Induced modification of the geometric phase of a qubit coupled to an XY spin chain by Dzyaloshinsky–Moriya interaction*

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We consider a qubit symmetrically and transversely coupled to an XY spin chain with Dzyaloshinsky–Moriya (DM) interaction in the presence of a transverse magnetic field. An analytical expression for the geometric phase of the qubit is obtained in the weak coupling limit. We find that the modification of the geometrical phase induced by the spin chain environment is greatly enhanced by DM interaction in the weak coupling limit around the quantum phase transition point of the spin chain.

Keywords: geometric phase, quantum phase transition, XY spin chain, Dzyaloshinsky–Moriya interaction

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1. Introduction

Since Berry[1] discovered the geometric phase (GP) of quantum systems that undergo an adiabatic cyclic evolution in curvature parameter spaces, the concept of GP has been extensively generalized to the non-adiabatic case,[2] the non-cyclic case,[3,4] nonunitary evolution,[5,6] as well as the degenerate state.[7] The potential application of GPs has been proposed for geometric quantum computation. This conjecture is based on the geometric feature of GPs: it depends only on the solid angle swept by the vector describing the time evolution of quantum systems in the parameter space. As a result, the GP may be less affected by uncontrolled fluctuations and can be utilized to implement geometric quantum computation that is resilient to stochastic control errors.[8–10]

On the other hand, quantum phase transitions (QPTs) exist in many-body quantum systems.[11] As a quantum critical phenomenon, the QPT happens at zero temperature, at which thermal fluctuations vanish. Thus, the QPT is driven only by quantum fluctuations. In recent years, the close relation between the geometric phase and quantum phase transition in many-body systems has been revealed.[12–18] Carollo and Pachos[12] showed that the geometric phase is very sensitive in controlling the parameters of the spin chain and can be exploited as a tool to detect regions of criticality. Yi and Wang[13] investigated the geometric phase induced in an auxiliary qubit by a one-dimensional XY spin chain, and found that the GP changes sharply around the QPT point, and tends to be close to zero, far from the critical points. It is also shown that the connection of GPs with the typical features of QPTs, such as the scaling feature and critical exponents, is not restricted to the XY spin chain model, but is universal in many-body quantum systems.[16]

In earlier stages, the investigation of GPs was restricted to closed quantum systems. In real situations, however, quantum systems are unavoidably coupled to their surroundings. Thus, it is natural to extend the concept of GPs from closed quantum systems to open quantum systems. In recent years, many approaches have been proposed for this purpose.[5,6,19–21] In general, an open quantum system is in mixed states. When we generalize the original GP concept, the first question is how to define the GP for mixed states. Uhlman[5,6] first studied this issue as a purely mathematical problem. Then, based on the experimental context of quantum interferometry, Sjöqvist *et al.*[19] introduced a definition of GP for mixed states undergoing unitary evolution. In Ref. [21], Tong and coworkers developed a kinematic approach to treating the GP for open quantum systems in nonunitary evolution caused by the coupling of the environment. Lombardo and Villar[22] calculated the spin-1/2 GP linearly coupled to a harmonic oscillator reservoir at arbitrary temperature, and estimated the time scale for experimentally measuring the GP. Yuan *et al.*[17] studied the GP of a qubit coupled to an antiferromagnetic spin environment, and found that the GP of the qubit changes abruptly to zero when the environment undergoes a spin-flop transition. Cucchietti *et al.*[23] reported the measurement of a spin-1/2 GP undergoing nonunitary evolution induced by the coupling to an environment with a NMR quantum simulator.

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In this paper, based on the definition of GP given in Ref. [21], we will investigate the GP of a qubit coupled to an XY spin chain. The Dzyaloshinsky–Moriya (DM) interaction[24,25] originating from the spin–orbit coupling is often present in the XY spin chain. Recently, a spin chain with the DM interaction has been reported by several groups.[26–28] Cheng and Liu[26] investigated the effect of the DM interaction on two spin qubits coupled to an XY spin chain, and showed that the DM interaction can slightly enhance the decay of the decoherence factor in the weak-coupling region. As is well known, a harmonic oscillator displays exact periodical motions without coupling to its surroundings. The periodical motion is damped and becomes quasi-cyclic when a harmonic oscillator is weakly coupled to the damping resources. The GPs of open quantum systems are defined in a quasi-cyclic motion path.[21] Thus, the GPs of open quantum systems should be considered in the weak-coupling limit. Now the question arises in our mind: does the DM interaction still affect the GP slightly in the weak-coupling limit? In the present paper, we will show that the DM interaction in an XY spin chain can dramatically modify the GP of a qubit coupled to the XY spin chain in the weak-coupling limit.

