Simulating regular wave propagation over a submerged bar by boundary element method model and Boussinesq equation model*

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(Received 11 June 2012; revised manuscript received 23 July 2012)

Numerical models based on the boundary element method and Boussinesq equation are used to simulate the wave transform over a submerged bar for regular waves. In the boundary-element-method model the linear element is used, and the integrals are computed by analytical formulas. The Boussinesq-equation model is the well-known FUNWAVE from the University of Delaware. We compare the numerical free surface displacements with the laboratory data on both gentle slope and steep slope, and find that both the models simulate the wave transform well. We further compute the agreement indexes between the numerical result and laboratory data, and the results support that the boundary-element-method model has a stable good performance, which is due to the fact that its governing equation has no restriction on nonlinearity and dispersion as compared with Boussinesq equation.

Keywords: numerical wave tank, boundary element method, Boussinesq equation

PACS: 47.11.–j, 47.11.Hj, 47.15.km, 47.35.Bb **DOI:** 10.1088/1674-1056/22/2/024701

1. Introduction

How to predict the propagation of nonlinear water waves and their interaction with the varying topography is essential to coastal engineering. In nearshore hydrodynamics, the Boussinesq equation (BE) always acts as the governing equation. Integrating the Navier–Stokes equation on the vertical coordinate[1] we can obtain the BE by introducing two small dimensionless parameters, i.e., nonlinearity (wave amplitude versus water depth, denoted as ε) and dispersion (water depth versus wave length, denoted as μ). It means that the BE reduces the wave problem from two-dimensional to one-dimensional, which makes its numerical implementation convenient.[2–5]

In the classical BE ε and μ^2 are assumed to have the same order, and the BE works only under the condition of relatively small wave amplitude and long wave length. Therefore, the main improvements of the BE concentrate on achieving the higher order of dispersion[6–9] which has already been extended to $O(\mu^4)$ or more. Meanwhile, the BE is applicable for the highly nonlinear waves after the famous work on saving the equation nonlinearity.[10,11]

Alternative to the BE, the boundary element method (BEM) is used not only to solve the fully nonlinear potential flow, but also to study the nearshore wave. One significant advantage of the BEM is its arbitrary boundary shape, which includes the free surface and bottom topography. Since the first success in the simulation of breaking waves by Longuet–Higgins and Cokelet,[12] the BEM has widely been used in the numerical simulation of both deep-water waves[13,14] and shallow-water waves.[15–19]

During the BEM implementation, we need to choose the element type and compute the element integration. For the linear element, we have an analytical formula to process the integration. For the higher order element, we always use the numerical treatment. Although the numerical integration method given by Grilli and Subramanya[20] is believed to be a successful method, it is still unclear in evaluating the analytical formula of the linear element, especially after Drimer and Agnon[16] and He *et al.*[17] have introduced a compatible condition. Hence, in the present work we calibrate the BEM model performance of He *et al.*[17] on the nearshore wave simulation, by comparing its numerical result with the laboratory data as well as the BE solutions.

The rest of the present paper is organized as follows. In Section 2 numerical models are briefly introduced. Then in Sections 3 and 4, numerical tests on wave transform over a submerged bar are presented for Luth *et al.*'s[21] experiment and Ohyama *et al.*'s[22] experiment, respectively. Section 5 is devoted to further discussion of the numerical results, and Section 6 shows some conclusions drawn from the present study.

*Project supported by the National Natural Science Foundation of China (Grant Nos. 41106019 and 41176016), the Knowledge Innovation Programs of the Chinese Academy of Sciences (Grant No. kzcx2-yw-201), the Public Science and Technology Research Funds Projects of Ocean (Grant No. 201105018), and the Natural Science Foundation of Jiangsu Province, China (Grant No. BK2012315).

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2. Model description

2.1. Boussinesq equation model

We use the well-known FUNWAVE model as our BE model. FUNWAVE was developed by Kirby *et al.*[23] in University of Delaware and has been widely tested and used as a stable Boussinesq model. The model is valid for the constant depth, planar slope and submerged bar.[11,23] It has also been successfully implemented in practical harbor design, tsunami risk assessment, etc.[24,25] We consider two types of BEs. One is an extended BE given by Nwogu[7] (called NBE), and the other is a fully nonlinear BE presented by Wei *et al.*[10] (named WBE). Nonlinearity and dispersion of the NBE have the orders of $O(\varepsilon)$ and $O(\mu^2)$ respectively, while the WBE has better nonlinearity and the same dispersion as the NBE.[10]

2.2. Boundary element method model

The present BEM model was first developed by Drimer and Agnon,[16] and later modified by Segre (1998, private communication) and He *et al.*[17] In the BEM model the linear element is used, a compatible condition on the node velocity is introduced, and the integration is solved by an analytical formula. The model is good for simulating waves on the constant depth and planar slope.[16–19,26,27]

3. Numerical results on the submerged bar with gentle slope

For regular waves passing over a submerged bar, waves decompose into higher frequency free waves due to the energy transform from the leading wave component to higher harmonics. A submerged bar is more difficult than a planar slope for a numerical model test because it requires capturing not only the nonlinear harmonic generation well, but also the releasing process of shorter waves behind the bar. A naturally submerged bar always has a gentle slope, while the artificial submerged bar always has a steep one. Following the considerations by Gobbi and Kirby,[11] we consider the cases of a gentle slope and steep slope of a submerged bar, which are referred to in Luth *et al.*'s[21] and Ohyama *et al.*'s[22] experiments respectively.

