Coherent nonlinear structures in dense electron–positron plasma

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It is shown that rarefactive-type double layer structures exist in ultradense electron–positron plasma. For this purpose, an extended Korteweg de Vries equation is derived and solved analytically in the low amplitude limit by employing the appropriate fluid equations. A strong influence of quantum degeneracy pressure of electrons and positrons, quantum diffraction effects and concentration of background positive ions on double layer is noticed. It is also pointed out that the amplitude and steepness of the double layer increases with an increase in ion concentration or ion charge number. The results are examined numerically for some interesting cases of dense plasmas with illustrations.

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1. Introduction

The physics of degenerate quantum plasmas has rapidly grown beyond the conventional plasmas found in space or laboratories; for an overview see, e.g., Refs. [1] and [2]. In such plasmas, quantum degeneracy effects are dominant at low temperatures and high densities. These are predominantly found in dense astrophysical objects (e.g., white and brown dwarfs, neutron stars, magnetars, and so on), core of giant planets (e.g., Jovian planets) and are aimed to be produced in the next generation of laser-based plasma compression schemes.[3–5] Theoretical models predict that the density, magnetic field, and temperature of such plasmas vary over a wide range of parameters. For instance, in dense astrophysical plasmas and intense laser-solid target interaction experiments, the degenerate electron number density may exceed the density of solid matter by many orders.[2,4,6] The magnetic field is estimated to vary from a few kilogauss to a few gigagauss (petagauss) in white dwarfs (neutron stars) and temperature as high as in fusion plasma. Quantum degeneracy and relativity effects are ubiquitous here because the average inter-Fermion distance is comparable to or less than the thermal de Broglie wavelength of electron $\lambda_{\rm Be}(=h/\sqrt{2\pi m_{\rm e}k_{\rm B}T})$, where $m_{\rm e}$ is the mass of electron, h is Planck's constant, $k_{\rm B}$ is the Boltzmann constant, and T is thermal temperature. Quantum effects of lighter species (electrons, positrons, holes, etc.) are more pronounced due to their smaller mass than the heavier species (ions), which may behave classically or quantum mechanically depending upon the degeneracy parameter, which is larger than unity for the quantum case. At extremely high densities, the electron Fermi energy $\epsilon_{\rm Fe}[=h^2/8m_{\rm e}(3n_{\rm e}/\pi)^{2/3}]$ becomes much larger than thermal energy $\sim k_{\rm B}T$. In this situation, the thermal pressure of electrons may be negligible as compared to the Fermi degeneracy pressure which arises due to the implications of Pauli's principle. The ratio of the typical momentum of thermal distribution and the limiting Fermi momentum ($p_{\rm Fe}$) of electron is small, i.e., $8\pi(3m_{\rm e}k_{\rm B}T)^{3/2}/3n_{\rm e}h^2 \leq 1$. Then, all the momentum states below $p_{\rm Fe}$ are filled while those above $p_{\rm Fe}$ are empty. Pauli's principle restricts the momentum states to the values $n(p) = 8\pi p^2/h^3$ when $p \leq p_{\rm Fe}$ and $n(p) = 0$ when $p > p_{\rm Fe}$, such that $n_{\rm e} = \int_0^{p_{\rm Fe}} n(p)\,{\rm d}p = 8\pi p_{\rm Fe}^3/3h^3$. Such degenerate electrons may be non-relativistic, relativistic or ultra-relativistic depending upon the ratio of the Fermi energy to the rest mass energy of electron. In compact astrophysical objects, the mass is imparted by the ions/nuclei and degeneracy pressure by the electrons which prevent them from collapse counterbalancing the inward pull of gravity. The plasma in their interior is degenerate and relativistic under such extreme conditions (e.g., the bulk electron number density in white dwarfs reaches upto 10^{30} cm^{-3}). However, the relativity effects diminish in the outer layers. It is believed that the most common constituents of white dwarf stars include fully ionized helium, carbon, and oxygen with an iron core and an outer layer of hydrogen.[7] These atoms have almost an equal number of protons and neutrons. The thermal energy of degenerate remnant stars produce observable luminosity. The estimates of star pulsations, radiation, and collective oscillations can provide an effective probe for the properties of inside matter.[8]

