Exact solutions of the Dirac equation with Pöschl–Teller double-ring-shaped Coulomb potential via the Nikiforov–Uvarov method

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Exact analytical solutions of the Dirac equation are reported for the Pöschl–Teller double-ring-shaped Coulomb potential. The radial, polar, and azimuthal parts of the Dirac equation are solved using the Nikiforov–Uvarov method, and the exact bound-state energy eigenvalues and corresponding two-component spinor wavefunctions are reported.

Keywords: Dirac equation, Pöschl–Teller double-ring-shaped Coulomb potential, Nikiforov–Uvarov method

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1. Introduction

There has been a continued interest in finding the analytical solutions of the various wave equations of quantum mechanics, including the Dirac, Klein–Gordon, Duffin–Kemmer–Petiau (DKP), and spinless-Salpeter equations for various potentials; see Refs. [1]– [29] and references therein. Such studies are motivated by the phenomenological outcomes of various potentials in different physical sciences, as well as the deeper insights provided by the analytical techniques in comparison with their numerical partners. A primary concern in such studies is interaction. Radial potentials, despite all their benefits, cannot account for some of the observations, such as ring-shaped molecules or deformed nuclei.[30,31] A good candidate for such cases is the Pöschl–Teller double-ring-shaped Coulomb (PTDRSC) potential[8] possessing the form

$$\begin{aligned} &V(r,\theta,\phi)\\ &= -\frac{\delta}{r}+\frac{1}{r^2}\left[\frac{B}{\sin^2\theta}+\frac{A(A-1)}{\cos^2\theta}\right]\\ &\quad+\frac{1}{r^2\sin^2\theta}\left[\frac{\alpha^2 D(D-1)}{\sin^2\alpha\phi}+\frac{\alpha^2 C(C-1)}{\cos^2\alpha\phi}\right], \end{aligned} \tag{1}$$

where $A, C, D > 1$; $\delta > 0$; $B > 0$; $\alpha = 1, 2, 3, \ldots$ are real positive parameters. When $A = C = D = 1$ and $B = 0$, the PTDRSC potential reduces to the Coulomb potential, which is frequently used in many studies. When $A = C = D = 1$, then the potential reduces to the so-called Hartmann potential, which is used in quantum chemistry.[31] When $C = D = 1$, the PTDRSC potential reduces to the double-ring-shaped Coulomb potential. This potential is noncentral; its axial symmetry and invariance under reflection make it a highly symmetric system. Moreover, the accidental degeneracies in its energy spectrum confirm that it possesses not only a geometrical symmetry, but also a dynamical symmetry. Indeed, these interesting features and the fact that exact solutions can be obtained for the quantum mechanical problem warrant a closer study of the double-ring-shaped oscillator. These potentials provide a useful theoretical ground for describing the interaction between the ring-shaped molecules and that between distorted nuclei. An important aspect of the use of the novel angle-dependent potential is to study the rotational–vibrational dynamics of a diatomic molecule in noncentral potentials.[34–42] Here, we intend to solve the Dirac equation under this potential. The structure of this paper is as follows. In Section 2, we give a brief introduction to the Dirac equation, rewrite it with scalar and vector PTDRSC potentials, and separate it in the spherical coordinates. In Section 3, the Nikiforov–Uvarov (NU) method[21,32,33] is briefly introduced, and in Section 4, we solve the radial, polar, and azimuthal parts of the Dirac equation, respectively. Finally, conclusions are given in Section 5.