This paper is organized as follows. In Section 2, the model under consideration is presented. In Section 3, an analytical expression of the GP is found in the weak-coupling limit, and the effect of the DM interaction on the GP is analytically and numerically investigated. Finally, a short summary is given in Section 4.

2. Model

We consider a central qubit symmetrically and transversely coupled to a cyclic XY spin chain with the DM interaction. The total Hamiltonian is expressed in the form (taking $\hbar = 1$ throughout this paper)

$$H = H_{\rm S} + H_{\rm E} + H_{\rm I}, \tag{1}$$

$$H_{\rm S} = \frac{\eta}{2}\sigma_{\rm S}^z, \tag{2}$$

$$H_{\rm E} = -\sum_{l=-M}^{M}\Big[\Big(\frac{1+\gamma}{2}\sigma_l^x\sigma_{l+1}^x + \frac{1-\gamma}{2}\sigma_l^y\sigma_{l+1}^y\Big) - \lambda\sigma_l^z - D\big(\sigma_l^x\sigma_{l+1}^y - \sigma_l^y\sigma_{l+1}^x\big)\Big], \tag{3}$$

$$H_{\rm I} = -\sigma_{\rm S}^z\sum_{l=-M}^{M} g\sigma_l^z, \tag{4}$$

where η is the transition frequency between the ground state and the excited state of the central qubit, γ and D characterize the anisotropy in the next-neighbor spin–spin interaction and the strength of the DM interaction, respectively, and g and λ are the coupling constants of the qubit to the spin chain and the transverse magnetic field, respectively. In addition, $\sigma_{\rm S}^z = |e\rangle\langle e| - |g\rangle\langle g|$ is the Pauli matrix of the central qubit, σ_l^α ($\alpha = x, y, z$) are the Pauli matrices for spin at the l-th site of the spin chain, and $M = L/2$ for even L and $M = (L-1)/2$ for odd L, where L is the size of the spin chain. The anisotropy XY spin chain model contains two special cases: for $0 < \gamma \leq 1$ it turns into a transverse Ising universality class XY spin chain, and it has a critical point at $|\lambda_{\rm c}| = 1$; for $\gamma = 0$ it is reduced to a transverse XX spin chain, which is critical for $|\lambda| \leq 1$.

Notice that $[H_{\rm S}, H_{\rm E} + H_{\rm I}] = 0$, the free-motion term of the central qubit in the Hamiltonian given by Eq. (1), can be removed by the frame rotation transformation $\exp(-{\rm i}\eta\sigma_{\rm S}^z t/2)$. Following the approach proposed in Ref. [11], we diagonalize the Hamiltonian $H_{\rm E} + H_{\rm I}$. Since $[\sigma_{\rm S}^z, \sigma_l^\alpha] = 0$, an operator-valued parameter $\Lambda = \lambda + g\sigma_{\rm S}^z$ is a conserved quantity. Thus, Λ can be treated as a c number when diagonalizing the Hamiltonian. Obviously, Λ has two eigenvalues: $\Lambda_n = \lambda + (-1)^n g$ ($n = 0, 1$).

By means of the Jordan–Wigner transformation, which converts the spin operators σ_l^α into the spinless fermionic operations c_l via the relation: $c_l = \prod_{n<l}\sigma_n^z(\sigma_l^x + {\rm i}\sigma_l^y)$, the Hamiltonian can be written as

$$H = \sum_{l=1}^{M}\Big[((c_l^+c_{l+1} + c_{l+1}^+c_l) + \gamma(c_l^+c_{l+1}^+ + c_{l+1}c_l)) + (2{\rm i}D(c_l^+c_{l+1} - c_{l+1}^+c_l) + \Lambda(1 - 2c_l^+c_l))\Big]. \tag{5}$$

Then, by introducing the Fourier transforms of the Fermionic operators described by

$$d_k = \frac{1}{\sqrt{L}}\sum_l c_l\,{\rm e}^{\frac{-{\rm i}2\pi lk}{L}},\ (k = -M, -M+1, \ldots, M),$$

the Hamiltonian given by Eq. (5) can be written in terms of the new Fermionic operators d_k and $d_k^\dagger$ as

$$H = \sum_k\big[2(\cos k - \Lambda + 2D\sin k)d_k^+d_k + {\rm i}\gamma\sin k(d_k^+d_{-k}^+ - d_{-k}d_k) + \Lambda\big]. \tag{6}$$