In recent years, a large number of investigations have been carried out in degenerate plasma dynamics by using different models employing the equation of state $P_{\rm e} \sim n_{\rm e}^{5/3}$, which is the non-relativistic limit of relativistic-degenerate plasma case in which $P_{\rm e} \sim n_{\rm e}^{4/3}$.[9–12] It was recognized early on that the underlying physics of nonlinear quantum-like equations can be better understood by transforming mathematical equations in hydrodynamic form. Such equations represent the evolution of quantum particle densities and mo-

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menta. Over the past few years, the studies of quantum plasmas in the framework of the hydrodynamic model have rapidly increased. This includes quantum ion-acoustic waves, and instabilities,[13,14] low frequency waves, and solitary structures in quantum dusty plasma,[15,16] solitons and shocks in quantum electron–positron–ion plasma,[17–19] and so on. The peculiarities of modeling nonlinear collective interactions in quantum plasmas have been pointed out recently in Refs. [2] and [20]. The authors have described the essential physics and salient features of dense degenerate quantum plasmas in a pedagogical way. The importance of quantum plasmas has also been recognized in ultrasmall electronic devices,[21] metallic and carbon nanostructures,[22,23] and plasmonics.[24]

The structural properties of ion-acoustic double layers (DLs) have been studied in classical plasmas both experimentally and theoretically.[25–27] Due to the significance of such structures in charged particle acceleration, the formation and propagation of DLs has also been investigated in multicomponent quantum plasmas in various settings.[28,29]

Electron–positron (ep) pair plasma is a class of equal-mass and absolute-charge particle plasmas believed to exist abundantly in the early universe and many astrophysical environments.[30,31] The next generation of ultrahigh-power lasers will enable new light-matter interactions leading to overdense ep plasmas in the laboratory reaching number density of $5\times10^{26}/\mathrm{cm}^3$.[32] The elementary physics of ep pair plasmas is different from ordinary matter having well-known properties.[9,33] The pair annihilation process is significant in such plasmas, which is analogous to recombination in electron–ion plasmas. For collective plasma effects to play a role, the annihilation time scale should be larger than the time scale of collective effects, which is the inverse of plasma frequency. It has also been pointed out in literature that the annihilation of electrons and positrons is relatively unimportant in classical,[34] as well as dense quantum plasmas.[35,36] The annihilation time τ_{an} for a nonrelativistic plasma is given by[37] $4\Theta/[3(1+6\Theta)n_{\mathrm{e,p}}\sigma_\tau c]$, where $\sigma_\tau = 6.65\times10^{-25}\ \mathrm{cm}^{-2}$ is the Thomson cross section, and c is the speed of light in vacuum. Here, the normalized thermal energy is defined as $\Theta = k_{\mathrm{B}}T_{\mathrm{Fe}}/mc^2$ which leads to $\zeta^2 \le \Theta \le 1$ where $\zeta = 7.29\times10^{-3}$ is the fine structure constant. In terms of temperature, $3\times10^5 < T_{\mathrm{F}}(K) < 5.9\times10^9$. Using $n_{\mathrm{e}0} = n_{\mathrm{p}0} = n_0$, $\Theta > 2.66\times10^{-19}n_0^{1/2}$ and the pair annihilation can be neglected for the density range $3.9\times10^{28} < n_0(\mathrm{cm}^{-3}) < 1.4\times10^{37}$, which has also been shown by Ali *et al.*,[35] recently. In this regime, the electron–positron annihilation time is much larger than the inverse of electron/positron plasma frequency $\omega_{\mathrm{pe,p}}$. In many situations of dense astrophysical plasmas, it is believed that the ions are also present in the background in addition to electron–positron (ep) pairs.[35] It makes the investigation of waves and nonlinear structures in quantum pair plasmas owing to the dynamics of degenerate electrons and positrons in the presence of ions background of practical interest. In this paper, we study the formation of electrostatic DL structures by using the Korteweg de Vries theory. The extended Korteweg de Vries (eKdV) equation is derived and solved analytically by employing the quantum hydrodynamic (QHD) model.

The paper is organized in the following way. The theoretical model to describe the electrostatic DL structures, and the linear wave analysis are given in Sections 2 and 3, respectively. In Section 4, the eKdV equation is derived and solution is obtained. The numerical results and discussions are presented in Section 5, and a summary is given in Section 6.

