that photon subtraction can be applied to improve entanglement between Gaussian states,[20,21] loophole-free tests of Bell's inequality,[22] and quantum computing.[23] Thus, photon subtraction can satisfy the requirement of quantum information protocols for long-distance communication. Nevertheless, it has been demonstrated by Parigi *et al.* that photon addition and subtraction are capable of probing quantum commutation rules. Recently, Hu *et al.*[24–26] studied the nonclassicality of the photon-added squeezed vacuum state (PASV), $a^{\dagger m}S(r)|0\rangle$ ($S(r)$ is the single-mode squeezing operator and r is the squeezing parameter[19]). A compact expression for the normalization factor of m-PASV, which is an m-order Legendre polynomial of squeezing parameter r, and the explicit analytical expression of the time evolution of the Wigner function (WF) of m-PASV in the thermal channel, have been derived. These results show that the WF always has a negative value for all parameters, r, if the decay time $\kappa t < (1/2)[\ln(2\bar{n}+2)/(2\bar{n}+1)]$. In Ref. [27], the nonclassicality of the photon-subtracted squeezed vacuum state (PSSV) $a^m S(r)|0\rangle$ was studied by Hu *et al.* The normalization factor of m-PSSV is also proved to be an m-order Legendre polynomial. The result of the time evolution of the WF of m-PSSV also shows that the WF always has a negative value for all parameters, r, if the decay time $\kappa t < (1/2)[\ln(2\bar{n}+2)/(2\bar{n}+1)]$. In particular, Lee and Nha investigated a coherent superposition $\mu a+\nu a^{\dagger}$ acting on continuous variable systems that can be used for quantum state engineering. It was shown that the operation of coherent superposition $\mu a+\nu a^{\dagger}$ can create nonclassicality out of a classical state with observable effects such as squeezing and sub-Poissonian statistics.[28]

In this paper, we apply the m-coherent superposition operation $(\mu a+\nu a^{\dagger})^m$ on the single-mode squeezed vacuum state (M-SSVS). This naturally includes the photon subtraction $(\nu=0)$ and the photon addition $(\mu=0)$ as its extremal cases. Specifically, we show the nonclassicality by the WF, and we resort to the negative part of a WF to judge whether the state is nonclassical or not. Finally, we derive the explicit analytical expression of the time evolution of the WF for the M-SSVS in the thermal channel, and discuss the loss

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of nonclassicality in reference to the negativity of the WF of M-SSVS in a thermal environment.

Our paper is arranged as follows. In Section 2, an M-SSVS, denoted as $(\mu a+\nu a^{\dagger})^m S(r)|0\rangle$, is introduced. The normalized factor is obtained as a Legendre polynomial being a remarkable result. In Section 3, the quantum statistical properties of the M-SSVS, such as photon-number distribution and the WF of the M-SSVS, are investigated, and in Section 4, the explicit analytical expression of the time evolution of the WF of the M-SSVS in the thermal channel is derived. The conclusions are given in the final section.

2. The normalization of the M-SSVS

The M-SSVS is defined as

$$|r,m\rangle = N_{r,m}(\mu a+\nu a^{\dagger})^m S(r)|0\rangle, \tag{1}$$

where $N_{r,m}$ is the normalization constant, and $S(r) = \exp[(ra^2 - ra^{\dagger 2})/2]$ is the single-mode squeezing operator, with r being the squeezing parameter. Noting that[24,29,30] $S^{\dagger}(r)aS(r) = a\cosh r - a^{\dagger}\sinh r$, and $S^{\dagger}(r)a^{\dagger}S(r) = a^{\dagger}\cosh r - a\sinh r$, equation (1) can be rewritten as

$$\begin{aligned} |r,m\rangle &= N_{r,m}S(r)[\mu S^{\dagger}(r)aS(r) + \nu S^{\dagger}(r)a^{\dagger}S(r)]^m|0\rangle \\ &= N_{r,m}S(r)[\mu' a + \nu' a^{\dagger}]^m|0\rangle, \end{aligned} \tag{2}$$

where $\mu' = \mu\cosh r - \nu\sinh r$, and $\nu' = \nu\cosh r - \mu\sinh r$. Using the operator identity