To diagonalize the Hamiltonian given by Eq. (6), we introduce the Bogoliubov transformation

$$a_{k,\Lambda} = \cos\frac{\theta_{k,\Lambda}}{2}d_k - {\rm i}\sin\frac{\theta_{k,\Lambda}}{2}d_{-k}^+. \tag{7}$$

Substituting Eq. (7) into Eq. (6), we find that equation (6) can be diagonalized if $\theta_{k,\Lambda}$ obeys the condition

$$\theta_{k,\Lambda} = \arctan\frac{\gamma\sin(2\pi k/L)}{\Lambda - \cos(2\pi k/L)}. \tag{8}$$

In this way, the diagonalized Hamiltonian has the form

$$H = \sum_k \Omega_{k,\Lambda}\Big(a_{k,\Lambda}^+a_{k,\Lambda} - \frac{1}{2}\Big), \tag{9}$$

with the energy spectrum

$$\Omega_{k,\Lambda} = 2\sqrt{\Big(\cos\frac{2\pi k}{L} - \Lambda\Big)^2 + \gamma^2\sin^2\frac{2\pi k}{L}} + 4D\sin\frac{2\pi k}{L}. \tag{10}$$

Since $g = 0$ and $\Lambda = \lambda$, replacing Λ with λ in Eqs. (7)–(10), we can obtain the normal modes $a_{k,\lambda}$ and $\theta_{k,\lambda}$, the di-

agonalized Hamiltonian, and the corresponding eigenenergies of the pure spin chain. It can be easily found that the normal modes $a_{k,\Lambda}$ dressed by the system–environment interaction are in association with the normal modes $a_{k,\lambda}$ of the pure environment as follows:

$$a_{k,\Lambda} = \cos\alpha_{k,\Lambda} a_{k,\lambda} - \mathrm{i}\sin\alpha_{k,\Lambda} a^{+}_{-k,\lambda}, \tag{11}$$

where

$$\alpha_{k,\Lambda} = (\theta_{k,\Lambda} - \theta_{k,\lambda})/2. \tag{12}$$

3. The geometric phase of the central qubit

Let us suppose that the central qubit is initially decoupled into the spin chain. The qubit is in a state below

$$|\varphi_{\mathrm{S}}(0)\rangle = \cos\frac{\beta}{2}|e\rangle + \sin\frac{\beta}{2}|g\rangle, \tag{13}$$

and the spin chain is in $|\varphi_{\mathrm{E}}(0)\rangle$. At time t, the qubit–environment coupled system unitarily evolves into the state described by the density matrix

$$\rho_{\mathrm{SE}}(t) = U(t)|\varphi_{\mathrm{S}}(0)\rangle|\varphi_{\mathrm{E}}(0)\rangle\langle\varphi_{\mathrm{E}}(0)|\langle\varphi_{\mathrm{S}}(0)|U^{\dagger}(t),$$

where $U(t) = \exp[-\mathrm{i}t(H_{\mathrm{S}} + H_{\mathrm{E}} + H_{\mathrm{I}})]$. By tracing $\rho_{\mathrm{SE}}(t)$ over the environmental variables, we obtain the reduced density matrix for the central qubit in the Schrödinger picture as follows:

$$\rho_{\mathrm{S}}(t) = \begin{pmatrix} \cos^2\frac{\beta}{2} & \frac{1}{2}\sin\beta\,\mathrm{e}^{-\mathrm{i}\eta t}F(t) \\ \frac{1}{2}\sin\beta\,\mathrm{e}^{\mathrm{i}\eta t}F^{*}(t) & \sin^2\frac{\beta}{2} \end{pmatrix}, \tag{14}$$

where the decoherence factor

$$F(t) = \langle\varphi_{\mathrm{E}}(0)|U^{+}_{\Lambda_1}(t)U_{\Lambda_0}(t)|\varphi_{\mathrm{E}}(0)\rangle, \tag{15}$$

and the unitary evolution operator $U_{\Lambda_{0,1}}$ are obtained from $U(t)$ by replacing Λ with $\lambda + g$ and $\lambda - g$, respectively. The decoherence factor describes the transition dynamics of a quantum two-level system from a pure state to a mixed one.[26,29,30]

