Efficient three-step entanglement concentration for an arbitrary four-photon cluster state*

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(Received 9 April 2012; revised manuscript received 28 September 2012)

We propose an entanglement concentration protocol to concentrate an arbitrary partially-entangled four-photon cluster state. As a pioneering three-step entanglement concentration scheme, our protocol only needs a single-photon resource to assist the concentration in each step, which makes this protocol more economical. With the help of the linear optical elements and weak cross-Kerr nonlinearity, one can obtain a maximally-entangled cluster state via local operations and classical communication. Moreover, the protocol can be iterated to obtain a higher success probability and is feasible under current experimental conditions.

Keywords: cluster state, entanglement concentration, cross-Kerr nonlinearity

PACS: 03.65.Ud, 03.67.Hk, 42.50.–p **DOI:** 10.1088/1674-1056/22/3/030305

1. Introduction

Quantum entanglement plays an important role in the field of quantum information. It is a crucial quantum resource with respect to various quantum information processing, such as quantum teleportation,[1] quantum dense coding,[2] quantum computation,[3] and quantum cryptography.[4–6] Most of the quantum information processing tasks are performed with only the maximally-entangled states.[7–11] However, in practical transmission and storage, the entanglement between quantum systems will inevitably interact with the environment, and the noise will degrade the entanglement. For this reason, many theoretical and experimental entanglement concentration protocols that aim to extract maximally-entangled states from some less-entangled states have been proposed during the past few decades. In 1996, Bennett *et al.* proposed an original entanglement concentration protocol (ECP) using collective measurements which are difficult to operate experimentally.[12] In 1999, Bose *et al.* proposed an entanglement concentration protocol based on entanglement swapping.[13] In 2001, Yamamoto *et al.* and Zhao *et al.* independently proposed two similar ECPs based on linear optics.[14,15] The ECPs based on cross-Kerr nonlinearity and a single-photon assistant were reported in Refs. [16] and [17].

Recently, more and more attention has been focused on the concentration for multi-particle systems. For a three-particle quantum system, there are two classes of states that cannot be converted into each other by local operations and classical communication (LOCC), namely, the GHZ state and the W state.[18] Generally speaking, the current concentration protocols for a two-particle system are usually suitable for a less-entangled GHZ state. However, these kinds of ECPs cannot be directly applied to the W state concentration. In light of this, many novel ECPs have been proposed to accomplish W state concentration in different physical systems.[19–23] As a universal resource in quantum computation and quantum information, the cluster state has been proven to be more immune to decoherence than the GHZ state[24], and can also be used as a powerful tool for performing nonlocality tests.[25,26] Because of these advantages, the cluster state has attracted more attention in theoretical research and practical applications. In 2005, a protocol was proposed by Zou *et al.* to generate a cluster state by using only linear optical elements.[27] Lately, Su *et al.* proposed an optimal protocol to generate a cluster state based on optical elements, cross-Kerr nonlinearity, and homodyne measurement.[28] Zhang *et al.* and Wang *et al.* independently proposed two schemes for implementing teleportation by using a cluster state as a quantum channel.[29,30] In 2012, Zhong *et al.* proposed a quantum information splitting scheme for an arbitrary three-qubit state by using a cluster state and a GHZ state as the quantum channels.[31] In order to guarantee the accuracy of the expected quantum communication tasks, maximally-entangled cluster states are required as quantum channels. Unfortunately, few schemes for concentrating multi-particle cluster states have

*Project supported by the National Natural Science Foundation of China (Grant Nos. 61068001 and 11264042), the Talent Program of Yanbian University, China (Grant No. 950010001), the National Science Foundation for Post-doctoral Scientists of China (Grant No. 2012M520612), and the Program for Chun Miao Excellent Talents of Department of Education of Jilin Province, China (Grant No. 201316).

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been proposed up to now.