2. Basic set of equations

Consider unmagnetized ultradense plasma of equal mass species, i.e., $m_j = m$, where $j =$ e and p for electrons and positrons, respectively. The mean rate of momentum transport per unit area gives rise to the degeneracy pressure. In the fully degenerate and relativistic limit, this pressure can be given by[3,38]

$$P_j = Af(r), \tag{1}$$

where

$$A = \frac{\pi}{3}\left(\frac{mc}{h}\right)^3 mc^2 (= 6\times10^{22}(\mathrm{cgs})),$$
$$f(r) = r\sqrt{1+r^2}(2r^2-3)+3\ln(r+\sqrt{1+r^2}),$$
$$r = p_{\mathrm{F}j}/mc \cong (n_j/5.9\times10^{29}\ \mathrm{cm}^{-3})^{1/3},$$

and Fermi relativistic momentum $p_{\mathrm{F}j} = (3h^3n_j/8\pi)^{1/3}$. Equation (1) was first used by Chandrasekhar to estimate the critical mass limit $M_\odot$ (also known as the Chandrasekhar limit or Chandrasekhar mass) of white dwarf stars which results from the degeneracy pressure of the relativistic electron gas. We consider the case of spinless particles and ignore the relativistic effects due to the smallness of electron/positron rest mass energy in comparison with the corresponding Fermi energy. Then, the equation of state (1) leads to,

$$P_j = Kn_j^\gamma, \tag{2}$$

where $K = (4\pi^2\hbar^2/5m)(3/8\pi)^{2/3}$, $\gamma = 5/3$, $\hbar = h/2\pi$, and $T_j \ll T_{\mathrm{F}j}(= \epsilon_{\mathrm{F}j}/k_{\mathrm{B}})$ with $T_{\mathrm{F}j}$ being Fermi temperature. In equilibrium, we assume that the stationary positive ions are present in background so that the charge neutrality condition $n_{\mathrm{e}0} = n_{\mathrm{p}0} + Z_{\mathrm{i}}n_{\mathrm{i}0}$ is satisfied, where Z_{i} is the charge state of ions and subscript 0 denotes the unperturbed value. The set of normalized equations describing the nonlinear electrostatic waves in one spatial dimension can be written as

$$\partial_t v_{\mathrm{e}} + v_{\mathrm{e}}\partial_x v_{\mathrm{e}} = \partial_x\varphi - (1/5n_{\mathrm{e}})\partial_x n_{\mathrm{e}}^{5/3} + (H_{\mathrm{e}}^2/2)\partial_x(\partial_x^2\sqrt{n_{\mathrm{e}}}/\sqrt{n_{\mathrm{e}}}), \tag{3}$$

$$\partial_t v_{\mathrm{p}} + v_{\mathrm{p}}\partial_x v_{\mathrm{p}} = -\partial_x\varphi - (\delta/5n_{\mathrm{p}})\partial_x n_{\mathrm{p}}^{5/3} + (\delta^{1/2}H_{\mathrm{p}}^2/2)\partial_x(\partial_x^2\sqrt{n_{\mathrm{p}}}/\sqrt{n_{\mathrm{p}}}), \tag{4}$$

$$\partial_t n_j + \partial_x(n_j v_j) = 0, \tag{5}$$

$$\partial_x^2\varphi = n_e - (1-\alpha)n_p - \alpha. \quad (6)$$

Here n_j, v_j, and φ are the normalized quantities representing the plasma number density, fluid velocity, and electrostatic wave potential, respectively. Further, $\delta = T_{Fp}/T_{Fe}$ and $\alpha = Z_i n_{i0}/n_{e0}$. The quantum diffraction effects of electrons and positrons are represented by the nondimensional parameters $H_e = \hbar\omega_{pe}/2k_B T_{Fe}$ and $H_p = \hbar\omega_{pp}/2k_B T_{Fp}$, respectively, with $\omega_{pj} = \sqrt{4\pi n_{j0}e^2/m}$ being the plasma frequency. The rescaling parameters used in the set of Eqs. (3)–(6) are defined in the following fashion

$$n_j' \to n_j/n_{j0},\ v_j' \to v_j/v_{Fj},\ \varphi' \to e\varphi/(2k_B T_{Fe}),$$
$$x' \to \omega_{pe}x/v_{Fj},\ t' \to t\omega_{pe}. \quad (7)$$

We have dropped the primes from the normalized perturbed quantities for simplicity.

