Melting phenomenon in magneto hydro-dynamics steady flow and heat transfer over a moving surface in the presence of thermal radiation

Reda G. Abdel-Rahman, M. M. Khader†, and Ahmed M. Megahed

Department of Mathematics, Faculty of Science, Benha University, Benha 13518, *Egypt*

(Received 4 July 2012; revised manuscript received 27 August 2012)

The Lie group method is applied to present an analysis of the magneto hydro-dynamics (MHD) steady laminar flow and the heat transfer from a warm laminar liquid flow to a melting moving surface in the presence of thermal radiation. By using the Lie group method, we have presented the transformation groups for the problem apart from the scaling group. The application of this method reduces the partial differential equations (PDEs) with their boundary conditions governing the flow and heat transfer to a system of nonlinear ordinary differential equations (ODEs) with appropriate boundary conditions. The resulting nonlinear system of ODEs is solved numerically using the implicit finite difference method (FDM). The local skin-friction coefficients and the local Nusselt numbers for different physical parameters are presented in a table.

Keywords: Lie group method, magneto hydro-dynamics, melting phenomenon, Newtonian fluid radiation

PACS: 02.20.Sv, 47.15.–x, 44.40.+a, 44.10.+i **DOI:** 10.1088/1674-1056/22/3/030202

1. Introduction

The melting heat transfer phenomenon has been a subject of intensive experimental and theoretical investigations for many years because it is closely related to a wide variety of technologically important processes. The melting of soil, the artificial freezing of ground for mining and construction purposes, the freezing of soil around the heat exchanger coils of a ground based pump, magma solidification, melting of permafrost, thawing of frozen grounds, the preparation of semiconductor-material thermal energy storage, the freeze treatment of sewage and the casting and welding of a manufacturing process are just a few important applications. Roberts[1] was the first to describe the melting phenomena of ice placed in a hot stream of air at a steady state. The similarity between melting and diffusional mass transfer or transpiration cooling problems was first noted by Yen and Tien.[2] Epstein and Cho[3] studied the melting heat transfer in a steady laminar flow over a stationary flat plate. Sparrow *et al*.[4] studied the velocity and temperature fields, the heat transfer rate, and the melting layer thickness in a melting region for natural convection. It is worth mentioning that important and new results on the flow and heat transfer with the melting phenomenon in a porous medium were presented by Kazmierczak *et al*.,[5] Bakier,[6] Gorla *et al*.,[7] Tashtouch,[8] and Cheng and Lin.[9] The melting process by the heat convection mechanism based on an unsteady state has been presented by Cheng and Lin.[10] Walker[11] studied the phenomenon of permafrost degradation in Alaska, which accelerates the green-house effect and is very critical to global warming. Cheng and Lin[12] studied the melting process on the heat mass transfer by the mixed convection mechanism based on an unsteady state. The boundary layer behavior on a moving continuous surface is an important type of flow occurring in many manufacturing processes such as metal extrusion, glass fiber production, hot rolling, continuous casting, and a polymer sheet extruded continuously from a die. It is worth mentioning that the flow and heat transfer characteristics on a moving flat plate in a parallel stream with a constant surface heat flux have been considered by Ishak *et al*.[13] Important and new results on a moving continuous surface were recently presented by Bachok *et al*.,[14] Ishak *et al*.,[15] and Chamkha *et al*.[16] Motivated by the work mentioned above, the main objective of this work is to study the effects of melting and thermal radiation on the magneto hydro-dynamics (MHD) steady laminar flow over a moving horizontal surface.

The Lie group analysis is a classical symmetry based method discovered by Norwegian mathematician Sophius Lie for finding invariant and similarity solutions.[17] This method unifies almost all known exact integration techniques for both ordinary and partial differential equations. A theoretical background to the Lie group method can be found in Refs. [18]–[21]. The classical symmetry method has been successfully applied to some different physical partial differential equations under potential symmetries, see Refs. [22]– [25].

The finite difference method (FDM) plays an important role in recent research in this field. It has been shown that this procedure is a powerful tool for solving various kinds of problems.[26–29] We introduce the implicit FDM to reduce the problem to a system of non-linear algebraic equations, which are solved by using the Newton iteration method.