2. The Dirac equation with scalar and vector potentials

The Dirac equation with scalar potential $S(r)$ and vector potential $V(r)$ reads ($h = c = 1$)[24]

$$\begin{aligned} &[\boldsymbol{\alpha}\cdot\boldsymbol{p}+\boldsymbol{\beta}(M+S(\boldsymbol{r}))-\mathrm{i}\boldsymbol{\beta}\boldsymbol{\alpha}\cdot\hat{\boldsymbol{r}}U(r)]\,\psi(\boldsymbol{r})\\ &=[E-V(\boldsymbol{r})]\,\psi(\boldsymbol{r}), \end{aligned} \tag{2}$$

where E, $\boldsymbol{p} = -\mathrm{i}\nabla$, and M denote the relativistic energy of the system, the three-dimensional momentum operator, and the mass of the fermionic particle, respectively. $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 & \boldsymbol{\sigma}_i \\ \boldsymbol{\sigma}_i & 0 \end{pmatrix} \quad \beta = \begin{pmatrix} \boldsymbol{I} & 0 \\ 0 & -\boldsymbol{I} \end{pmatrix}, \quad i = 1, 2, 3, \tag{3}$$

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where $\boldsymbol{I}$ is the 2×2 unitary matrix, and the spin matrices are

$$\boldsymbol{\sigma}_1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\ \boldsymbol{\sigma}_2=\begin{pmatrix}0&-\mathrm{i}\\\mathrm{i}&0\end{pmatrix},\ \boldsymbol{\sigma}_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}. \tag{4}$$

In the Pauli–Dirac representation,

$$\boldsymbol{\Psi}(\boldsymbol{r})=\begin{pmatrix}\varphi(\boldsymbol{r})\\\chi(\boldsymbol{r})\end{pmatrix}. \tag{5}$$

Substituting Eqs. (3)–(5) into Eq. (2) gives

$$\boldsymbol{\sigma}\cdot\boldsymbol{p}\chi(\boldsymbol{r})=[E-V(\boldsymbol{r})-M-S(\boldsymbol{r})]\varphi(\boldsymbol{r}), \tag{6a}$$
$$\boldsymbol{\sigma}\cdot\boldsymbol{p}\phi(\boldsymbol{r})=[E-V(\boldsymbol{r})+M+S(\boldsymbol{r})]\chi(\boldsymbol{r}). \tag{6b}$$

When scalar potential $S(\boldsymbol{r})$ is equal to vector potential $V(\boldsymbol{r})$, equation (6a) becomes

$$\boldsymbol{\sigma}\cdot\boldsymbol{p}\chi(\boldsymbol{r})=[E-M-2V(\boldsymbol{r})]\phi(\boldsymbol{r}), \tag{7a}$$
$$\chi(\boldsymbol{r})=\frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+M}\varphi(\boldsymbol{r}). \tag{7b}$$

Substituting Eq. (7b) into Eq. (7a) yields[24]

$$[p^2+2(E+M)V(\boldsymbol{r})]\varphi(\boldsymbol{r})=[E^2-M^2]\varphi(\boldsymbol{r}). \tag{8}$$

Now, by inserting the PTDRSC potential into Eq. (8), we have

$$\left(-\boldsymbol{\nabla}^2+2(E_{nlm}+M)\left\{-\frac{\delta}{r}+\frac{1}{r^2}\left[\frac{B}{\sin^2\theta}+\frac{A(A-1)}{\cos^2\theta}\right]+\frac{1}{r^2\sin^2\theta}\left[\frac{\alpha^2D(D-1)}{\sin^2\alpha\phi}+\frac{\alpha^2C(C-1)}{\cos^2\alpha\phi}\right]\right\}+M^2-E_{nlm}^2\right)\varphi_{nlm}(\boldsymbol{r})=0. \tag{9}$$

To proceed further, let us consider a solution of the form[8]

$$\phi_{nlm}(\boldsymbol{r})=\frac{U_{nlm}(r)}{r}\frac{H_l(\theta)}{(\sin\theta)^{1/2}}\Phi_m(\phi). \tag{10}$$