In this paper, we present a pioneering three-step ECP to concentrate an arbitrary partially-entangled four-photon cluster state into a canonical maximally-entangled four-photon cluster state $(|HHHH\rangle+|HHVV\rangle+|VVHH\rangle-|VVVV\rangle)/2$. Here, H and V denote the horizontal and vertical linear polarizations, respectively. In this protocol, we only need some single-photon resources to assist the concentration. With the aid of some optical elements and weak cross-Kerr nonlinearity, our protocol does not require sophisticated single-photon detectors. Moreover, the protocol considers cyclic utilization to enhance the probability of success.

The remainder of this paper is organized as follows. In Section 2, we first review the key element of our ECP, i.e., the parity check detector (PCD) based on weak cross-Kerr nonlinearity. In Section 3, we describe our ECP in detail and explicitly calculate the success probability. In Section 4, a discussion and summary are given.

2. Review of PCD based on weak cross-Kerr nonlinearity

Before describing the ECP, we first explain the principle of PCD based on cross-Kerr nonlinearity. The Hamiltonian of the cross-Kerr nonlinearity is written as[32]

$$\hat{H}_{\rm K}=\hbar\chi\hat{n}_{\rm a}\hat{n}_{\rm c}. \tag{1}$$

Here, $\hat{n}_{\rm a}$ and $\hat{n}_{\rm c}$ are the photon number operators for modes a and c, respectively. χ is the coupling strength of the nonlinearity, which is decided by the property of the nonlinear material. We consider two single photons $|\psi\rangle_1=(|H\rangle_1+|V\rangle_1)/\sqrt{2}$ and $|\psi\rangle_2=(|H\rangle_2+|V\rangle_2)/\sqrt{2}$ coupled with a probe mode in coherent state $|\alpha\rangle$, as shown in Fig. 1. The state $|\psi\rangle_1$ is in spatial mode $\rm a_1$, and $|\psi\rangle_2$ is in spatial mode $\rm b_1$. The combined system with the coherent state under the nonlinear interaction, can be written as

$$\begin{aligned}&|\psi\rangle_1\otimes|\psi\rangle_2\otimes|\alpha\rangle\\ =\;&\frac{1}{\sqrt{2}}(|H\rangle_1+|V\rangle_1)\otimes\frac{1}{\sqrt{2}}(|H\rangle_2+|V\rangle_2)|\alpha\rangle\\ \longrightarrow\;&\frac{1}{2}(|HH\rangle_{12}+|VV\rangle_{12})|\alpha\rangle+\frac{1}{2}(|HV\rangle_{12}|\alpha\,{\rm e}^{-2{\rm i}\theta}\rangle\\ &+|VH\rangle_{12}|\alpha\,{\rm e}^{2{\rm i}\theta}\rangle).\end{aligned} \tag{2}$$

From Eq. (2), it can be seen that the even-parity states $|HH\rangle_{12}$ and $|VV\rangle_{12}$ lead to the coherent state picking up no phase shift. The odd-parity states $|HV\rangle_{12}$ and $|VH\rangle_{12}$ lead to the coherent state picking up -2θ and 2θ phase shifts, respectively. With the help of an X quadrature homodyne measurement, we can distinguish the even-parity states from the odd-parity states according to the phase shift of the coherent state.[32] A diagram of PCD based on weak cross-Kerr nonlinearity is shown in Fig. 1.

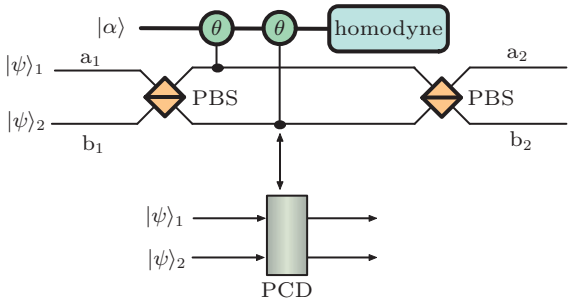


Fig. 1. (color online) The principle of PCD based on weak cross-Kerr nonlinearity.[33] The polarizing beam splitter (PBS) transmits the horizontal polarization $|H\rangle$ and reflects the vertical polarization $|V\rangle$. Homodyne represents an X quadrature homodyne measurement.