3. Linear wave analysis

To obtain the linear dispersion relation, we assume that the quantities in Eqs. (3)–(6) follow sinusoidal perturbations proportional to $\exp[i(kx-\omega t)]$, where ω and k are the wave frequency and wave vector, respectively. It leads to the dispersion relation

$$\omega^4 - \left[(v_{Fe}^2\beta_1 + v_{Fp}^2\beta_2)k^2 + (2-\alpha)\omega_{pe}^2\right]\omega^2 + \omega_{pe}^2 v_{Fp}^2 k^2\beta_2 + \omega_{pe}^2 v_{Fe}^2 k^2\beta_1(1-\alpha) + v_{Fe}^2 v_{Fp}^2 k^4\beta_1\beta_2 = 0, \quad (8)$$

where $v_{Fj}(=(2E_{Fj}/m)^{1/2})$ is the Fermi speed, $\lambda_{Fj} = v_{Fj}/\omega_{pj}$ is the Fermi length, $\beta_1 = 1/3 + H_e^2\lambda_{Fe}^2k^2/4$, and $\beta_2 = 1/3 + H_p^2\lambda_{Fp}^2k^2/4$. For pure degenerate electron–positron plasma, $\alpha = 0$, $T_{Fe} = T_{Fp}$ and equation (8) gives rise to the Langmuir wave in quantum ep plasmas. On the other hand, the presence of ions leads to $n_{e0} \neq n_{p0}$ and $T_{Fe} > T_{Fp}$. Then, the two solutions of Eq. (8) can be written as

$$\omega^2 = (1/2)\left[(v_{Fe}^2\beta_1 + v_{Fp}^2\beta_2)k^2 + (2-\alpha)\omega_{pe}^2 \pm \sqrt{disc}\right], \quad (9)$$

where

$$disc = \left[(v_{Fe}^2\beta_1 + v_{Fp}^2\beta_2)k^2 + (2-\alpha)\omega_{pe}^2\right]^2 - 4\left[\omega_{pe}^2 v_{Fp}^2 k^2\beta_2 + \omega_{pe}^2 v_{Fe}^2 k^2\beta_1(1-\alpha) + v_{Fe}^2 v_{Fp}^2 k^4\beta_1\beta_2\right]. \quad (10)$$

The $\pm$ signs in Eq. (9) correspond to fast Langmuir and slow acoustic type waves, respectively, which is similar to the classical pair plasmas.[39] However, both cases are different physically.

4. Derivation of the extended Korteweg-de Vries equation

Let us introduce the stretching of space and time coordinates in Eqs. (3)–(6) as follows:[28]

$$\xi = \epsilon(x - \lambda t) \text{ and } \tau = \epsilon^3 t, \quad (11)$$

where ϵ is a small parameter and λ is the normalized phase speed. The plasma parameters are expanded in the power of ϵ as

$$\begin{bmatrix} n_j \\ v_j \\ \varphi \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \epsilon n_{j1} \\ \epsilon v_{j1} \\ \epsilon\varphi_1 \end{bmatrix} + \begin{bmatrix} \epsilon^2 n_{j2} \\ \epsilon^2 v_{j2} \\ \epsilon^2\varphi_2 \end{bmatrix} + \cdots. \quad (12)$$

Using Eqs. (11) and (12) in Eqs. (3)–(6), we obtain different orders of parameter ϵ. The lowest order $O(\epsilon)$ gives $n_{e1} = \varphi_1/(1/3-\lambda^2)$, $n_{p1} = -\varphi_1/(\delta/3-\lambda^2)$, $v_{e1} = \lambda\varphi_1/(1/3-\lambda^2)$, and $v_{p1} = -\lambda\varphi_1/(\delta/3-\lambda^2)$. Then, the normalized phase velocity λ becomes