The separation of variables in Eq. (9) results in

$$\frac{\mathrm{d}^2U_{nlm}(r)}{\mathrm{d}r^2}+\left[\frac{2(E_{nlm}+M)\delta}{r}+\frac{1/4-l^2}{r^2}-M^2+E_{nlm}^2\right]\times U_{nlm}(r)=0, \tag{11a}$$
$$\frac{\mathrm{d}^2H_l(\theta)}{\mathrm{d}\theta^2}+\left[\frac{1/4-2(E_{nlm}+M)B-m^2}{\sin^2\theta}-\frac{2(E_{nlm}+M)A(A-1)}{\cos^2\theta}+l^2\right]H_l(\theta)=0, \tag{11b}$$
$$\frac{\mathrm{d}^2\Phi_m(\phi)}{\mathrm{d}\phi^2}+\left[\frac{-2(E_{nlm}+M)\alpha^2D(D-1)}{\sin^2\alpha\phi}-\frac{2(E_{nlm}+M)\alpha^2C(C-1)}{\cos^2\alpha\phi}+m^2\right]\Phi_m(\phi)=0, \tag{11c}$$

where $l=0,1,2,\ldots$, and $m=0,\pm1,\pm2,\ldots$

3. Nikiforov–Uvarov method

As frequently noted, this mathematical technique solves the second-order differential equations. Let us consider the following differential equation:[21,32,33]

$$\left\{\frac{\mathrm{d}^2}{\mathrm{d}s^2}+\frac{\alpha_1-\alpha_2s}{s(1-\alpha_3s)}\frac{\mathrm{d}}{\mathrm{d}s}+\frac{1}{[s(1-\alpha_3s)]^2}[-\xi_1s^2+\xi_2s-\xi_3]\right\}\times\psi=0. \tag{12}$$

According to the NU method, the eigenfunctions and eigenenergies are obtained via[21,32,33]

$$\psi(s)=s^{\alpha_{12}}(1-\alpha_3s)^{-\alpha_{12}-\alpha_{13}/\alpha_3}\times P_n^{(\alpha_{10}-1,\alpha_{11}/\alpha_3-\alpha_{10}-1)}(1-2\alpha_3s), \tag{13}$$

$$\alpha_2n-(2n+1)\alpha_5+(2n+1)(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8})+n(n-1)\alpha_3+\alpha_7+2\alpha_3\alpha_8+2\sqrt{\alpha_8\alpha_9}=0, \tag{14}$$

where

$$\begin{aligned}&\alpha_4=\frac{1}{2}(1-\alpha_1),\ \alpha_5=\frac{1}{2}(\alpha_2-2\alpha_3),\ \alpha_6=\alpha_5^2+\xi_1,\\&\alpha_7=2\alpha_4\alpha_5-\xi_2,\ \alpha_8=\alpha_4^2+\xi_3,\\&\alpha_9=\alpha_3\alpha_7+\alpha_3^2\alpha_8+\alpha_6,\ \alpha_{10}=\alpha_1+2\alpha_4+2\sqrt{\alpha_8},\\&\alpha_{11}=\alpha_2-2\alpha_5+2(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8}),\ \alpha_{12}=\alpha_4+\sqrt{\alpha_8},\\&\alpha_{13}=\alpha_5-(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8}).\end{aligned} \tag{15}$$

In the rather more special case of $\alpha_3=0$,

$$\lim_{\alpha_3\to0}P_n^{(\alpha_{10}-1,\frac{\alpha_{11}}{\alpha_3}-\alpha_{10}-1)}(1-2\alpha_3s)=L_n^{\alpha_{10}-1}(\alpha_{11}s), \tag{16a}$$
$$\lim_{\alpha_3\to0}(1-\alpha_3s)^{-\alpha_{12}-\frac{\alpha_{13}}{\alpha_3}}=\mathrm{e}^{\alpha_{13}s}. \tag{16b}$$

From Eq. (14), we find

$$\psi=s^{\alpha_{12}}e^{\alpha_{13}s}L_n^{\alpha_{10}-1}(\alpha_{11}s). \tag{17}$$

4. Solutions

4.1. Solution of the radial equation

To obtain the solution of Eq. (11a), we first introduce the parameters

$$\begin{aligned}A_r&=2(E_{nlm}+M)\delta,\\B_r&=1/4-l^2,\\C_r&=E_{nlm}^2-M^2,\end{aligned} \tag{18}$$

and bring Eq. (11a) into the form

$$\frac{\mathrm{d}^2U_{nlm}(r)}{\mathrm{d}r^2}+\left(\frac{A_r}{r}+\frac{B_r}{r^2}+C_r\right)U_{nlm}(r)=0. \tag{19}$$