†Corresponding author. E-mail: mohamedmbd@yahoo.com

© 2013 Chinese Physical Society and IOP Publishing Ltd

2. Formulation of the problem

Consider a moving surface melting at a steady state, which emerges from a narrow slit at the origin, as shown in Fig. 1. The x axis is chosen along the plane of the surface, and the y axis is taken to be normal to the plane. We assume that the surface is moving with velocity U. It is also assumed that the temperature of the melting surface is T_m and the temperature in the free stream condition is T_∞, where $T_\infty > T_m$. The fluid is assumed to have a low electrical conductivity over the continuous moving surface under the influence of a uniform transverse magnetic field with field intensity B_0. Moreover, it is assumed that the viscous dissipation and the heat generation and absorption can be neglected.

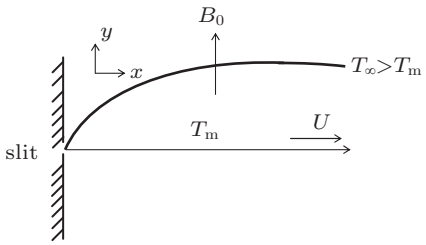


Fig. 1. Boundary layer on a moving continuous surface.

The velocity and temperature fields of the steady liquid flow obey the following boundary layer equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2 u}{\rho x}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_\mathrm{p}}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_\mathrm{p}}\left(\frac{\partial q_\mathrm{r}}{\partial y}\right), \tag{3}$$

with the boundary conditions

$$u=U, \quad T=T_\mathrm{m} \quad \text{at} \quad y=0, \tag{4}$$

$$u\to 0, \quad T\to T_\infty \quad \text{as} \quad y\to\infty, \tag{5}$$

$$\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0}=\rho[\lambda+c_\mathrm{s}(T_\mathrm{m}-T_0)]v(x,0), \tag{6}$$

where x and y are the Cartesian coordinates along and normal to the surface, respectively, u and v are the velocity components along the x and y directions, respectively, ρ is the fluid density, μ is the dynamic viscosity, T is the temperature of the fluid, κ is the thermal conductivity, q_r is the radiative heat flux, c_p is the specific heat at constant pressure, σ is the electric conductivity, U is the surface velocity, λ is the latent heat of the fluid, and c_s is the heat capacity of the solid surface. Equation (6) states that the heat conducted to the melting surface is equal to the heat of melting plus the sensible heat required to raise the solid temperature T_0 to its melting temperature T_m, see Refs. [3] and [13].

Using the Rosseland approximation for radiation,[30] we have $q_\mathrm{r}=\frac{-4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}$, where σ^* is the Stefan–Boltzman constant, and k^* is the mean absorption coefficient. Following Raptis,[31] we assume that the temperature difference within the flow is such that T^4 may be expanded in a Taylor series. Expanding T^4 about T_∞ and neglecting higher orders, we get $T^4=4T_\infty^3T-3T_\infty^4$. Therefore, equation (3) becomes

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_\mathrm{p}}\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma^* T_\infty^3}{3\rho c_\mathrm{p}k^*}\frac{\partial^2 T}{\partial y^2}. \tag{7}$$

The non-dimensional system function $\psi(x,y)$ is introduced to have

$$u=\frac{\partial\psi}{\partial y}, \quad v=-\frac{\partial\psi}{\partial x}, \tag{8}$$

which satisfies Eq. (1) identically. If we introduce the non-dimensional temperature θ defined by $\theta=\frac{T-T_\mathrm{m}}{T_\infty-T_\mathrm{m}}$, equations (2) and (7) respectively become

$$H_1=\psi_y\psi_{xy}-\psi_x\psi_{yy}-\frac{\mu}{\rho}\psi_{yyy}+\frac{\sigma B_0^2}{\rho x}\psi_y=0, \tag{9}$$

$$H_2=\psi_y\theta_x-\psi_x\theta_y-\left(\frac{\kappa}{\rho c_\mathrm{p}}+\frac{16\sigma^* T_\infty^3}{3\rho c_\mathrm{p}k^*}\right)\theta_{yy}=0, \tag{10}$$

with the boundary conditions

$$\psi_y=U, \quad \theta=0 \quad \text{at} \quad y=0, \tag{11}$$

$$\psi_y\to 0, \quad \theta\to 1 \quad \text{as} \quad y\to\infty, \tag{12}$$

$$(T_\infty-T_\mathrm{m})\theta_y\big|_{y=0}+\rho\left[\lambda+c_\mathrm{s}(T_\mathrm{m}-T_0)\right]\psi_{xy=0}=0, \tag{13}$$

where the subscripts denote the partial derivatives.