When the qubit is decoupled from the spin chain, i.e., $g = 0$, $F(t)$ equals 1. When the qubit is coupled with the spin chain, the value of $|F(t)|$ is less than 1 and the coherence of the qubit (the off-diagonal matrix elements of the reduced density matrix (Eq. 14)) is diminished. Then, the qubit evolves in phase damping.[31]

The ground state of the spin chain is defined as $a_{k,\lambda}|G\rangle_{\lambda} = 0$. It can be written in the form

$$|G\rangle_{\lambda} = \prod_{k>0}^{M}\Big(\cos\frac{\theta_{k,\lambda}}{2}|0\rangle_k|0\rangle_{-k} + \mathrm{i}\sin\frac{\theta_{k,\lambda}}{2}|1\rangle_k|1\rangle_{-k}\Big),$$

where $|0\rangle_k$ and $|1\rangle_k$ are the vacuum and single excitation states of the k-th pure environment mode $a_{k,\lambda}$, respectively. Then, according to Eq. (11), $|G\rangle_{\lambda}$ can be further written as a BCS-like state

$$|G\rangle_{\lambda} = \prod_{k>0}^{M}(\cos\alpha_{k,\Lambda} - \mathrm{i}\sin\alpha_{k,\Lambda}a^{+}_{k,\Lambda}a^{+}_{-k,\Lambda})|G\rangle_{\Lambda}, \tag{16}$$

where $|G\rangle_{\Lambda}$ is the ground state of the Hamiltonian (9) and satisfies the condition $a_{k,\Lambda}|G\rangle_{\Lambda} = 0$.

Assume that the spin chain is initially in its ground state (16). Substituting Eq. (16) into Eq. (15), one can find an explicit expression for the decoherence factor

$$\begin{aligned} |F(t)| &= \prod_{k>0}^{M}|\sin\alpha_{k,\Lambda_0}\sin\alpha_{k,\Lambda_1}\cos\Delta\alpha\,\mathrm{e}^{-\mathrm{i}\Delta\Omega t} \\ &\quad - \cos\alpha_{k,\Lambda_0}\sin\alpha_{k,\Lambda_1}\sin\Delta\alpha\,\mathrm{e}^{\mathrm{i}\delta\Omega t} \\ &\quad + \sin\alpha_{k,\Lambda_0}\cos\alpha_{k,\Lambda_1}\sin\Delta\alpha\,\mathrm{e}^{-\mathrm{i}\delta\Omega t} \\ &\quad + \cos\alpha_{k,\Lambda_0}\cos\alpha_{k,\Lambda_1}\cos\Delta\alpha\,\mathrm{e}^{\mathrm{i}\Delta\Omega t}| \\ &= \prod_{k>0}^{M}F_k(t), \end{aligned} \tag{17}$$

where $\Delta\alpha = \alpha_{k,\Lambda_0} - \alpha_{k,\Lambda_1}$, $\Delta\Omega = \Omega_{k,\Lambda_0} - \Omega_{k,\Lambda_1}$, and $\delta\Omega = \Omega_{k,\Lambda_0} + \Omega_{k,\Lambda_1}$. The values of Ω_{k,Λ_n} and α_{k,Λ_n} are given by replacing Λ with Λ_n in Eqs. (10)–(12), respectively.

According to the GP definition of an open quantum system, which is proposed in Ref. [21], the GP acquired by the qubit in a nonunitary evolution is given by

$$\begin{aligned} \Phi &= \arg\Big(\sum_k\sqrt{\varepsilon_k(0)\varepsilon_k(T)}\langle\varepsilon_k(0)|\varepsilon_k(T)\rangle \\ &\quad \times\exp\Big(-\int_0^T\langle\varepsilon_k(t)|\partial/\partial t|\varepsilon_k(t)\rangle\,\mathrm{d}t\Big)\Big), \end{aligned} \tag{18}$$

where $\varepsilon_k(t)$ and $|\varepsilon_k(t)\rangle$ are the eigenvalues and the corresponding eigenstates of the reduced density matrix (14), and $T = 2\pi/\eta$ denotes a time evolution cycle of the qubit when it is isolated from the environment. If the qubit is coupled to the spin chain, the evolution of its state is no more periodic. However, if the coupling is weak, we may consider a quasi cyclic path $\mathbf{P}: t \in [0, T]$.[21]