$$(\mu a+\nu a^{\dagger})^m = \left(-\mathrm{i}\sqrt{\frac{\mu\nu}{2}}\right)^m : H_m\left(\mathrm{i}\sqrt{\frac{\mu}{2\nu}}a+\mathrm{i}\sqrt{\frac{\nu}{2\mu}}a^{\dagger}\right) : , \tag{3}$$

where the symbol : : denotes the normally ordered product of the Bose operators, and $H_m(x)$ is an m-order single-variable Hermite polynomial,[31,32] we can rewrite Eq. (2) as

$$|r,m\rangle = N_{r,m}AS(r)H_m(\mathrm{i}B^{1/2}a^{\dagger})|0\rangle, \tag{4}$$

where $A = [-\mathrm{i}(\mu'\nu'/2)^{1/2}]^m$, and $B = \nu'/(2\mu')$. The above equation tells us that the M-SSVS is equivalent to a squeezed Hermite-excited vacuum state.

Noting that

$$1 = \langle r,m.|r,m\rangle = N_{r,m}^2|A|^2\langle 0|H_m(-\mathrm{i}B^{1/2}a)H_m(\mathrm{i}B^{1/2}a^{\dagger})|0\rangle, \tag{5}$$

and using the generating function

$$H_m(x) = [\partial^m/(\partial t^m)]\exp(2xt - t^2)|_{t=0},$$

we obtain

$$\begin{aligned} 1 &= N_{r,m}^2|A|^2\frac{\partial^m}{\partial t^m}\frac{\partial^m}{\partial \tau^m}\exp(-t^2-\tau^2) \\ &\quad\times\langle 0|\exp(-2\mathrm{i}B^{1/2}at)\exp(2\mathrm{i}B^{1/2}a^{\dagger}\tau)|0\rangle_{t=\tau=0}. \end{aligned} \tag{6}$$

Considering

$$\exp(\mu a)\exp(\nu a^{\dagger}) = \exp(\mu\nu)\exp(\nu a^{\dagger})\exp(\mu a),$$

and the following formula:[27]

$$\frac{\partial^{2m}}{\partial\tau^m\partial t^m}\exp\left(-\tau^2-t^2+\frac{2xt\tau}{(x^2-1)^{1/2}}\right) = \frac{2^m m!}{(x^2-1)^{m/2}}P_m(x), \tag{7}$$

where $P_m(x)$ is the Legendre polynomial, equation (8) has been replaced by

$$\begin{aligned} 1 &= N_{r,m}^2|A|^2\frac{\partial^m}{\partial t^m}\frac{\partial^m}{\partial\tau^m}\exp(-t^2-\tau^2+4Bt\tau)|_{t=\tau=0} \\ &= N_{r,m}^2|A|^2 2^m m!(4B^2-1)^{m/2}P_m\left(\sqrt{\frac{4B^2}{4B^2-1}}\right). \end{aligned} \tag{8}$$

Thus, we can obtain the the normalization constant of the M-SSVS as

$$N_{r,m}^{-2} = 2^m m!|A|^2(4B^2-1)^{m/2}P_m\left(\sqrt{\frac{4B^2}{4B^2-1}}\right). \tag{9}$$

When $\mu = 0$ and $\nu = 1$, or $\mu = 1$ and $\nu = 0$, we have

$$\begin{aligned} N_{r,m}^{-2} &= m!\cosh^m r P_m(\cosh r), \\ N_{r,m}^{-2} &= (-\mathrm{i})^m m!(\sinh^m r)P_m(\mathrm{i}\sinh r), \end{aligned}$$

which are the normalization constants of the photon-added squeezed vacuum states,[29] and the photon-subtraction squeezed vacuum states, respectively.