3. Symmetry group analysis

A symmetry admitted by Eqs. (9) and (10) is a transformation mapping any solution of Eqs. (9) and (10) into another solution of the equations. The symmetry transformation, to be useful, must be defined in terms of the frame. We consider the Lie point symmetry characterized by an infinitesimal generator of the form

$$\begin{aligned}\chi \;=\; &\xi(x,y,\psi,\theta)\frac{\partial}{\partial x}+\zeta(x,y,\psi,\theta)\frac{\partial}{\partial y}\\ &+\Phi(x,y,\psi,\theta)\frac{\partial}{\partial\psi}+\Theta(x,y,\psi,\theta)\frac{\partial}{\partial\theta},\end{aligned} \tag{14}$$

with the invariant surface conditions

$$\begin{aligned}&\zeta(x,y,\psi,\theta)\psi_x+\zeta(x,y,\psi,\theta)\psi_y-\Phi=0,\\ &\zeta(x,y,\psi,\theta)\theta_x+\zeta(x,y,\psi,\theta)\theta_y-\Theta=0.\end{aligned} \tag{15}$$

The infinitesimal generator (14) corresponds to a one-parameter (ε) Lie group of point transformations

$$\begin{aligned}\bar{x}&=x+\varepsilon\,\xi(x,y,\psi,\theta)+\mathrm{O}(\varepsilon^2),\\ \bar{y}&=y+\varepsilon\,\zeta(x,y,\psi,\theta)+\mathrm{O}(\varepsilon^2),\\ \bar{\psi}&=\psi+\varepsilon\,\Phi(x,y,\psi,\theta)+\mathrm{O}(\varepsilon^2),\end{aligned}$$

$$\bar{\theta} = \theta + \varepsilon\Theta(x, y, \psi, \theta) + \mathrm{O}(\varepsilon^2), \tag{16}$$

where ξ, ζ, Φ, and Θ are infinitesimals of the group. The prolongation extension of Eq. (16) to an infinitesimal generator acting on all derivatives of ψ and θ is given by

$$\begin{aligned}\chi^2 = {} & \chi + \Phi^x\frac{\partial}{\partial\psi_x} + \Phi^y\frac{\partial}{\partial\psi_y} + \Theta^x\frac{\partial}{\partial\theta_x} + \Theta^y\frac{\partial}{\partial\theta_y} \\ & + \Phi^{yy}\frac{\partial}{\partial\psi_{yy}} + \Theta^{yy}\frac{\partial}{\partial\theta_{yy}} \\ & + \Phi^{yx}\frac{\partial}{\partial\psi_{xy}} + \Phi^{yyy}\frac{\partial}{\partial\psi_{yyy}},\end{aligned} \tag{17}$$

where Φ^x, Φ^y, Θ^x, Θ^y, Φ^{yy}, Θ^{yy}, Φ^{yx}, and Φ^{yyy} are given by

$$\begin{aligned}
\Phi^x &= \mathrm{D}_x(\Phi) - u_x\mathrm{D}_x(\xi) - u_y\mathrm{D}_x(\zeta), \\
\Phi^y &= \mathrm{D}_y(\Phi) - \psi_x\mathrm{D}_y(\xi) - \psi_y\mathrm{D}_y(\zeta), \\
\Theta^x &= \mathrm{D}_x(\Theta) - \theta_x\mathrm{D}_x(\xi) - \theta_y\mathrm{D}_x(\zeta), \\
\Theta^y &= \mathrm{D}_y(\Theta) - \theta_x\mathrm{D}_y(\xi) - \theta_y\mathrm{D}_y(\zeta), \\
\Phi^{yy} &= \mathrm{D}_y(\Phi^y) - \psi_{xy}\mathrm{D}_y(\xi) - \psi_{yy}\mathrm{D}_y(\zeta), \\
\Theta^{yy} &= \mathrm{D}_y(\Theta^y) - \theta_{xy}\mathrm{D}_y(\xi) - \theta_{yy}\mathrm{D}_y(\zeta), \\
\Phi^{yx} &= \mathrm{D}_x(\Phi^y) - \psi_{xx}\mathrm{D}_x(\xi) - \psi_{yx}\mathrm{D}_x(\zeta), \\
\Phi^{yyy} &= \mathrm{D}_y(\Phi^{yy}) - \psi_{xyy}\mathrm{D}_y(\xi) - \psi_{yyy}\mathrm{D}_y(\zeta),
\end{aligned} \tag{18}$$