By comparing Eq. (19) with Eq. (12), we find

$$\begin{aligned}&\alpha_1=\alpha_2=\alpha_3=0,\ \xi_1=-C_r,\ \xi_2=A_r,\ \xi_3=-B_r,\\&\alpha_4=\frac{1}{2},\ \alpha_5=0,\ \alpha_6=\xi_1,\ \alpha_7=-\xi_2,\\&\alpha_8=\frac{1}{4}+\xi_3,\ \alpha_9=\xi_1,\end{aligned}$$

$$\alpha_{10} = 1+2\sqrt{\frac{1}{4}+\xi_3},\ \alpha_{11} = 2\sqrt{\xi_1},$$
$$\alpha_{12} = \frac{1}{2}+\sqrt{\frac{1}{4}+\xi_3},$$
$$\alpha_{13} = -\sqrt{\xi_1}, \tag{20}$$

which, from Eq. (14), determine the radial energy eigenvalues as

$$(2n+1)\sqrt{M^2-E_{nlm}^2}-2(E_{nlm}+M)\delta + 2\sqrt{l^2(M^2-E_{nlm}^2)}=0, \tag{21}$$

or

$$E_{nlm} = \frac{M(4n^2+4l+4l^2+1-4\delta^2+4n+8nl)}{4\delta^2+1+4l+4n+4l^2+8nl+4n^2}. \tag{22}$$

From Eq. (17), the radial eigenfunction is found to be

$$U_{nlm}(r) = r^{\frac{1}{2}+\sqrt{\frac{1}{4}-B_r}}\,\mathrm{e}^{-\sqrt{-C_r}r}L_n^{2\sqrt{\frac{1}{4}-B_r}}(2\sqrt{-C_r}r). \tag{23}$$

4.2. Solution of the polar equation

In this case, we have

$$\frac{\mathrm{d}^2H_l(\theta)}{\mathrm{d}\theta^2}+\left(\frac{A_\theta}{\cos^2\theta}+\frac{B_\theta}{\sin^2\theta}+C_\theta\right)H_l(\theta)=0, \tag{24}$$

where

$$A_\theta = -2(E_{nlm}+M)A(A-1),$$
$$B_\theta = 1/4-2(E_{nlm}+M)B-m^2,$$
$$C_\theta = l^2. \tag{25}$$

Using transformation $z=\cos^2\theta$, we write Eq. (24) as

$$\frac{\mathrm{d}^2H_l(z)}{\mathrm{d}z^2}+\frac{(1/2-z)}{z(1-z)}\frac{\mathrm{d}H_l(z)}{\mathrm{d}z}+\frac{1}{(z(1-z))^2} \times\left[\frac{-C_\theta z^2}{4}+\frac{(-A_\theta+B_\theta+C_\theta)z}{4}+\frac{A_\theta}{4}\right]H_l(z)=0. \tag{26}$$

By comparing Eq. (26) with Eq. (12), we find

$$\alpha_1=\frac{1}{2},\ \alpha_2=1,\ \alpha_3=1,\ \xi_1=\frac{C_\theta}{4},$$
$$\xi_2=\frac{(-A_\theta+B_\theta+C_\theta)}{4},\ \xi_3=-\frac{A_\theta}{4},$$
$$\alpha_4=\frac{1}{4},\ \alpha_5=-\frac{1}{2},\ \alpha_6=\frac{1}{4}+\xi_1,\ \alpha_7=-\frac{1}{4}-\xi_2,$$
$$\alpha_8=\frac{1}{16}+\xi_3,\ \alpha_9=\frac{1}{16}+\xi_1-\xi_2+\xi_3,$$
$$\alpha_{10}=1+2\sqrt{\frac{1}{16}+\xi_3},$$
$$\alpha_{11}=2+2\left(\sqrt{\frac{1}{16}+\xi_1-\xi_2+\xi_3}+\sqrt{\frac{1}{16}+\xi_3}\right),$$
$$\alpha_{12}=\frac{1}{4}+\sqrt{\frac{1}{16}+\xi_3},$$
$$\alpha_{13}=-\frac{1}{2}-\left(\sqrt{\frac{1}{16}+\xi_1-\xi_2+\xi_3}+\sqrt{\frac{1}{16}+\xi_3}\right). \tag{27}$$