3. The quantum statistical properties of the M-SSVS

3.1. The photon-number distribution of the M-SSVS

Now we shall discuss the photon-number distribution (PND) of the M-SSVS. We begin with the following equation:

$$p(n) = \langle n|m,r\rangle. \tag{10}$$

The PND can be expressed in the form[32–34]

$$P(n) = |p(n)|^2. \tag{11}$$

We substitute Eq. (4) into Eq. (10), and then obtain

$$p(n) = C_{r,m}\sum_{l=0}^{\min[m,n]}\frac{(2\mathrm{i}B^{1/2}\operatorname{sech}r)^{m-2l}[-(\tanh r)/2]^l(-2B\tanh r-1)^{(n-m)/2+l}}{l!(m-2l)![(n-m)/2+l]!}, \tag{12}$$

where $C_{r,m} = N_{r,m}Am!\sqrt{n!}\operatorname{sech}^{1/2}r$. Here, we used $\int \mathrm{d}z_1^2|z_1\rangle\langle z_1|/\pi = 1$, and

$$\int\frac{\mathrm{d}^2z}{\pi}\exp(\zeta|z|^2+\xi z+\eta z^*+fz^2+gz^{*2}) = \frac{1}{\sqrt{\zeta^2-4fg}}\exp\left[\frac{-\zeta\xi\eta+\xi^2g+\eta^2f}{\zeta^2-4fg}\right], \tag{13}$$

where $\mathrm{Re}(\zeta\pm f\pm g) < 0$, and $\mathrm{Re}(\zeta^2-4fg)/(\zeta\pm f\pm g) < 0$. Finally, by substituting Eqs. (12) and (13) into Eq. (11), the PND of the M-SSVS can be derived as

$$P(n) = C_{r,m}^2\left|\sum_{l=0}^{\min[m,n]}\frac{(2\mathrm{i}B^{1/2}\operatorname{sech}r)^{m-2l}[-(\tanh r)/2]^l(-2B\tanh r-1)^{(n-m)/2+l}}{l!(m-2l)![(n-m)/2+l]!}\right|^2, \tag{14}$$

where $m-2l$ and $(n-m)/2+l$ are positive integers.

When $\mu=0$ and $\nu=1$, or $\mu=1$ and $\nu=0$, we have

$$P(n)=\frac{m!n!\sinh^m r\,\mathrm{sech}\,r\tanh^n r}{2^m P_m(\cosh r)}\left|\sum_{l=0}^{\min[m,n]}\frac{(-1)^{(2l+n)/2}2^{(2m-n-2l)/2}\sinh^{2l-m}r}{l!(m-2l)![(n-m)/2+l]!}\right|^2,$$

and

$$P(n)=\frac{m!n!\tanh^n r}{(2\mathrm{i})^m\cosh^{m+1}rP_m(\mathrm{i}\sinh r)}\left|\sum_{l=0}^{\min[m,n]}\frac{(-1)^{(m+n-2m+4l)/2}2^{(2m+4l-n)/2}}{l!(m-2l)![(n-m)/2+l]!}\right|^2,$$

which are the PNDs of the photon-added squeezed vacuum states and the photon-subtracted squeezed vacuum states, respectively.

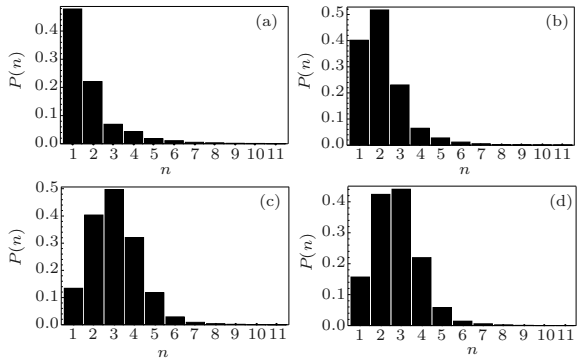


Fig. 1. The PND of M-SSVS for (a) $m=1$, $r=0.3$, (b) $m=2$, $r=0.3$, (c) $m=4$, $r=0.3$, and (d) $m=2$, $r=0.2$. In all the plots, $\mu=1$, and $\nu=1$.

In order to better understand it, we plot the curve of the results from Eq. (14) in the phase space. In Fig. 1, we can see that the wave peak moves right with the increase in m at $r=0.3$ (see Figs. 1(a) and 1(c)). The wave peak moves right with the decrease of r with the same m (see Figs. 1(b) and 1(d)). The reason for this is that the m-coherent superposition operation $(\mu a+\nu a^\dagger)^m$ acting on the single-mode squeezed vacuum state enhances the nonclassical property.