with D_x and D_y being the operators of total differentiation with respect to x and y, respectively. The operator χ is a point symmetry of Eqs. (9) and (10) if

$$\chi^2(H_i)|_{H_i=0} = 0, \quad i = 1, 2. \tag{19}$$

Equation (19) is called the invariant conditions and implies that

$$\begin{aligned}
&\psi_x\Phi^{xy} + \psi_{xy}\Phi^y - \psi_{yy}\Phi^x - \psi_x\Phi^{yy} - \frac{\mu}{\rho}\Phi^{yyy} \\
&- \frac{\sigma B_0^2}{\rho x^2}\xi\psi_y + \frac{\sigma B_0^2}{\rho x}\Phi^y + \Phi u_y = 0,
\end{aligned} \tag{20}$$

$$\psi_y\Theta^x + \theta_x\Phi^y - \frac{\kappa}{\rho c_\rho}\Theta^{yy} - \Phi^x\theta_y - \Theta^y\psi_x = 0. \tag{21}$$

Substituting Φ^x, Φ^y, Θ^x, Θ^y, Φ^{yy}, Θ^{yy}, Φ^{yx}, and Φ^{yyy} into Eqs. (20) and (21) and setting all the coefficients of like derivatives of ψ and θ to zero, we obtain a system of determining equations for infinitesimals ξ, ζ, Φ, and Θ. Solving these determining equations, we find that functions ξ, ζ, Φ and Θ should be in the following forms:

$$\begin{aligned}
&\xi = 2c_1x, \quad \zeta = c_1y + \gamma(x), \\
&\Phi = c_1\psi + c_2, \quad \Theta = c_3 + c_4\theta,
\end{aligned} \tag{22}$$

where c_1, c_2, c_3, and c_4 are arbitrary constants, and $\gamma(x)$ is an arbitrary function of x. Applying the invariant condition (19) on the boundary conditions (11)–(13), we get $\gamma(x) = c_2 = c_3 = c_4 = 0$.

The knowledge of the infinitesimals ξ, ζ, Φ, and Θ of Eqs. (9) and (10) and boundary conditions (11)–(13) enable us to obtain a reduction of variables in Eqs. (9) and (10), which can be obtained by solving the characteristic equations

$$\frac{\mathrm{d}x}{2c_1x} = \frac{\mathrm{d}y}{c_1y} = \frac{\mathrm{d}\psi}{c_1\psi} = \frac{\mathrm{d}\theta}{0}. \tag{23}$$

These may be solved to obtain

$$\eta = \frac{y}{\sqrt{x}}, \quad \psi = \sqrt{x}G(\eta), \quad \theta = h(\eta). \tag{24}$$

Substituting Eq. (24) into Eqs. (9) and (10), we obtain a system of nonlinear ordinary differential equations in the following form:

$$\frac{\mu}{\rho}G''' + \frac{1}{2}GG'' - \frac{\sigma B_0^2}{\rho}G' = 0, \tag{25}$$

$$\left(\frac{\kappa}{\rho c_\mathrm{p}} + \frac{16\sigma^* T_\infty^3}{3\rho c_\mathrm{p}k^*}\right)h'' + \frac{1}{2}Gh' = 0. \tag{26}$$

The appropriate boundary conditions (11)–(13) are expressed as

$$G'(0) = U, \quad h(0) = 0, \quad G'(\infty) = 0, \quad h(\infty) = 1, \tag{27}$$

$$h'(0) + \left(\frac{[\lambda + c_\mathrm{s}(T_\mathrm{m} - T_0)]}{T_\infty - T_\mathrm{m}}\right)\frac{\rho}{\kappa}G(0) = 0. \tag{28}$$

We introduce the following transformations for η, G, and $h(\eta)$ in Eqs. (25)–(27):

$$\begin{aligned}
&\eta^* = \sqrt{\frac{U\rho}{2\mu}}\,\eta, \quad G(\eta) = \sqrt{\frac{2\mu U}{\rho}}\,f(\eta^*), \\
&h(\eta) = \sqrt{\frac{2\mu U}{\rho}}\,\theta(\eta^*).
\end{aligned} \tag{29}$$