The instantaneous eigenvectors and the corresponding eigenvalues of the reduced density matrix (14) are found to be

$$|\varepsilon_{\pm}(t)\rangle = \mathrm{e}^{-\mathrm{i}\eta t}\sin\frac{\beta_t}{2}|e\rangle \pm \cos\frac{\beta_t}{2}|g\rangle, \tag{19}$$

$$\varepsilon_{\pm}(t) = \frac{1}{2}\Big(1 \pm \sqrt{\cos^2\beta + \sin^2\beta|F(t)|^2}\Big), \tag{20}$$

where

$$\beta_t = 2\arctan\frac{\sqrt{\cot^2\beta + |F(t)|^2} + \cot\beta}{|F(t)|}. \tag{21}$$

Substituting Eqs. (19) and (20) into Eq. (18), we obtain the geometric phase of the central qubit

$$\Phi = \eta\int_0^{2\pi/\eta}\mathrm{d}t\sin^2\frac{\beta_t}{2}. \tag{22}$$

If the qubit is decoupled from the spin chain, i.e., $g = 0$, the well-known result $\Phi = \pi(1 + \cos\beta)$ is recovered from Eq. (22). When the qubit is coupled to the spin chain, the decoherence factor is crucial in Eqs. (19) and (20), the eigenvalues and eigenvectors are changed, and thus the GP will be modified. In order to obtain a basic and clear concept about

the effect of the DM interaction in the spin chain on the GP acquired by the qubit in the quasi cycle, we first make an approximate analysis on the GP.

As shown in Eqs. (19) and (20), the effects of environmental coupling on the eigenvalues and eigenstates of the reduced density matrix of the qubit are fully included in the decoherence factor. Then, modification of the GP of the qubit induced by the environment is determined by the decoherence factor. Therefore, we first analyze the decoherence factor. To this end, following Ref. [29], we use the cut-off approximation, and assume that the main contributions to the multi products come from the terms with a smaller k than a cut-off number K_c, which determines the largest energy scale of the environment. Then, one defines the partial product

$$|F(t)|_c = \prod_{k>0}^{K_c} F_k, \tag{23}$$

and the corresponding partial sum

$$S(t) = \ln|F(t)|_c = -\sum_{k>0}^{K_c} |\ln F_k|.$$

For small k, expanding Eq. (10) at $k=0$ and keeping the terms up to the first power of k, we have

$$\Omega_{k,\Lambda_n} \approx 2|\Lambda_n - 1| + \frac{8\pi k}{L} D, \tag{24}$$

and

$$\sin(2\alpha_{k,\Lambda_n}) \approx \frac{(-1)^{n+1} 2\gamma\pi k g}{L|(\Lambda_n - 1)(\lambda - 1)|}, \tag{25}$$

$$\sin^2(\alpha_{k,\Lambda_0} - \alpha_{k,\Lambda_1}) \approx \frac{4\gamma^2\pi^2 k^2 g^2}{L^2(\Lambda_0 - 1)^2(\Lambda_1 - 1)^2}. \tag{26}$$

In this way, the partial sum $S(t)$ can be written approximately in the form

$$\begin{aligned} S(t) \approx -\frac{8\pi^2\gamma^2 g^2 t^2}{L^2(\lambda-1)^2} \sum_{k>0}^{K_c} k^2 \Big[& \frac{(|\lambda - g - 1| + 4\pi k D/L)^2}{(\lambda - g - 1)^2} \\ & + \frac{(|\lambda + g - 1| + 4\pi k D/L)^2}{(\lambda + g - 1)^2} \\ & + 2\frac{(|\lambda - g - 1| + 4\pi k D/L)(|\lambda + g - 1| + 4\pi k D/L)}{|(\lambda - g - 1)(\lambda + g - 1)|} \Big]. \end{aligned} \tag{27}$$