3.2. The Wigner function of M-SSVS

In order to discuss the decoherence properties of M-SSVS in a thermal environment, we shall derive the analytical expression of the WF for M-SSVS. The WF is a distribution function. In classical physics, the distribution function has no negative region in the phase space. While in quantum physics, the WF may present a negative region in the phase space. Thus, we can use the negative region to check whether a state has a nonclassicality.[35] For the single-mode case, the Wigner operator is defined as

$$\Delta(q,p)\to\Delta(\alpha,\alpha^*)=\frac{\mathrm{e}^{2|\alpha|^2}}{\pi}\int\frac{\mathrm{d}^2z}{\pi}|z\rangle\langle -z|\,\mathrm{e}^{-2(z\alpha^*-z^*\alpha)}. \quad (15)$$

In the coherent state, $\alpha=(q+\mathrm{i}p)/\sqrt{2}$, and $|z\rangle=\exp[-2(za^\dagger-z^*a)]|0\rangle$. Noticing that the single-mode squeezing operator $S(r)$ has its natural expression in coordinate representation, it follows that

$$S^\dagger(r)\Delta(q,p)S(r)=\Delta(q\,\mathrm{e}^{-r},p\,\mathrm{e}^r). \quad (16)$$

Thus, the WF of M-SSTS can be expressed as follows:[36]

$$W(q,p)=\langle r,m|\Delta(q,p)|r,m\rangle. \quad (17)$$

Substituting Eqs. (4), (15), and (16) into Eq. (17), we can then put Eq. (17) into the following form:

$$W(q,p)=N_{r,m}^2|A|^2\langle 0|H_m(-\mathrm{i}B^{1/2}a)\Delta(q\,\mathrm{e}^{-r},p\,\mathrm{e}^r):H_m(\mathrm{i}B^{1/2}a^\dagger):|0\rangle=N_{r,m}^2|A|^2\frac{\mathrm{e}^{-2|\beta|^2}}{\pi}\frac{\partial^m}{\partial t^m}\frac{\partial^m}{\partial\tau^m}\mathrm{e}^{2\bar\beta t-4Bt\tau+2\bar\beta^*\tau-t^2-\tau^2}\Big|_{\tau=t=0}, \quad (18)$$

where $\bar\beta=-2\mathrm{i}B^{1/2}\beta$, and $\beta=(q\,\mathrm{e}^{-r}+\mathrm{i}p\,\mathrm{e}^r)/\sqrt{2}=\alpha\cosh r-\alpha^*\sinh r$. In the last step, we used the integration formula (13).

Expanding the exponential term $\exp(-4Bt\tau)$ in Eq. (18) into a sum series and using

$$H_m(x)=[\partial^m/(\partial t^m)]\exp(2xt-t^2)|_{t=0},$$

we have

$$W(q,p)=\frac{|A|^2\,\mathrm{e}^{-2|\beta|^2}}{\pi N_{r,m}^{-2}}\sum_{l=0}^{m}\frac{(m!)^2(-4B)^l}{l![(m-l)!]^2}|H_{m-l}(\bar\beta)|^2, \quad (19)$$

where we have used

$$\frac{\mathrm{d}}{\mathrm{d}x^l}H_n(x)=\frac{2^l n!}{(n-l)!}H_{n-l}(x). \quad (20)$$

When $\mu=0$ and $\nu=1$, or $\mu=1$ and $\nu=0$, equation (18) becomes

$$W(q,p)=\frac{\mathrm{e}^{-2|\beta|^2}\sinh^m r}{\pi 2^m P_m(\cosh r)}\sum_{l=0}^{m}\frac{m!(-2\coth r)^l}{l![(m-l)!]^2}|H_{m-l}(\bar\beta)|^2, \quad (21)$$

and

$$W(q,p)=\frac{\mathrm{i}^m\mathrm{e}^{-2|\beta|^2}\cosh^m r}{\pi 2^m m!P_m(\mathrm{i}\sinh r)}\times\sum_{l=0}^{m}\frac{(m!)^2(-2\tanh r)^l}{l![(m-l)!]^2}|H_{m-l}(\bar\beta)|^2, \quad (22)$$

which are the WF of the photon-added squeezed vacuum states and the photon-subtracted squeezed vacuum states, respectively.