$$\lambda = \sqrt{(1-\alpha+\delta)/3(2-\alpha)}. \quad (13)$$

The next higher order of ϵ leads to

$$n_{e2} - (1/2)(1/9-3\lambda^2)(1/3-\lambda^2)^{-3}\varphi_1^2 + (1/3-\lambda^2)^{-1}\varphi_2 = 0, \quad (14)$$
$$n_{p2} - (1/2)(\delta/9-3\lambda^2)(\delta/3-\lambda^2)^{-3}\varphi_1^2 - (\delta/3-\lambda^2)^{-1}\varphi_2 = 0, \quad (15)$$
$$v_{e2} - ((1-\alpha+\delta)/3(2-\alpha))^{1/2} \times \left[(1/3-\lambda^2)^{-1}\varphi_2 + (1/3-\lambda^2)^{-3}\{(1/2)(1/9-3\lambda^2) - (1/3-\lambda^2)\}\varphi_1^2\right] = 0, \quad (16)$$
$$v_{p2} - ((1-\alpha+\delta)/3(2-\alpha))^{1/2} \times \left[-(\delta/3-\lambda^2)^{-1}\varphi_2 + (\delta/3-\lambda^2)^{-3}\{(1/2)(\delta/9-3\lambda^2) - (\delta/3-\lambda^2)\}\varphi_1^2\right] = 0, \quad (17)$$

and

$$\left[(1/18-3\lambda^2/2)(1/3-\lambda^2)^{-3} - (1-\alpha)(\delta/18-3\lambda^2/2)(\delta/3-\lambda^2)^{-3}\right]\varphi_1^2 = 0. \quad (18)$$

It is to be noted that the term in the square bracket in Eq. (18) should be at least of the first order of ϵ because $\varphi_1 \neq 0$. It means that equation (18) is now third order in ϵ, which should be added in the same order terms of Eq. (6).

From the next higher order terms of ϵ, we have

$$-\lambda\partial_\xi v_{e3} - \partial_\xi\varphi_3 + (1/3)\partial_\xi n_{e3} = -\partial_\tau v_{e1} - \partial_\xi(v_{e1}v_{e2}) + (1/9)\partial_\xi(n_{e1}n_{e2}) - (2/27)n_{e1}^2\partial_\xi n_{e1} + (H_e^2/4)\partial_\xi^3 n_{e1}, \quad (19)$$
$$-\lambda\partial_\xi v_{p3} + \partial_\xi\varphi_3 + (\delta/3)\partial_\xi n_{p3} = -\partial_\tau v_{e1} - \partial_\xi(v_{p1}v_{p2}) + (\delta/9)\partial_\xi(n_{p1}n_{p2}) - (2\delta/27)n_{e1}^2\partial_\xi n_{e1} + (\delta^{1/2}H_p^2/4)\partial_\xi^3 n_{p1}, \quad (20)$$
$$-\lambda\partial_\xi n_{e3} + \partial_\xi v_{e3} + \partial_\tau n_{e1} + \partial_\xi(n_{e1}v_{e2}) + \partial_\xi(n_{e2}v_{e1}) = 0, \quad (21)$$
$$-\lambda\partial_\xi n_{p3} + \partial_\xi v_{p3} + \partial_\tau n_{p1} + \partial_\xi(n_{p1}v_{p2}) + \partial_\xi(n_{p2}v_{p1}) = 0, \quad (22)$$

and

$$n_{e3} - (1-\alpha)n_{p3} - \partial_\xi^2\varphi_1 + \left[(1/3-\lambda^2)^{-3}(1/18-3\lambda^2/2) - (1-\alpha)(\delta/18-3\lambda^2/2)(\delta/3-\lambda^2)^{-3}\right]\varphi_1^2 = 0. \quad (23)$$

Equations (19)–(22) lead to

$$-\partial_\xi \varphi_3 + (1/3-\lambda^2)\partial_\xi n_{e3} = y_1, \tag{24}$$

and

$$\partial_\xi \varphi_3 + (\delta/3-\lambda^2)\partial_\xi n_{p3} = -y_2, \tag{25}$$

where

$$\begin{aligned} y_1 &= -2\lambda\partial_\tau n_{e1} - 2\partial_\xi(v_{e1}v_{e2}) + (1/9-\lambda^2)\partial_\xi(n_{e1}n_{e2}) \\ &\quad -(2/27)n_{e1}^2\partial_\xi n_{e1} - (H_e^2/4)\partial_\xi^3 n_{e1}, \end{aligned}$$

and

$$\begin{aligned} y_2 &= 2\lambda\partial_\tau n_{p1} + 2\partial_\xi(v_{p1}v_{p2}) - (\delta/9-\lambda^2)\partial_\xi(n_{p1}n_{p2}) \\ &\quad -(2\delta/27)n_{p1}^2\partial_\xi n_{p1} - (H_p^2/4)\partial_\xi^3 n_{p1}. \end{aligned}$$