Thus, substituting Eq. (27) into Eq. (14), we find

$$n'+(2n'+1)/2+(2n'+1)\left(\sqrt{\frac{1}{16}-\frac{B_\theta}{4}}+\sqrt{\frac{1}{16}-\frac{A_\theta}{4}}\right) + n'(n'-1)-\frac{1}{4}-\frac{(-A_\theta+B_\theta+C_\theta)}{4}+2\left(\frac{1}{16}-\frac{A_\theta}{4}\right) + 2\sqrt{\left(\frac{1}{16}-\frac{A_\theta}{4}\right)\left(\frac{1}{16}-\frac{B_\theta}{4}\right)}=0, \tag{28}$$

where

$$\begin{aligned} l = {} & (1/2)\Big[(16n'+5+16n'\sqrt{2BE_{nlm}+2BM+m^2}+8n'\sqrt{1+8E_{nlm}A^2-8E_{nlm}A+8MA^2-8MA} \\ & \times 8\sqrt{2BE_{nlm}+2BM+m^2}+4\sqrt{1+8E_{nlm}A^2-8E_{nlm}A+8MA^2-8MA}+16n'^2+8E_{nlm}A^2-8E_{nlm}A+8MA^2 \\ & -8MA+8BE_{nlm}+8BM+4m^2+4\sqrt{(1+8E_{nlm}A^2-8E_{nlm}A+8MA^2-8MA)(2BE_{nlm}+2BM+m^2)}\Big]^{1/2}. \end{aligned} \tag{29}$$

Substituting Eq. (27) into Eq. (13), we obtain

$$H_l(\theta) = (\cos^2\theta)^{\frac{1}{4}+\sqrt{\frac{1}{16}-\frac{A_\theta}{4}}}(1-\cos^2\theta)^{\frac{1}{4}+\sqrt{\frac{1}{16}-\frac{B_\theta}{4}}} \times P_n^{\left(2\sqrt{\frac{1}{16}-\frac{A_\theta}{4}},2\sqrt{\frac{1}{16}-\frac{B_\theta}{4}}\right)}(1-2\cos^2\theta). \tag{30}$$

4.3. Solution of the azimuthal equation

To obtain the solution of Eq. (11c), we first introduce the parameters

$$A_\varphi = -2(E_{nlm}+M)\alpha^2C(C-1),$$
$$B_\varphi = -2(E_{nlm}+M)\alpha^2D(D-1),$$
$$C_\varphi = m^2 \tag{31}$$

to write Eq. (11c) as

$$\frac{\mathrm{d}^2\Phi_m(\varphi)}{\mathrm{d}\varphi^2}+\left(\frac{A_\varphi}{\cos^2\alpha\varphi}+\frac{B_\varphi}{\sin^2\alpha\varphi}+C_\varphi\right)\Phi_m(\varphi)=0. \tag{32}$$

We now apply the transformation $z=\cos^2\alpha\phi$ to bring Eq. (32) into the form for the NU method

$$\frac{\mathrm{d}^2\Phi_m(z)}{\mathrm{d}z^2}+\frac{(1/2-z)}{z(1-z)}\frac{\mathrm{d}\Phi_m(z)}{\mathrm{d}z}+\frac{1}{(z(1-z))^2} \times\left[\frac{-C_\phi z^2}{4\alpha^2}+\frac{(-A_\phi+B_\phi+C_\phi)z}{4\alpha^2}+\frac{A_\phi}{4\alpha^2}\right]\Phi_m(z)=0, \tag{33}$$