Equations (25)–(28) then take the following form:

$$f''' + ff'' - M_\mathrm{a}f' = 0, \tag{30}$$

$$\frac{1}{Pr}(1 + R)\theta'' + f\theta' = 0, \tag{31}$$

$$f'(0) = 1, \quad \theta(0) = 0, \quad Prf(0) + M\theta'(0) = 0, \tag{32}$$

$$f' \to 0, \quad \theta \to 1 \quad \text{as} \quad \eta \to \infty, \tag{33}$$

where the primes denote the differentiation with respect to η, $M_\mathrm{a} = \frac{2\sigma B_0^2}{U\rho}$ is the magnetic number, $Pr = \frac{\mu c_\mathrm{p}}{\kappa}$ is the Prandtl number, $M = \frac{c_\mathrm{p}(T_\infty - T_\mathrm{m})}{\lambda + c_\mathrm{p}(T_\mathrm{m} - T_0)}$ is the dimensionless melting parameter, and $R = \frac{16\sigma^* T_0^3}{3k^*\kappa}$ is the radiation parameter.

The physical quantities of interest are the skin-friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_\mathrm{f} = \frac{\tau_w}{\rho U^2/2}, \quad Nu_x = \frac{xq_\mathrm{w}}{\kappa(T_\infty - T_\mathrm{m})}, \tag{34}$$

where τ_w and q_w are the shear stress and the heat transfer from the surface of the plate, respectively, and are given by

$$\tau_w = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -\kappa \left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{35}$$

Using the non-dimensional variables (24) and (29), we obtain

$$C_f = \sqrt{2} Re_x^{\frac{-1}{2}} f''(0), \quad Nu_x = -\frac{1}{\sqrt{2}} Re_x^{\frac{1}{2}} \theta'(0), \tag{36}$$

where $Re_x = \rho U x/\mu$ is the local Reynolds number.

4. Finding solution with FDM

In this section, we use the finite difference method to solve Eqs. (30) and (31) with the boundary conditions (32) and (33). This method has been tested for accuracy and efficiency by solving different problems.[16,26]

To implement the FDM, we simplify Eqs. (30)–(33) by using the transformation $f'(\eta) = v(\eta)$ to rewrite this system in the following form:

$$f' - v = 0, \tag{37}$$
$$v'' + fv' - M_a v = 0, \tag{38}$$
$$(1+R)\theta'' + Prf\theta' = 0, \tag{39}$$

with the boundary conditions

$$Prf(0) + M\theta'(0) = 0, \quad v(0) = 1,$$
$$\theta(0) = 0, \quad v(\infty) = 0, \quad \theta(\infty) = 1. \tag{40}$$

In the finite difference method, the space of solution's domain $(0, \mu_\infty)$ is discretized. We will use $\Delta\eta = \hbar > 0$ as the grid size in the η direction, $\Delta\eta = \mu_\infty/N$, with $\eta_i = i\hbar$ for $i = 0, 1, \ldots, N$. Define $f_i = f(\eta_i)$, $v_i = v(\eta_i)$, and $\theta_i = \theta(\eta_i)$. Let F_i, V_i, and Θ_i denote the numerical values of f, v, and θ at the i-th node, respectively. We have

$$f'|_i = \frac{f_{i+1} - f_{i-1}}{2\hbar} + O(\hbar^2),$$
$$v'|_i = \frac{v_{i+1} - v_{i-1}}{2\hbar} + O(\hbar^2),$$
$$\theta'|_i = \frac{\theta_{i+1} - \theta_{i-1}}{2\hbar} + O(\hbar^2), \tag{41}$$
$$v''|_i = \frac{v_{i+1} - 2v_i + v_{i-1}}{\hbar^2} + O(\hbar^2),$$
$$\theta''|_i = \frac{\theta_{i+1} - 2\theta_i + \theta_{i-1}}{\hbar^2} + O(\hbar^2). \tag{42}$$