Solving Eqs. (23)–(25) by employing first order solutions and Eqs. (13)–(18), we readily obtain the eKdV equation as follows:

$$A\,\partial\tau\varphi + B\,\partial_\xi\varphi^2 + C\,\partial_\xi\varphi^3 + D\,\partial_\xi^3\varphi = 0, \tag{26}$$

where the coefficients of Eq. (26) are defined as

$$A = 2\lambda\left[(1-\alpha)(\delta/3-\lambda^2)^{-2} + (1/3-\lambda^2)^{-2}\right], \tag{27}$$

$$\begin{aligned} B &= (1/2)\left[(1-\alpha)(\delta/9-3\lambda^2)(\delta/3-\lambda^2)^{-3}\right. \\ &\quad \left. -(1/9-3\lambda^2)(1/3-\lambda^2)^{-3}\right], \end{aligned} \tag{28}$$

$$\begin{aligned} C &= (1/2)\left[(1/3-\lambda^2)^{-5}\{(1/9-3\lambda^2)^2\right. \\ &\quad +4(1/81+\lambda^2)(1/3-\lambda^2)\} \\ &\quad -(1-\alpha)(\delta/3-\lambda^2)^{-5}\{(\delta/9-3\lambda^2)^2 \\ &\quad \left. +4(\delta/81+\lambda^2)(\delta/3-\lambda^2)\}\right], \end{aligned} \tag{29}$$

$$\begin{aligned} D &= 1-(1/4)(1/3-\lambda^2)^{-2}H_e^2 \\ &\quad -(\delta^2/4)(\delta/3-\lambda^2)^{-2}H_p^2, \end{aligned} \tag{30}$$

with $\varphi_1 \equiv \varphi$. The coefficients A, B, C, and D are functions of plasma number density. The role of static ion concentration and ion charge is significant and will be discussed later.

To solve Eq. (26), we introduce the transformation $\eta = \xi - v_0\tau$, with v_0 being the constant speed of the frame moving with the structure.[28] Using the boundary conditions: $\varphi \to 0$ and $\mathrm{d}\varphi/\mathrm{d}\eta \to 0$ at infinity, equation (26) leads to,

$$(1/2)(\mathrm{d}\varphi/\mathrm{d}\eta)^2 + V(\varphi) = 0, \tag{31}$$

where pseudo-potential function (Sagdeev-like potential) $V(\varphi)$ is given by

$$V(\varphi) = -(v_0A/2D)\varphi^2 + (B/3D)\varphi^3 + (C/4A)\varphi^4. \tag{32}$$

For the DL solution to exist, $V(\varphi)$ and $V'(\varphi)$ must vanish for $\varphi = 0$ and $\varphi = \varphi_m$, whereas, $V''(\varphi) < 0$ at the same points. Then, $V(\varphi)$ reduces to

$$\begin{aligned} V(\varphi) &= (C/4A)\varphi^2(\varphi_m - \varphi)^2 \\ &= (C/4A)\varphi^2(2B/3C + \varphi)^2. \end{aligned} \tag{33}$$

Here, $\varphi_m = -2B/3C$ is the maximum potential and $v_0 = -(C/2A)\varphi_m^2$ is the constant speed. Thus, the steady state DL solution of Eq. (31) can be written as