and the required parameters are

$$\alpha_1=\frac{1}{2},\ \alpha_2=1,\ \alpha_3=1,\ \xi_1=\frac{C_\varphi}{4\alpha^2},$$
$$\xi_2=\frac{(-A_\varphi+B_\varphi+C_\varphi)}{4\alpha^2},\ \xi_3=-\frac{A_\varphi}{4\alpha^2},\ \alpha_4=\frac{1}{4},$$
$$\alpha_5=-\frac{1}{2},\ \alpha_6=\frac{1}{4}+\xi_1,\ \alpha_7=-\frac{1}{4}-\xi_2,$$
$$\alpha_8=\frac{1}{16}+\xi_3,\ \alpha_9=\frac{1}{16}+\xi_1-\xi_2+\xi_3,$$
$$\alpha_{10}=1+2\sqrt{\frac{1}{16}+\xi_3},$$
$$\alpha_{11}=2+2(\sqrt{\frac{1}{16}+\xi_1-\xi_2+\xi_3}+\sqrt{\frac{1}{16}+\xi_3}),$$
$$\alpha_{12}=\frac{1}{4}+\sqrt{\frac{1}{16}+\xi_3},$$
$$\alpha_{13}=-\frac{1}{2}-(\sqrt{\frac{1}{16}+\xi_1-\xi_2+\xi_3}+\sqrt{\frac{1}{16}+\xi_3}). \quad (34)$$

Thus, substituting Eq. (34) in Eq. (14), we find

$$n''+(2n''+1)/2+(2n''+1)\left(\sqrt{\frac{1}{16}-\frac{B_\phi}{4\alpha^2}}+\sqrt{\frac{1}{16}-\frac{A_\phi}{4\alpha^2}}\right)+n''(n''-1)$$
$$-\frac{1}{4}-\frac{(-A_\phi+B_\phi+C_\phi)}{4\alpha^2}+2\left(\frac{1}{16}-\frac{A_\phi}{4\alpha^2}\right)$$
$$+\sqrt{\left(\frac{1}{16}-\frac{A_\phi}{4\alpha^2}\right)\left(\frac{1}{16}-\frac{B_\phi}{4\alpha^2}\right)}=0. \quad (35)$$

Using Eq. (35), m is found as

$$\begin{aligned}m &= (1/2)\{6-8E_{nlm}D+16n''\\ &+8n''\sqrt{1+8D^2E_{nlm}-8E_{nlm}D+8D^2M-8MD}\\ &+8n''\sqrt{1+8C^2E_{nlm}-8E_{nlm}C+8C^2M-8MC}\\ &+4\sqrt{1+8D^2E_{nlm}-8E_{nlm}D+8D^2M-8MD}\\ &+4\sqrt{1+8C^2E_{nlm}-8E_{nlm}C+8C^2M-8MC}\\ &+16n''^2+8C^2E_{nlm}-8E_{nlm}C+8C^2M-8MC\\ &+8D^2E_{nlm}+8D^2M-8MD+2[1+128C^2E_{nlm}D^2M\\ &-128C^2E_{nlm}MD-128E_{nlm}CD^2M+128E_{nlm}CMD\\ &-8E_{nlm}C-8MC-8E_{nlm}D-8MD+64C^2E_{nlm}^2D^2\\ &-64C^2E_{nlm}^2D-64E_{nlm}^2CD^2+64E_{nlm}^2CD+64C^2M^2D^2\\ &-64C^2M^2D-64M^2CD^2+64M^2CD+8C^2E_{nlm}\\ &+8C^2M+8D^2E_{nlm}+8D^2M]^{1/2}\}^{1/2}D. \quad (36)\end{aligned}$$

Substituting Eq. (34) into Eq. (13), we obtain

$$\Phi_m(\phi)=(\cos^2\alpha\phi)^{\frac{1}{4}+\sqrt{\frac{1}{16}-\frac{A_\phi}{4\alpha^2}}}(1-\cos^2\alpha\phi)^{\frac{1}{4}+\sqrt{\frac{1}{16}-\frac{B_\phi}{4\alpha^2}}}$$
$$\times P_n^{\left(2\sqrt{\frac{1}{16}-\frac{A_\phi}{4\alpha^2}},2\sqrt{\frac{1}{16}-\frac{B_\phi}{4\alpha^2}}\right)}(1-2\cos^2\alpha\phi). \quad (37)$$