3. Cluster state concentration with linear optics and weak cross-Kerr nonlinearity

In the following, let us describe the partially-entangled four-photon cluster state concentration process in detail. With the PCD shown in Fig. 1, a schematic diagram of our ECP for less-entangled cluster states is shown in Fig. 2. Suppose that the less-entangled four-photon cluster state is initially described as

$$\begin{aligned}|{\rm Cluster}\rangle_{1234}\;=\;&a|HHHH\rangle_{1234}+b|HHVV\rangle_{1234}\\ &+c|VVHH\rangle_{1234}-d|VVVV\rangle_{1234},\end{aligned} \tag{3}$$

where $a^2+b^2+c^2+d^2=1$. Without loss of generality, let a, b, c, and d be real. Photons 1 and 4 belong to Alice, while photons 2 and 3 belong to Bob and Charlie, respectively.

In the first step, Alice introduces an additional photon in the polarization state

$$|\psi_1\rangle_5=\frac{d}{\sqrt{a^2+d^2}}|H\rangle_5+\frac{a}{\sqrt{a^2+d^2}}|V\rangle_5,$$

and let photons 4 and 5 pass through the PCD. The combined system with the coherent state evolves as

$$\begin{aligned}&|{\rm Cluster}\rangle_{1234}\otimes|\psi_1\rangle_5\otimes|\alpha\rangle\longrightarrow\\ &\Big(\frac{ad}{\sqrt{a^2+d^2}}|HHHHH\rangle_{12345}+\frac{ab}{\sqrt{a^2+d^2}}|HHVVV\rangle_{12345}\\ &+\frac{cd}{\sqrt{a^2+d^2}}|VVHHH\rangle_{12345}\\ &-\frac{ad}{\sqrt{a^2+d^2}}|VVVVV\rangle_{12345}\Big)|\alpha\rangle\\ &+\Big(\frac{bd}{\sqrt{a^2+d^2}}|HHVVH\rangle_{12345}\\ &-\frac{d^2}{\sqrt{a^2+d^2}}|VVVVH\rangle_{12345}\Big)|\alpha\,{\rm e}^{2{\rm i}\theta}\rangle\\ &+\Big(\frac{a^2}{\sqrt{a^2+d^2}}|HHHHV\rangle_{12345}\\ &+\frac{ac}{\sqrt{a^2+d^2}}|VVHHV\rangle_{12345}\Big)|\alpha\,{\rm e}^{-2{\rm i}\theta}\rangle.\end{aligned} \tag{4}$$

Through a general X homodyne measurement, if the probe mode is in the coherent state $|\alpha\rangle$, the five-photon state will be projected into

$$\begin{aligned}|\Phi_{10}\rangle = & \frac{ad}{\sqrt{2a^2d^2+a^2b^2+c^2d^2}}|HHHHH\rangle_{12345} \\ & + \frac{ab}{\sqrt{2a^2d^2+a^2b^2+c^2d^2}}|HHVVV\rangle_{12345} \\ & + \frac{cd}{\sqrt{2a^2d^2+a^2b^2+c^2d^2}}|VVHHH\rangle_{12345} \\ & - \frac{ad}{\sqrt{2a^2d^2+a^2b^2+c^2d^2}}|VVVVV\rangle_{12345}, \end{aligned} \quad (5)$$

with a probability of $p_1 = (2a^2d^2+a^2b^2+c^2d^2)/(a^2+d^2)$. Otherwise, if the probe mode is in $|\alpha e^{\pm 2i\theta}\rangle$, the five-photon state will be

$$\begin{aligned}|\varphi\rangle = & a'|HHHHV\rangle_{12345} + b'|HHVVH\rangle_{12345} \\ & + c'|VVHHV\rangle_{12345} - d'|VVVVH\rangle_{12345}, \end{aligned} \quad (6)$$

with

$$a' = \frac{a^2}{\sqrt{a^2+d^2}}, \quad b' = \frac{bd}{\sqrt{a^2+d^2}},$$

$$c' = \frac{ac}{\sqrt{a^2+d^2}}, \quad d' = \frac{d^2}{\sqrt{a^2+d^2}}. \quad (7)$$