$$\varphi = (\varphi_m/2)(1-\tanh(-C/8D)^{1/2}\varphi_m\eta). \tag{34}$$

5. Numerical analysis and discussion

In this section, we discuss the conditions of existence and properties of DL structures. The existence of DL structures requires $CD > 0$ showing either C or $D < 0$ as evident from Eq. (34). In order to decide whether the DL is compressive or rarefactive, one has to check the sign of coefficient B in the expression of φ_m. If $B > 0$, φ_m is negative and a rarefactive DL will be formed whereas for negative B, a compressive DL will exist. It is noticed that C is always positive for singly charged ions, and hence D should be negative in this case. Similarly, B is always positive resulting in the formation of the rarefactive DL only. This happens in dense e–p–i plasma situations because, $n_{i0} \neq 0$, $n_{p0} < n_{e0}$, and $T_{Fp} < T_{Fe}$ since the Fermi temperature is a function of density in quantum plasmas. For the existence of a DL, the system must have dispersion which appears due to deviation from quasi-neutrality as shown in Poisson's equation. In quantum plasmas, when quantum parameters are assumed to be zero, i.e., $H_e = H_p = 0$, then the dispersive term does not vanish in Eq. (26) and the DL solution still exist due to contribution from electrostatic field. However, in quasi-neutral case, the dispersion effects comes from the quantum diffraction term. The DLs in such system are formed due to the balance between harmonic nonlinearities and the quantum diffraction effects. On the other hand, in a classical quasi-neutral plasmas, the DL structures ceases to exist. In what follows, we numerically examine the foregoing results and effects of quantum diffraction as well as ion concentration and ion charge on the formation and behavior of an electrostatic DL by using the typical parameters of the degenerate plasmas found in the outer layers of dense astrophysical objects.[18,35] The electron and ion densities are assumed as $n_{e0} \simeq 6\times10^{28}$ cm^{-3} and $n_{i0} \simeq 0.2n_{e0}$ with $Z_i = 1$. This leads to the average interparticle distance $n_{e0}^{-1/3} \simeq 2.5\times10^{-10}$ cm, $H_e \simeq 0.081$, and $H_p \simeq 0.085$. The parametric range in which the DL solution (30) exists is found by using the values described above. The coefficients A, B, C, and D of the eKdV equation given in Eqs. (27)–(30) are functions of plasma number density only and $H_j \propto n_{j0}^{-1/6}$ for j-th species. A slight change in the ion number density modify the DL structure significantly. It is noticed that $BC > 0$ for all values of n_{i0}, which gives $\varphi_m < 0$. It is also noticed that D assumes negative values below a certain critical value of n_{i0} only, i.e., $n_{crit} \simeq 2.63\times10^{28}$ cm^{-3} which leads to $CD < 0$ (where C is always positive). Beyond n_{crit}, D becomes positive, $CD > 0$ and DL does not exist. However, when the electron number density is increased, n_{crit} shifts to lower values. Similarly, the region of existence of DL shifts to lower ion concentration for larger Z_i as shown in Fig. 1. The plot of C/D against n_{i0} shows that

D is negative within a certain range of n_{i0} values. Figures 2 and 3 show the variation of DL potential with electron and ion number density, respectively. It is seen that the width of the DL structure decreases (increases) with electron (ion) concentration. Figure 4 shows the variation of pseudo-potential with electron concentration. It is shown that amplitude and width of $V(\varphi)$ increases with H_e for fixed ion number density. On the other hand, the amplitude of $V(\varphi)$ increases but width decreases with n_{i0} keeping n_{e0} fixed as shown in Fig. 5.

It is evident that the solution (30) mainly depends on the potential across the DLs. In our case, potential is smaller, i.e., $|\varphi_m| \ll 1$ leading to the formation of weak DL. The DL solution of eKdV equation, in a co-moving frame with speed v_0 is stable since it is obtained as the asymptotic evolution of a given initial condition.

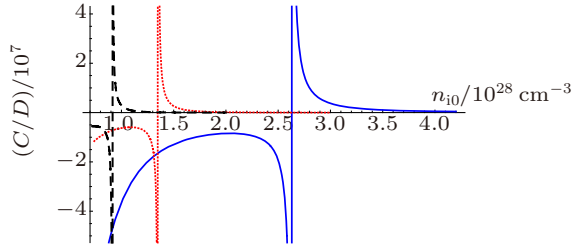


Fig. 1. (color online) Plot showing the existence region of rarefactive DL with the variation of ion charge number Z_i. Solid, dotted, and dashed curves represent $Z_i = 1$, 2, and 3, respectively, where $n_{e0} = 8 \times 10^{28}$ cm^{-3}.

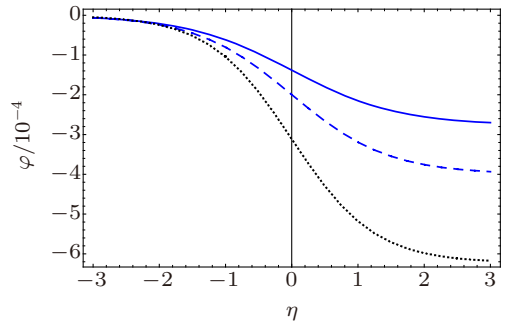


Fig. 2. (color online) The electrostatic DL potential φ is plotted against η for various values of electron number density. Dotted line represents $n_{e0} \simeq 4 \times 10^{28}$ cm^{-3}, dashed line represents 5×10^{28} cm^{-3}, and solid line represents $n_{e0} = 6 \times 10^{28}$ cm^{-3}, with $n_{i0} \simeq 1 \times 10^{28}$ cm^{-3}.