The main step is to discretize the system of ordinary differential equations (37)–(39) in space by using the FDM. To do this, we substitute Eqs. (41) and (44) into Eqs. (37)–(39) and neglect the truncation errors, the resulting algebraic equations take the form ($i = 1, 2, \ldots, N-1$)

$$F_{i+1} - F_{i-1} - 2\hbar V_i = 0, \tag{43}$$
$$V_{i+1} - 2V_i + V_{i-1} + 0.5\hbar F_i(V_{i+1} - V_{i-1}) - \hbar^2 M_a V_i = 0, \tag{44}$$
$$(1+R)(\Theta_{i+1} - 2\Theta_i + \Theta_{i-1})$$
$$+ 0.5\hbar Pr F_i(\Theta_{i+1} - \Theta_{i-1}) = 0. \tag{45}$$

The boundary conditions are

$$F_0 = \frac{-M}{2\hbar Pr}\Theta_1, \quad V_0 = 1, \quad \Theta_0 = 0, \quad V_N = 0, \quad \Theta_N = 1. \tag{46}$$

The system of Eqs. (43)–(46) is the numerical scheme of the finite difference method and represents a nonlinear system of algebraic equations in variables F_i, V_i, and Θ_i. In our calculation, we use the Mathematica package and the Newton iteration method with a suitable initial solution to obtain the numerical solution.

5. Results and discussion

In this work, we study the effects of the melting phenomenon and thermal radiation on the magneto hydrodynamic flow and heat transfer. Figure 2 reveals the velocity distributions for different values of melting parameter M. It is observed that increasing the melting parameter M leads to an increase in both the velocity and the boundary layer thickness. Increasing the melting parameter M causes a decrease in the temperature distribution, as observed from Fig. 3. This behavior means that a small melting parameter M is due to the free stream temperature T_∞ approximately equal to the surface temperature T_m, i.e., $T_\infty \simeq T_m$, which implies an enhancement in the temperature distribution. The presence of a magnetic field in an electrically conducting fluid introduces a force called the Lorentz force which acts against the flow if the magnetic field is applied in the normal direction as in our problem. This resistive force tends to slow down the flow, so the effect of decreasing the magnetic parameter M_a is to increase the velocity as shown in Fig. 4. Increasing the magnetic parameter M_a also leads to a decrease in the temperature of the fluid inside the boundary layer, as shown in Fig. 5. Figures 6 and 7 show the effects of the Prandtl number Pr on the velocity and temperature profiles. It is shown in Fig. 6 that the velocity increases within the boundary layer as the Prandtl number Pr decreases, but the temperature distribution is observed to increase as the Prandtl number Pr increases, as shown in Fig. 7. The effects of the thermal radiation parameter R on the velocity profiles are plotted in Fig. 8. It is evident from this figure that the presence of thermal radiation inversely affects the activity of the fluid velocity. This can be seen clearly from the velocity, which decreases as the radiation parameter R increases. In addition, it is found that an increase in R leads to a decrease in the conduction effect and the thermal boundary layer, as illustrated in Fig. 9.

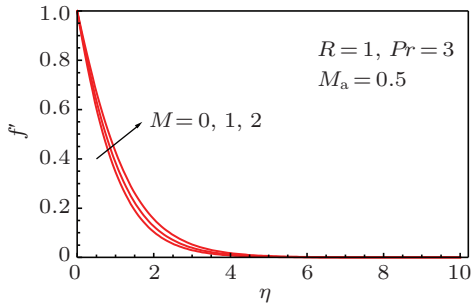


Fig. 2. (color online) Velocity distributions for various values of M.

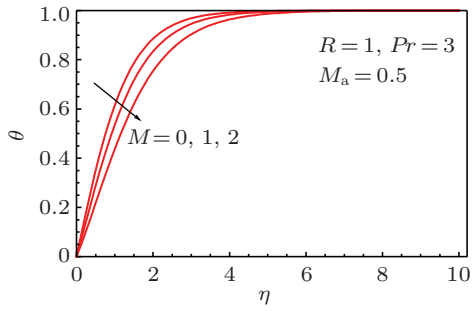


Fig. 3. (color online) Temperature distributions for various values of M.

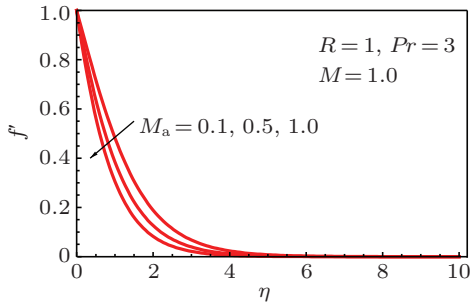


Fig. 4. (color online) Velocity distributions for various values of M_a.