In the following, Alice chooses the case where the probe mode picks up no phase shift and rotates the polarization of photon 5 with the half-wave plate (HWP_{45} in Fig. 2), and state (5) is changed into

$$\begin{aligned}|\Phi_{11}\rangle = & \frac{1}{\sqrt{2}}\left[\frac{ad}{\sqrt{2a^2d^2+a^2b^2+c^2d^2}}\right. \\ & \times(|HHHH\rangle_{1234} - |VVVV\rangle_{1234}) \\ & + \frac{ab}{\sqrt{2a^2d^2+a^2b^2+c^2d^2}}|HHVV\rangle_{1234} \\ & \left. + \frac{cd}{\sqrt{2a^2d^2+a^2b^2+c^2d^2}}|VVHH\rangle_{1234}\right]|H\rangle_5 \\ & + \frac{1}{\sqrt{2}}\left[\frac{cd}{\sqrt{2a^2d^2+a^2b^2+c^2d^2}}|VVHH\rangle_{1234}\right. \\ & - \frac{ab}{\sqrt{2a^2d^2+a^2b^2+c^2d^2}}|HHVV\rangle_{1234} \\ & + \frac{ad}{\sqrt{2a^2d^2+a^2b^2+c^2d^2}} \\ & \left.\times(|HHHH\rangle_{1234} + |VVVV\rangle_{1234})\right]|V\rangle_5. \end{aligned} \quad (8)$$

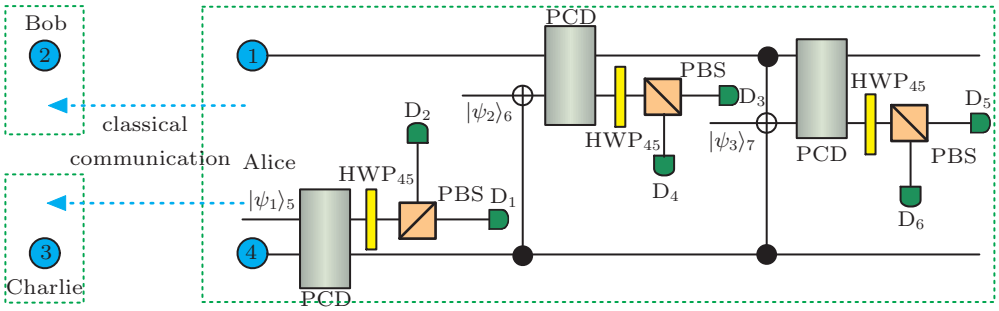


Fig. 2. (color online) Schematic diagram of entanglement concentration for an arbitrary four-photon cluster state with three PCDs. The PBS represents the polarizing beam splitter which is used to transmit the horizontal polarization $|H\rangle$ and reflect the vertical polarization $|V\rangle$. HWP rotates the polarization of the state by 45°. D_i is a single-photon detector. $|\psi_i\rangle_5$ represents the state of the ancillary single photon which is introduced each time.

After passing through a PBS, photon 5 is detected by two conventional photon detectors D_1 and D_2. If D_1 clicks, equation (8) becomes

$$\begin{aligned}|\Phi_{12}\rangle = & \frac{ad}{\sqrt{2a^2d^2+a^2b^2+c^2d^2}}|HHHH\rangle_{1234} \\ & + \frac{ab}{\sqrt{2a^2d^2+a^2b^2+c^2d^2}}|HHVV\rangle_{1234} \\ & + \frac{cd}{\sqrt{2a^2d^2+a^2b^2+c^2d^2}}|VVHH\rangle_{1234} \\ & - \frac{ad}{\sqrt{2a^2d^2+a^2b^2+c^2d^2}}|VVVV\rangle_{1234}. \end{aligned} \quad (9)$$

Otherwise, if D_2 clicks, Alice should let photon 4 go through a $(\pi/2)$-phase shifter whose function is $|V\rangle \longrightarrow -|V\rangle$ to obtain $|\Phi_{12}\rangle$.