If K_c is small, then in the thermodynamical limit $L \to \infty$, one may ignore all the terms related to L^{-l} $(l > 3)$ in Eq. (27). Consequently, in a short time, when $\lambda \to 1$, we have

$$|F(t)|_c \simeq e^{-(\tau_1 + \tau_2)t^2}, \tag{28}$$

where

$$\tau_1 = 8E(K_c)\gamma^2 g^2/(\lambda - 1)^2, \tag{29}$$

$$\tau_2 = 32F(K_c)\gamma^2 g D/(\lambda - 1)^2, \tag{30}$$

and

$$E(K_c) = \frac{2\pi^2 K_c(K_c + 1)(2K_c + 1)}{3L^2}, \tag{31}$$

$$F(K_c) = \frac{2\pi^3 K_c^2 (K_c + 1)^2}{L^3}. \tag{32}$$

Since $|F(t)|_c \geq |F(t)|$, from the above approximate results we find that the decoherence factor would decay in a Gaussian-type manner in a short time near the critical point $\lambda_c = 1$.[26,29,30] The closer to the critical point, the larger the decay constants become, and then the decoherence process is greatly accelerated around the critical point. This sensitivity of the decoherence process to the quantum phase transition of the environment has been found previously.[26,29,30] As shown in Ref. [28], the decay rate of the decoherence factor is determined by the two constants τ_1 and τ_2, which are proportional to g^2 and gD, respectively. Since the parameter g is adjustable experimentally, we have a situation where $D > g$ can be realized. As a result, τ_2 may play a major role in the decoherence process in the weak coupling limit ($0 < g \ll 1$ and $D > g$).

Substituting Eq. (28) into Eq. (22), keeping all the terms up to the first order of the coupling strength g, and completing the integral (22), we obtain

$$\Phi = \pi(1 + \cos\beta) + \frac{64\pi^3\gamma^2 g F(K_c)\sin\beta\sin 2\beta}{3\eta^2(\lambda - 1)^2} D. \tag{33}$$

Obviously, in the weak coupling limit, the GP modification induced by the spin chain is proportional to the DM interaction strength and the criticality factor $(\lambda - 1)^{-2}$. Thus, we may expect that the DM interaction can greatly enhance the GP modification in the weak coupling limit around the critical point. It is also noticed that the GP modification becomes zero at the points $\beta = 0$, $\pi/2$, and π, is positive when $\beta < \pi/2$ and negative when $\beta > \pi/2$, and changes the sign around $\beta = \pi/2$.

In order to obtain the entire picture about the effect of the interaction of the DM on the GP, we now turn to a numerical investigation of the GP according to Ref. [22].

In Fig. 1, the GP is plotted as a function of the initial state coefficient β with various values of the DM interaction strength in the weak coupling limit. Obviously, all the variation features of the GP with parameter β are the same, as expected by Eq. (33). For a fixed value of parameter β, except for $\beta = 0$, $\pi/2$, and π, the GP modification is zero, as shown in Fig. 1, and the modification of the GP is enhanced by the DM interaction. In order to make this point more clear, in Fig. 2 the GP is plotted as a function of the DM interaction strength D with fixed values of β. It is clearly shown that the GP linearly changes when the DM interaction is weak, as expected by Eq. (33), but approaches 2π (0) as the DM interaction becomes strong when $\beta < (>)$ π. In Fig. 3, the GP is given as a function of the coupling strength g. It can be observed that all the variation features of the GP with g are similar to those shown in Fig. 2. As expected by Eq. (33), the variation slope increases as the DM interaction becomes strong.

In Fig. 4, the GP is plotted as a function of λ with different values of the DM interaction strength. It is observed that the variation of the GP can be greatly sharpened around the

critical point $\lambda = 1$ by enhancing the DM interaction strength. One may also notice that the GP changes asymmetrically as a function of λ around the critical point.

As pointed out above, the GP is modified by making use of the decoherence factor. In Figs. 5 and 6, the time evolution of the decoherence factor is shown for different values of the DM interaction strength and λ, respectively. As shown in Fig. 5, the decoherence process is accelerated by enhancing the DM interaction. Comparing Fig. 4 with Fig. 5, we find that the faster the coherence factor decays, the sharper the GP varies around the critical point $\lambda = 1$. As shown in Fig. 6, the decoherence factor decays monotonously with time when $\lambda = 1$, and displays different oscillation behaviors in the two cases, $\lambda > 1$ and $\lambda < 1$. It is the different behavior around $\lambda = 1$ that results in the asymmetrical variation of the GP around $\lambda = 1$.

In Fig. 7, the GP is plotted as a function of g for various values of the size of the spin chain. It is noticed that the effect of the DM interaction on the GP is enhanced as the size of the spin increases.

In a word, in the presence of the DM interaction in the XY spin chain, the variation of the GP becomes sharp around the critical point $\lambda = 1$ in the weak coupling limit due to the accelerated decoherence process.