Based on Eq. (19), we show the WF plots in the phase space. In Fig. 2, for the squeezing parameter $r=3$ with a different photon-added number m when $\mu=1$ and $\nu=1$, one

can clearly see that there is a negative region of the WF in the phase space, which implies the nonclassicality of this state. In addition, the squeezing effect in one of the quadratures is clearly seen in Fig. 2(a), which is more evidence of the nonclassicality of this state. The WF has its minimum value for $m = 1$ and $m = 3$ at the center of the phase space ($p = q = 0$, see Figs. 2(a) and 2(c)). This case is not true for $m = 2$ and $m = 4$ (see Figs. 2(b) and 2(d)). Here, we consider the existence of negative parts of the WF function as proof of nonclassicality.

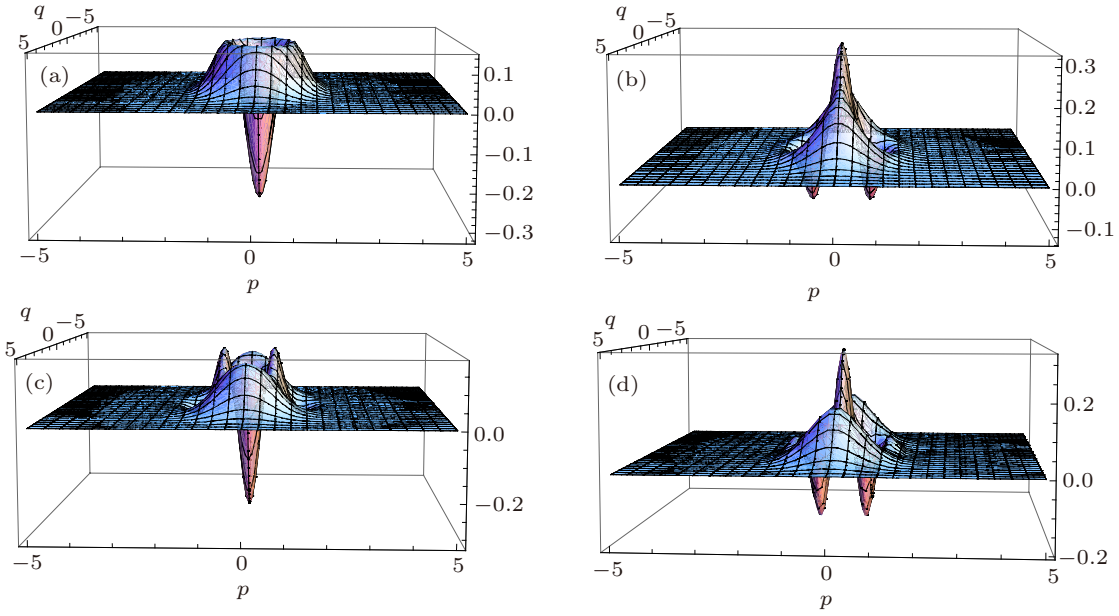


Fig. 2. (color online) The Wigner functions of M-SSVS for $\mu = \nu = 1$, $r = 0.3$ with (a) $m = 1$, (b) $m = 2$, (c) $m = 3$, and (d) $m = 4$.

4. The decoherence of M-SSVS in a thermal environment

When the M-SSVS evolves in the thermal channel, the evolution of the density matrix can be described by the master equation[2]

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \kappa(\bar{n}+1)(2a\rho a^{\dagger} - a^{\dagger}a\rho - \rho a^{\dagger}a) + \kappa\bar{n}(2a^{\dagger}\rho a - aa^{\dagger}\rho - \rho aa^{\dagger}), \tag{23}$$

where κ represents the dissipative coefficient, and η denotes the average thermal photon number of the environment. When $\eta = 0$, equation (23) reduces to the master equation describing the M-SSVS channel. The evolution of the WF is governed by the following integration equation:[31]

$$W(\zeta,\zeta^{*},t) = \frac{2}{(2\bar{n}+1)T}\int \frac{\mathrm{d}^2\alpha}{\pi} W(\alpha,\alpha^{*},0) \times \exp\left[-\frac{2|\zeta-\alpha\,\mathrm{e}^{-\kappa t}|^2}{(2\bar{n}+1)T}\right], \tag{24}$$

where $W(\alpha,\alpha^{*},0)$ is the WF of the initial state, and $T = 1-\exp(-2\kappa t)$. Equation (24) is simply the evolution equation of the WF in the thermal channel. Thus, by performing the integration with an initial value, the WF of M-SSVS may be obtained.