The second step is an analogy with the first one. Alice introduces another photon in the polarization state

$$|\psi_2\rangle_6 = \frac{b}{\sqrt{b^2+d^2}}|H\rangle_6 + \frac{d}{\sqrt{b^2+d^2}}|V\rangle_6,$$

and the whole five-photon system can be written as

$$\begin{aligned}|\Phi_2\rangle = & |\Phi_{12}\rangle\otimes|\psi_2\rangle_6 \\ = & \frac{abd}{\sqrt{(b^2+d^2)(2a^2d^2+a^2b^2+c^2d^2)}} \\ & \times(|HHHHH\rangle_{12346} + |HHVVV\rangle_{12346} \\ & - |VVVVH\rangle_{12346}) \\ & + \frac{cd^2}{\sqrt{(b^2+d^2)(2a^2d^2+a^2b^2+c^2d^2)}}|VVHHV\rangle_{12346} \\ & + \frac{ad^2}{\sqrt{(b^2+d^2)(2a^2d^2+a^2b^2+c^2d^2)}} \\ & \times(|HHHHV\rangle_{12346} - |VVVVV\rangle_{12346}) \end{aligned}$$

$$+\frac{ab^2}{\sqrt{(b^2+d^2)(2a^2d^2+a^2b^2+c^2d^2)}}|HHVVH\rangle_{12346}$$
$$+\frac{bcd}{\sqrt{(b^2+d^2)(2a^2d^2+a^2b^2+c^2d^2)}}$$
$$\times|VVHHH\rangle_{12346}. \quad (10)$$

At the beginning, Alice carries out a controlled-NOT gate on photons 4 and 6, taking photon 4 as the control qubit and photon 6 as the target qubit. Then, Alice performs a similar operation on photons 1 and 6 as that on photons 4 and 5 in the first step, followed by the detection of photon 6, as shown in Fig. 2.

If D_3 clicks, then the original four-photon state becomes

$$|\Phi_{20}\rangle = \frac{ab}{\sqrt{3a^2b^2+c^2d^2}}(|HHHH\rangle_{1234}+|HHVV\rangle_{1234}-|VVVV\rangle_{1234})+\frac{cd}{\sqrt{3a^2b^2+c^2d^2}}|VVHH\rangle_{1234}. \quad (11)$$

Otherwise, if D_4 clicks, then Alice should let photon 1 go through a $(\pi/2)$-phase shifter to obtain $|\Phi_{20}\rangle$. The success probability of step two is

$$p_2=\frac{3a^2b^2d^2+c^2d^4}{(b^2+d^2)(2a^2d^2+a^2b^2+c^2d^2)}.$$

In the last step, Alice should introduce the third additional photon in the polarization state

$$|\psi_3\rangle_7=\frac{cd}{\sqrt{a^2b^2+c^2d^2}}|H\rangle_7+\frac{ab}{\sqrt{a^2b^2+c^2d^2}}|V\rangle_7,$$

and the whole five-photon system can be written as

$$|\Phi_3\rangle = |\Phi_{23}\rangle\otimes|\psi_3\rangle_7$$
$$= \frac{abcd}{\sqrt{(3a^2b^2+c^2d^2)(a^2b^2+c^2d^2)}}\times(|HHHHH\rangle_{12347}+|HHVVH\rangle_{12347}+|VVHHV\rangle_{12347}-|VVVVH\rangle_{12347})+\frac{c^2d^2}{\sqrt{(3a^2b^2+c^2d^2)(a^2b^2+c^2d^2)}}|VVHHH\rangle_{12347}+\frac{a^2b^2}{\sqrt{(3a^2b^2+c^2d^2)(a^2b^2+c^2d^2)}}\times(|HHHHV\rangle_{12347}+|HHVVV\rangle_{12347}-|VVVVV\rangle_{12347}). \quad (12)$$