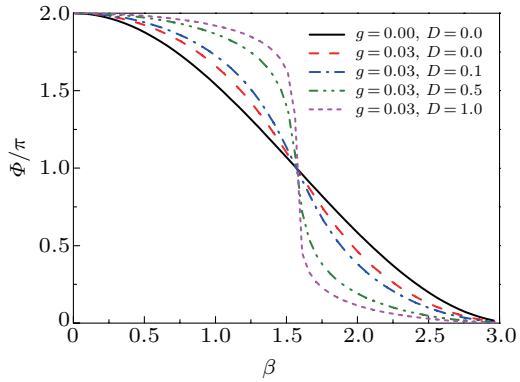


Fig. 1. (color online) The GP scaled by π as a function of the initial state coefficient β with different values of the DM interaction strength and $\lambda = 1.0$, $\gamma = 0.4$, $L = 351$, and $\eta = \pi$.

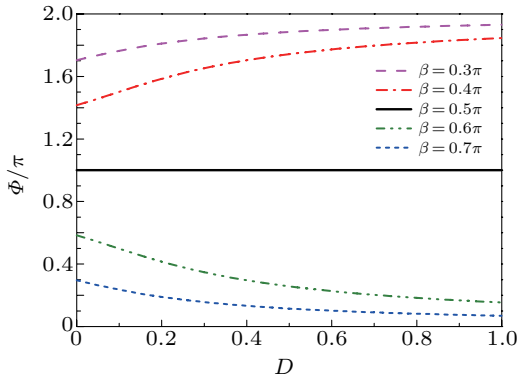


Fig. 2. (color online) The GP as a function of the DM interaction D at the critical point with different values of β, and $L = 351$, $\eta = \pi$, $\gamma = 0.4$, and $g = 0.03$.

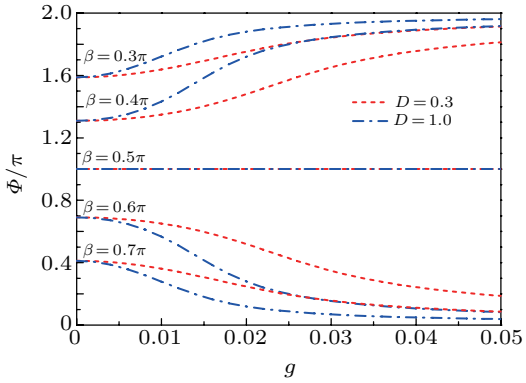


Fig. 3. (color online) The GP as a function of g with different values of β and D. The other parameters are $L = 351$, $\lambda = 1.0$, $\eta = \pi$, and $\gamma = 0.4$.

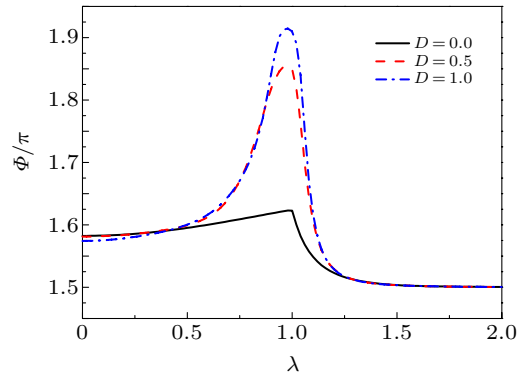


Fig. 4. (color online) The GP as a function of λ for various values of the DM interaction strength with $L = 351$, $\eta = \pi$, $g = 0.03$, and $\gamma = 0.4$.

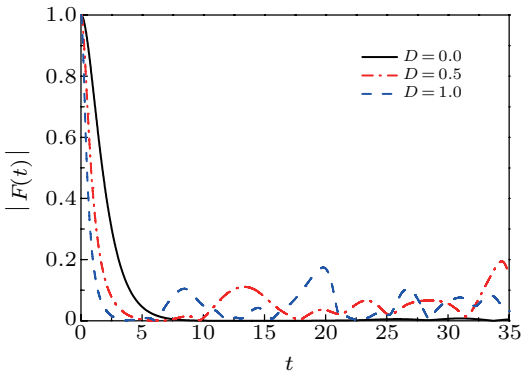


Fig. 5. (color online) The time evolution of the decoherence factor for various values of the DM interaction strength with $\gamma = 0.4$ and $L = 351$ at the critical point.