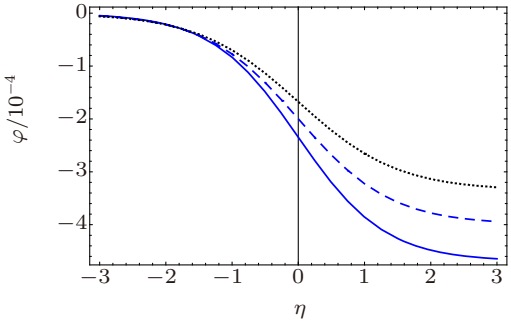


Fig. 3. (color online) Electrostatic potential φ is plotted versus η for different values of ion number density with fixed electron concentration. Solid (dashed) line represents $n_{i0} = 3 \times 10^{28}$ cm^{-3} (2×10^{28} cm^{-3}) and dotted line represents $n_{i0} \simeq 1 \times 10^{28}$ cm^{-3} with $n_{e0} \simeq 6 \times 10^{28}$ cm^{-3}.

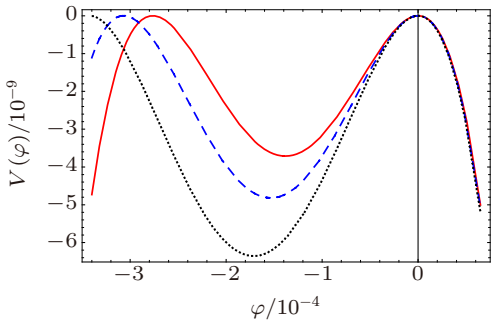


Fig. 4. (color online) The effect of electron quantum diffraction is shown on Sagdeev-like potential $V(\varphi)$ by plotting $V(\varphi)$ against φ. Solid (dashed) line represents $n_{e0} = 6 \times 10^{28}$ cm^{-3} (5.7×10^{28} cm^{-3}) and dotted line is for $n_{e0} = 5.4 \times 10^{28}$ cm^{-3} keeping the ion concentration constant ($n_{i0} = 1 \times 10^{28}$ cm^{-3}).

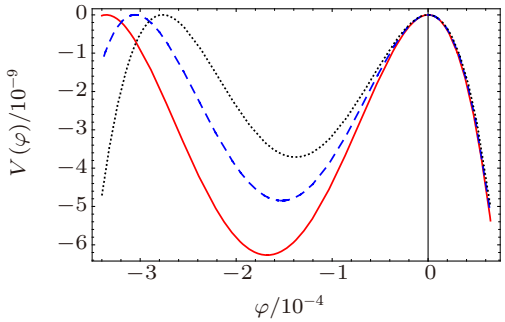


Fig. 5. (color online) Plot of Sagdeev-like potential $V(\varphi)$ versus φ with n_{i0}. Solid (dashed) line represents $n_{i0} = 1.1 \times 10^{28}$ cm^{-3} (1.05×10^{28} cm^{-3}) and dotted line is for $n_{i0} = 1.1 \times 10^{28}$ cm^{-3} for electron number density being the same as in Fig. 3.

6. Summary

To summarize, we have investigated the existence and formation of low amplitude DLs in a dense unmagnetized ep plasma in the presence of small fraction of stationary ions in the background. By employing the QHD model, we have derived an eKdV equation and found its analytical solution. The conditions of existence and behavior of DLs in such plasmas is discussed, and the influence of ion number density and quantum effects (degeneracy and diffraction) of electrons and positrons is pointed out. It is seen that only a rarefactive DL can exist in this system beyond a critical value of ion number density. This critical value shifts to lower ion concentration for heavier ions. The quantum parameters H_e and H_p, and static ion concentration significantly affect the amplitude and steepness of the DL structures. These parameters also show a strong influence on $V(\varphi)$.

The variation of potential across the DL can accelerate, decelerate, and reflect particles entering from the sides. The significant charged particle acceleration to high energies in the dense astrophysical objects, e.g., white dwarf stars has been pointed out in some recent observations.[40] The acceleration of electron–positron plasma in a neutron star's crust is also dictated by DLs.[41] The present study is general which aims to understand some aspects of DL formation in very dense ep plasma environments.

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