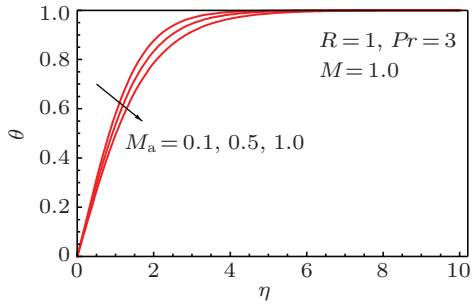


Fig. 5. (color online) Temperature distributions for various values of M_a.

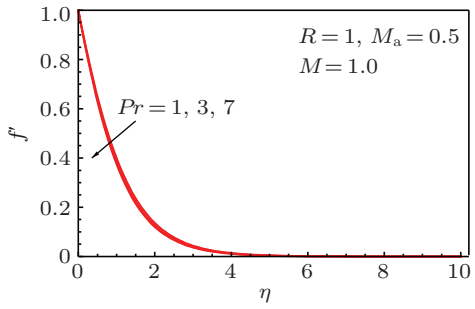


Fig. 6. (color online) Velocity distributions for various values of Pr.

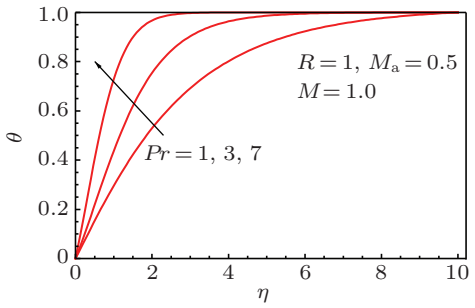


Fig. 7. (color online) Temperature distributions for various values of Pr.

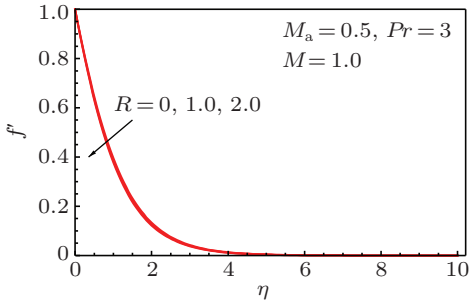


Fig. 8. (color online) Velocity distributions for various values of R.

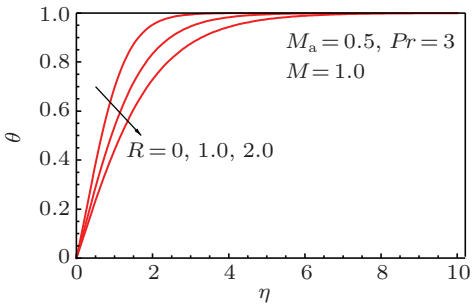


Fig. 9. (color online) Temperature distributions for various values of R.

Table 1. The $f''(0)$ and $-\theta'(0)$ for various values of M, M_a, Pr, and R.

M	M_a	Pr	R	$f''(0)$	$-\theta'(0)$
0.0	0.5	3	1	–0.90584	–0.73599
1.0	0.5	3	1	–0.81047	-0.56894
2.0	0.5	3	1	–0.75176	–0.46907
1.0	0.1	3	1	–0.57603	–0.61487
1.0	0.5	3	1	–0.81047	–0.56894
1.0	1.0	3	1	–1.03937	–0.52373
1.0	0.5	1	1	–0.77527	–0.26455
1.0	0.5	3	1	–0.81047	–0.56894
1.0	0.5	7	1	–0.83542	–0.96741
1.0	0.5	3	0.0	–0.78413	–0.72579
1.0	0.5	3	1.0	–0.81047	–0.56894
1.0	0.5	3	2.0	–0.82736	–0.46637

Table 1 presents the skin-friction coefficient in terms of $f''(0)$ and the rate of heat transfer in terms of $-\theta'(0)$ for various values of M, M_a, Pr, and R. From this table, we can observe that the negative value of $-\theta'(0)$ means that the heat flows from the fluid to the solid surface. Thus, increasing the

melting parameter M decreases both the absolute value of the heat transfer rate at the solid–fluid interface $-\theta'(0)$ and the absolute value of the skin friction coefficient $f''(0)$, but the reverse is true for the Prandtl number Pr. Increasing both the magnetic number M_a and the radiation parameter R causes an enhance of the absolute value of the skin friction coefficient and a fall in the absolute value of the heat transfer rate.