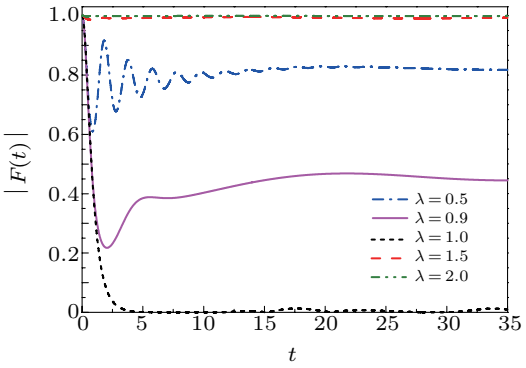


Fig. 6. (color online) The time evolution of the decoherence factor for different values of λ with $D = 0.3$. The other parameters are the same as those in Fig. 5.

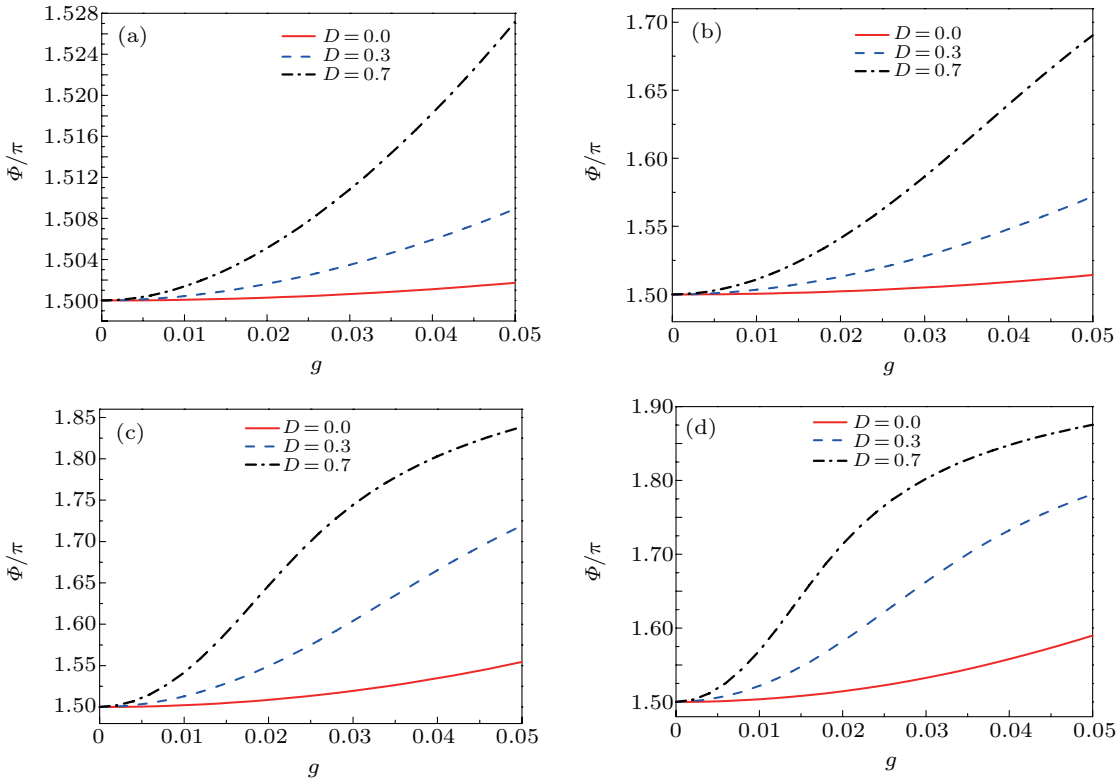


Fig. 7. (color online) The GP as a function of the coupling constant g at the critical point with different values of the DM interaction. (a) $L = 101$, (b) $L = 801$, (c) $L = 3001$, and (d) $L = 5001$. The other parameters are $\beta = \pi/3$, $\eta = 8\pi$, and $\gamma = 0.3$.

4. Conclusions

We considered a qubit coupled to an XY chain with the DM interaction, and investigated the effect of the DM interaction on the geometric phase of the central qubit. An analytical expression for the geometric phase of the qubit is obtained in the weak coupling limit. We found that the modification of the geometrical phase induced by the spin chain environment is very sensitive to and greatly enhanced by the DM interaction around the quantum phase transition point of the spin chain.

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