In a similar way, substituting Eq. (19) into Eq. (24) and the generating function of single-variable Hermite polynomials (20), we have

$$\begin{aligned} &W(\zeta,\zeta^{*},t) \\ &= \frac{N}{\sqrt{E}}\exp\left[\frac{-2|\zeta|^2}{(2\bar{n}+1)T} + \frac{C|D|^2}{E} + \frac{(D^{*2}+D^2)\sinh 2r}{E}\right] \\ &\times \sum_{l=0}^{m}\sum_{k=0}^{m-l}\frac{(m!)^2(-4B)^l G^{m-l-k}I^k}{l!k![(m-l-k)!]^2}\left|H_{m-l-k}\left(\frac{J}{G^{1/2}}\right)\right|^2, \end{aligned} \tag{25}$$

where we have set

$$\begin{aligned} &N = \frac{2N_{r,m}^2|A|^2}{\pi(2\bar{n}+1)T}, \quad C = \frac{2\,\mathrm{e}^{-2\kappa t}}{(2\bar{n}+1)T} + 2\cosh 2r, \\ &D = \frac{2\zeta\,\mathrm{e}^{-\kappa t}}{(2\bar{n}+1)T}, \quad E = C^2 - 4\sinh^2 2r, \\ &G = 1 - \frac{C+2\cosh 2r}{E}8B\sinh 2r,\ J = \frac{1}{E}(2CF - 4F^{*}\sinh 2r), \\ &I = \frac{16B(C\cosh 2r + 2\sinh^2 2r)}{E}. \end{aligned} \tag{26}$$

Equation (25) is simply the analytical expression of WF for the M-SSVS in the thermal channel. It is obvious that the WF loses its Gaussian property due to the presence of single-variable Hermite polynomials. In particular, at the initial time ($t = 0$), equation (25) reduces to Eq. (19). When $\mu = 0$ and $\nu = 1$, equation (25) is the WF of the photon-added squeezed vacuum states in the thermal environment, and when $\mu = 1$ and $\nu = 0$, equation (25) is the WF of the photon-subtracted squeezed vacuum states in the thermal environment.

While for $m = 1$, equation (25) has the following form:

$$W(\zeta,\zeta^*,t) = \frac{|2J|^2 + I - 4B}{2B\sqrt{E}\pi(2\bar{n}+1)T} \exp\left[\frac{-2|\zeta|^2}{(2\bar{n}+1)T}\right] \times \exp\left[\frac{C|D|^2}{E} + \frac{(D^{*2}+D^2)\sinh 2r}{E}\right]. \quad (27)$$

When $\zeta = 0$, then $J = 0$ and $E > 0$. When $(I - 4B)/B = 4(C\cosh 2r + 2\sinh^2 2r)/E - 1 < 0$, we have the following condition:

$$\kappa t < \kappa t_c = \frac{1}{2}\ln\frac{2\bar{n}+2}{2\bar{n}+1}. \quad (28)$$

A negative region always exists for the WF in the phase space, which is independent of the squeezing parameter r and parameters μ and ν. The WF of M-SSVS is also always positive in the whole phase space when κt exceeds the threshold value κt_c. This similar result to that of PASV and PSSV tells us that PASV and PSSV can be contained in M-SSVS. When we set μ or ν to be zero, the PASV and PSSV can be obtained in M-SSVS.

Based on Eq. (25), we show the plots of the decoherence in the phase space. In Fig. 3, we can see that the negative part decreases with the increase in κt, and that the negative part is divided into two parts when m is even. In Fig. 4, we also see that the negative part decreases with the increase in $\bar{n}$. In Fig. 5, the negative part decreases with the increase in m. All the decoherence figures show that the negative part will disappear with increasing parameter values.