6. Conclusion

The problem of MHD flow and heat transfer over a moving horizontal surface with a melting mechanism based on a steady state in the presence of thermal radiation is considered. The governing partial differential equations are converted into a system of ordinary differential equations by similarity transformation before being solved numerically using an implicit finite difference method. The main results of this study provide information regarding velocity, temperature, skin-friction coefficient, and the local Nusselt number. Furthermore, the effects of the melting, the radiation, the magnetic parameters and the Prandtl number on the flow and heat transfer characteristics are thoroughly examined. Moreover, the boundary layer thickness and the velocity are found to increase as the melting parameter increases, but the reverse is true for both the magnetic and the radiation parameters.

References

[1] Roberts L 1958 *J. Fluid Mech.* **4** 505
[2] Yen Y C and Tien C 1963 *J. Geophys. Res.* **68** 3673
[3] Epstein M and Cho D H 1976 *J. Heat Transfer* **98** 531
[4] Sparrow E M, Patankar S V and Ramadhyani S 1977 *ASME J. Heat Transfer* **99** 520
[5] Kazmierczak M, Poulikakos D and Sadowski D 1987 *Int. Comm. Heat Mass Transfer* **14** 507
[6] Bakier A Y 1997 *Transport in Porous Media* **29** 127
[7] Gorla R S R, Mansour M A, Hassanien I A and Bakier A Y 1999 *Transport in Porous Media* **36** 245
[8] Tashtouch B 2005 *Eng. Convers Manage* **46** 2566
[9] Cheng W T and Lin C H 2007 *Int. J. Heat Mass Transfer* **50** 3026
[10] Cheng W T and Lin C H 2006 *Int. J. Eng. Sci.* **44** 1023
[11] Walker G 2007 *Nature* **446** 718
[12] Cheng W T and Lin C H 2008 *Int. Commun. Heat Mass Transfer* **35** 1350
[13] Ishak A, Nazar R and Pop I 2008 *Heat Mass Transfer* **45** 563
[14] Bachok N, Ishak A and Pop I 2010 *Phys. Lett. A* **374** 4075
[15] Ishak A, Nazar R, Bachok N and Pop I 2010 *Heat Mass Transfer* **46** 463
[16] Chamkha A J, Ahmed S E and Aloraier A S 2010 *Int. J. Phy. Sci.* **5** 1212
[17] Lie S 1984 *Matrh Annalan* **24** 52
[18] Ibragimove N H 1994 *Handbook of Lie Group Analysis of Differential Equations II. Application in Engineering and Physical Science* (Boca Raton: CRC Press)
[19] Hill J 1992 *Differential Equations and Group Methods for Science and Engineers* (Boca Raton: CRC Press)
[20] Olver P J 1993 *Application of Lie Groups to Differential Equations Graduate Text in Mathematics* Vol. **107** (2nd edn.) (New York: Springer)
[21] Bluman G W and Cole J D 1974 *Similarity Methods for Differential Equations: Applied Mathematical Science* Vol. **13** (New York: Springer)
[22] Johnpillai I K, McCue S W and Hill J M 2005 *J. Math. Anal. Appl.* **301** 135
[23] Kalpakides V K and Balassas K G 2004 *Int. J. Nonlinear Mechanics* **39** 1659
[24] Saied E A and Abdel-Rahman R G 1998 *J. Phys. Sco. Japan.* **68** 360
[25] Abdel-Rahman R G 2006 *J. Quantitative Spectroscopy & Radiative Transfer* **98** 1
[26] Sweilam N H, Khader M M and Nagy A M 2011 *Comput. Appl. Maths.* **235** 2832
[27] Butcher J C 2003 *Numerical Methods for Ordinary Differential Equations* (New York: John Wiley & Sons)
[28] Tanvir M, Faisal R and Islam A K M 2006 *ARPN Journal of Engineering and Applied Sciences* **1** 13
[29] Johnston H and Liu J G 2002 *Journal of Computational Physics* **180** 120
[30] Raptis A 1998 *Int. J. Heat Mass Transfer* **41** 2865
[31] Raptis A 1999 *Int. Comm. Heat Mass Transfer* **26** 889