Then Alice carries out a Toffoli gate on photons 1, 4, and 7, taking photons 1, 4 as the control qubits and photon 7 as the target qubit. The whole system state evolves as

$$|\Phi_{30}\rangle = \frac{abcd}{\sqrt{(3a^2b^2+c^2d^2)(a^2b^2+c^2d^2)}}\times(|HHHHH\rangle_{12347}+|HHVVH\rangle_{12347}+|VVHHV\rangle_{12347}-|VVVVV\rangle_{12347})+\frac{c^2d^2}{\sqrt{(3a^2b^2+c^2d^2)(a^2b^2+c^2d^2)}}|VVHHH\rangle_{12347}+\frac{a^2b^2}{\sqrt{(3a^2b^2+c^2d^2)(a^2b^2+c^2d^2)}}\times(|HHHHV\rangle_{12347}+|HHVVV\rangle_{12347}-|VVVVH\rangle_{12347}). \quad (13)$$

Next, let photons 1 and 7 experience the same operation as photons 4 and 5 in the first step, followed by the detection of photon 7, as shown in Fig. 2. If D_5 clicks, then the original four-photon state is left in

$$|\Phi_{31}\rangle = \frac{1}{2}(|HHHH\rangle_{1234}+|HHVV\rangle_{1234}+|VVHH\rangle_{1234}-|VVVV\rangle_{1234}). \quad (14)$$

Otherwise, if D_6 clicks, then Alice should let photon 1 go through a $(\pi/2)$-phase shifter to obtain $|\Phi_{31}\rangle$. The success probability of the third step is

$$p_3=\frac{4a^2b^2c^2d^2}{(3a^2b^2+c^2d^2)(a^2b^2+c^2d^2)}.$$

So far, we have fully explained our ECP process. The total success probability of sharing a nonlocal maximally-entangled cluster state among Alice, Bob, and Charlie is

$$P=p_1p_2p_3=\frac{4a^2b^2c^2d^4}{(a^2+d^2)(b^2+d^2)(a^2b^2+c^2d^2)}. \quad (15)$$

In our above discussion, we divide the ECP into three steps by introducing three ancillary photons. We only choose the same case where the probe mode picks up no phase shift after each PCD. In fact the discarded items in the three steps can also be reused as initial states for the next concentration process. It is obvious to see that Eq. (6) has the same form as Eq. (3). That is to say, the odd-parity states in each step can also be reused as the initial state for extracting the maximally-entangled cluster state. By iterating each step, we can obtain a higher success probability. The new success probability in each step can be calculated under the iteration condition.

In the first step, we obtain the success probability in each round as

$$P_1^1 = \frac{2a^2d^2+a^2b^2+c^2d^2}{a^2+d^2},$$
$$P_1^2 = \frac{(2a^2d^2+a^2b^2+c^2d^2)a^2d^2}{(a^2+d^2)(a^4+d^4)},$$
$$P_1^3 = \frac{(2a^2d^2+a^2b^2+c^2d^2)a^6d^6}{(a^2+d^2)(a^4+d^4)(a^8+d^8)},$$
$$\vdots$$
$$P_1^m = \frac{(2a^2d^2+a^2b^2+c^2d^2)a^{2^m-2}d^{2^m-2}}{(a^2+d^2)(a^4+d^4)\cdots(a^{2^m}+d^{2^m})}. \quad (16)$$

Here, subscripts $1,2,\ldots,m$ denote the iteration numbers.