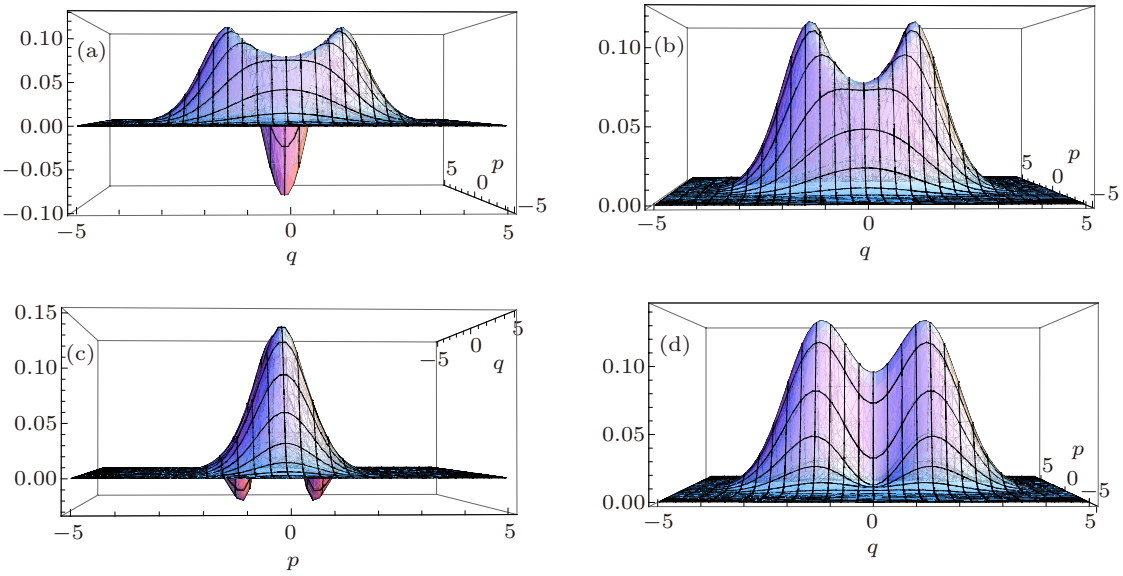


Fig. 3. (color online) The Wigner functions of M-SSVS in the thermal environment for $\mu = \nu = 1$, $r = 0.3$, and $\bar{n} = 1$ with (a) $m = 1$, $\kappa t = 0.05$, (b) $m = 2$, $\kappa t = 0.05$, (c) $m = 1$, $\kappa t = 0.1$, and (d) $m = 2$, $\kappa t = 0.1$.