Therefore, $\varphi_{nlm}(r,\theta,\phi)$ is

$$\varphi_{nlm}(\boldsymbol{r})=\frac{U_{nlm}(r)}{r}\frac{H_l(\theta)}{(\sin\theta)^{1/2}}\Phi_m(\phi)$$
$$=\frac{1}{r}\left[r^{\frac{1}{2}+\sqrt{\frac{1}{4}-B_r}}\mathrm{e}^{-\sqrt{-C_r}r}L_n^{2\sqrt{\frac{1}{4}-B_r}}(2\sqrt{-C_r}r)\right]\frac{1}{(\sin\theta)^{1/2}}\left[P_n^{\left(2\sqrt{\frac{1}{16}-\frac{A_\theta}{4}},2\sqrt{\frac{1}{16}-\frac{B_\theta}{4}}\right)}(1-2\cos^2\theta)(\cos^2\theta)^{\frac{1}{4}+\sqrt{\frac{1}{16}-\frac{A_\theta}{4}}}\right.$$
$$\left.\times(1-\cos^2\theta)^{\frac{1}{4}+\sqrt{\frac{1}{16}-\frac{B_\theta}{4}}}\right](\cos^2\alpha\phi)^{\frac{1}{4}+\sqrt{\frac{1}{16}-\frac{A_\phi}{4\alpha^2}}}(1-\cos^2\alpha\phi)^{\frac{1}{4}+\sqrt{\frac{1}{16}-\frac{B_\phi}{4\alpha^2}}}P_n^{\left(2\sqrt{\frac{1}{16}-\frac{A_\phi}{4\alpha^2}},2\sqrt{\frac{1}{16}-\frac{B_\phi}{4\alpha^2}}\right)}(1-2\cos^2\alpha\phi). \quad (38)$$

and the total wavefunction obtained from Eq. (7b) is

$$\begin{aligned}\Psi_{nlm}(r,\theta,\phi) &= \frac{1}{r}\left[r^{\frac{1}{2}+\sqrt{\frac{1}{4}-B_r}}\mathrm{e}^{-\sqrt{-C_r}r}L_n^{2\sqrt{\frac{1}{4}-B_r}}(2\sqrt{-C_r}r)\right]\frac{1}{(\sin\theta)^{1/2}}P_n^{\left(2\sqrt{\frac{1}{16}-\frac{A_\phi}{4\alpha^2}},2\sqrt{\frac{1}{16}-\frac{B_\phi}{4\alpha^2}}\right)}(1-2\cos^2\alpha\phi)\\ &\times\left[(\cos^2\theta)^{\frac{1}{4}+\sqrt{\frac{1}{16}-\frac{A_\theta}{4}}}(1-\cos^2\theta)^{\frac{1}{4}+\sqrt{\frac{1}{16}-\frac{B_\theta}{4}}}P_n^{(2\sqrt{\frac{1}{16}-\frac{A_\theta}{4}},2\sqrt{\frac{1}{16}-\frac{B_\theta}{4}})}(1-2\cos^2\theta)\right]\\ &\times(\cos^2\alpha\phi)^{\frac{1}{4}+\sqrt{\frac{1}{16}-\frac{A_\phi}{4\alpha^2}}}(1-\cos^2\alpha\phi)^{\frac{1}{4}+\sqrt{\frac{1}{16}-\frac{B_\phi}{4\alpha^2}}}\left(\frac{1}{\frac{\sigma.p}{E_{nlm}+M}}\right). \quad (39)\end{aligned}$$

5. Conclusion

We obtained exact solutions of the radial, polar, and azimuthal parts of the Dirac equation for the PTDRSC potential by using the Nikiforov–Uvarov method. We found that the problem is solved simply without having to deal with any numerical programming. Our solution yields the results for the Coulomb and Hartmann potentials as special cases and can be directly applied to the study of deformed nuclei.

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