In the same manner, we calculate the success probabilities in the second and third steps as

$$P_2^n = \frac{(3a^2b^2+c^2d^2)b^{2^n-2}d^{2^n}}{(2a^2d^2+a^2b^2+c^2d^2)(b^2+d^2)(b^4+d^4)\cdots(b^{2^n}+d^{2^n})},$$
$$P_3^t = \frac{4a^{2^t}b^{2^t}c^{2^t}d^{2^t}}{(3a^2b^2+c^2d^2)(a^2b^2+c^2d^2)(a^4b^4+c^4d^4)\cdots(a^{2^t}b^{2^t}+c^{2^t}d^{2^t})}. \quad (17)$$

Here, subscript 2 (3) denotes the second (third) step, and superscript $n(t)$ denotes the iteration number. Therefore, the total success probability of obtaining this maximally-entangled cluster state is

$$P_{\text{total}} = \sum_{m=1}^{\infty} P_1^m \sum_{n=1}^{\infty} P_2^n \sum_{t=1}^{\infty} P_3^t. \quad (18)$$

We plotted the total success probability of this ECP as shown in Fig. 3. Here, we suppose $c = d = 1/2$, and the independent variable $a \in (0, \sqrt{2}/2)$. Figure 3 indicates that the total success probability increases with the iteration number. At the same time, it is shown that all success probability monotonically increase with a, when $a \in (0, 1/2)$. They both have a maximal value when $a = 1/2$.

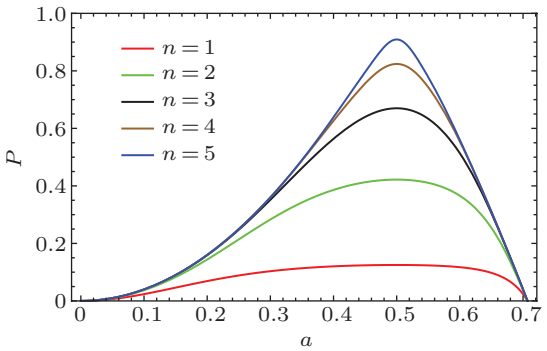


Fig. 3. (color online) The total success probability P of obtaining a maximally-entangled cluster state as a function of the initial coefficient a. Here, we suppose $c = d = 1/2$, and $a \in (0, \sqrt{2}/2)$. The variable n denotes the iteration number. The five curves in different colors denote the corresponding total success probabilities under different numbers of iterations.

4. Discussion and summary

In this section, let us outline some of the practical issues regarding the experimental feasibility of our protocol. First, it is clear that the feasibility of the present ECP depends on the accuracy of the PCD. The X quadrature homodyne measurement is the crucial ingredient in the PCD. Because of the fact that the coherent states of the probe mode with different phase shifts are not completely orthogonal, these states cannot be distinguished determinately. However, the probability of this error occurring is on the order of 10^{-3}, as long as $\alpha\theta^2 > 6.9$.[34] It has been suggested recently that the nonlinearities may be on the order of 10^{-2} with electromagnetically induced transparency.[35] That is to say, in the regime of weak cross-Kerr nonlinearity, we can implement the PCD with a high precision. Second, the feasibility of our protocol can also be influenced by the realization of quantum control gates, including a controlled-NOT gate and a Toffoli gate. Fortunately, many protocols have been provided for the realization of controlled-NOT [36–39] and Toffoli [40] gates using linear optics systems. With the aid of weak cross-Kerr nonlinearity, an almost deterministic controlled-NOT gate was realized by Nemoto *et al.* in 2004.[32] Following the same technique, Lin *et al.* proposed a protocol to realize the Toffoli gate in 2009.[41] Additionally, the corresponding success probability is 2/23. In the latter, their group proposed an almost deterministic protocol to realize the Toffoli gate by combining the controlled-path gate with the merging gate.[42] To sum up, our protocol has been shown to be feasible experimentally with the current technology.

In summary, we presented an ECP to concentrate an arbitrary partially-entangled four-photon cluster state. This protocol is a pioneering three-step entanglement concentration scheme and involves linear optical elements and nonlinear optical cross-Kerr nonlinearity. Moreover, our protocol only needs a single-photon resource to assist the concentration in each step, which makes it more economical. In addition, with PCD, it does not require the use of sophisticated single-photon detectors and can be iterated to obtain a higher success probability. All these advantages make our protocol feasible and useful in practical applications.

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