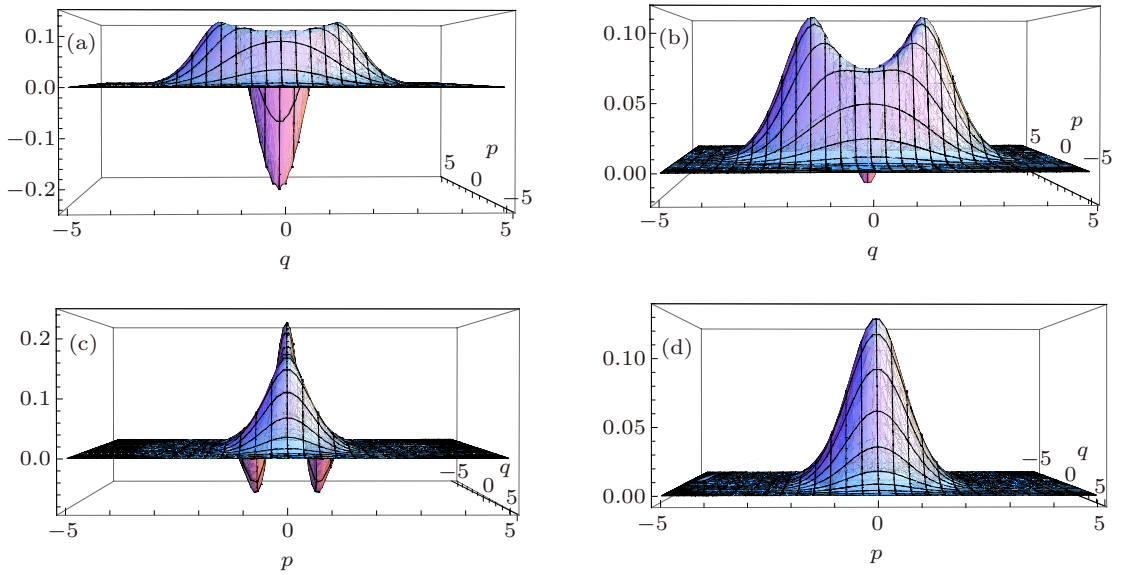


Fig. 4. (color online) The Wigner functions of M-SSVS in the thermal environment for $\mu = \nu = 1$, $r = 0.3$, $\kappa t = 0.05$ with (a) $m = 1$, $\bar{n} = 0$, (b) $m = 2$, $\bar{n} = 2$, (c) $m = 1$, $\bar{n} = 0$, and (d) $m = 2$, $\bar{n} = 2$.

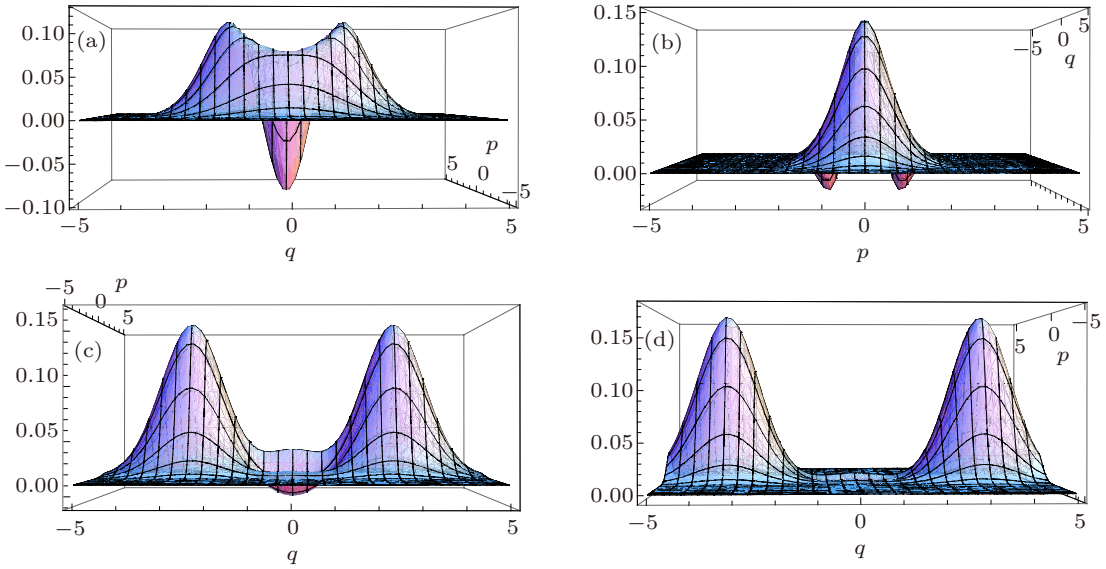


Fig. 5. (color online) The Wigner functions of M-SSVS in the thermal environment for $\mu = \nu = 1$, $r = 0.3$, $\kappa t = 0.05$, $\bar{n} = 2$ with (a) $m = 1$, (b) $m = 2$, (c) $m = 5$, and (d) $m = 10$.

5. Conclusions

In this paper, the statistical properties and decoherence of arbitrary-number photon-subtracted and photon-added squeezed vacuum states in a thermal environment are investigated. The normalization of M-SSVS, which is an m-order Legendre polynomial, is derived. The PND and WF are calculated. In particular, the PND and WF results show that M-SSVS is a nonclassical state. The explicit analytical expression of the time evolution of the WF of the M-SSVS in the thermal channel is also derived, and it can be seen that the negative part of the WF decreases with the increase in the parameters from the decoherence plots. It is of importance to note that the WF always has a negative region, which is independent of the squeezing parameter r and the parameters μ and ν if the decay time $\kappa t < \ln[(2\bar{n}+2)/(2\bar{n}+1)]/2$ for the M-SSVS.

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