In the laboratory experiment of Luth *et al.*[21] the submerged bars have varying gentle slopes (the front slope is 1:20 and the back slope is 1:10) as shown in Fig. 1. We consider two different incident wave cases. In the first case (denoted by case a) the wave amplitude is 0.01 m and the wave period is 2.02 s, and in the second case (denoted by case c) the wave amplitude is 0.0205 m and the wave period is 1.01 s. During the BE model implementation, we choose $\Delta x = 0.05$ and $\Delta t = 0.01$. In the BEM model simulation, we set 25 nodes per wave length.

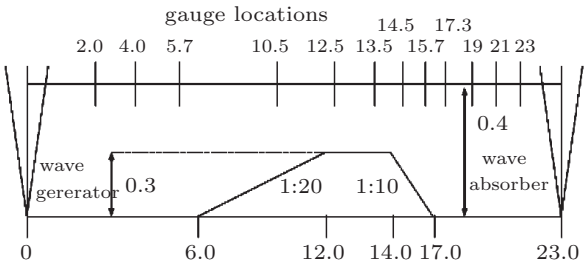


Fig. 1. Schematic plot of wave tank of Luth *et al.* (1994). All dimensions are in meters.

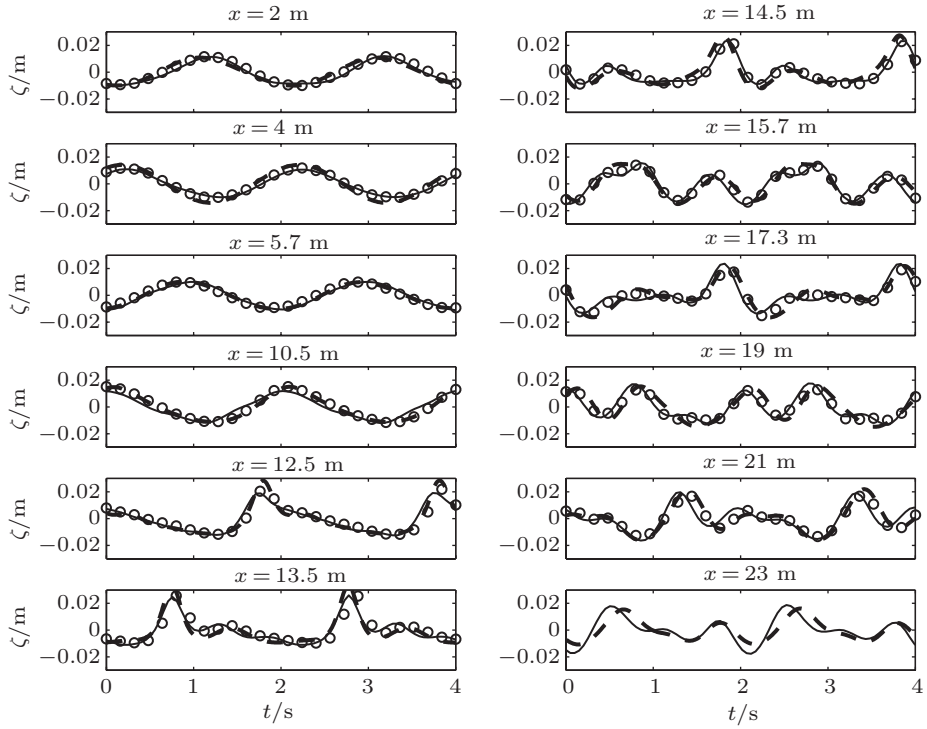


Fig. 2. Comparisons of time series of wave elevation with those in case a of Luth *et al.*'s (1994) experimental data at several gauge locations. NBE result is denoted with dashed line, BEM result is represented with solid line, data are indicated with circles.

Figures 2 and 3 show comparisons in laboratory data between NBE, WBE, and BEM models for case a. The NBE, WBE, and BEM models perform quite well for all the gauges. It means NBE, WBE, and BEM all capture the wave transform of this case, characterized by an incident long wave with relatively low nonlinearity ($\varepsilon = 0.025$ and $\mu = 0.67$).

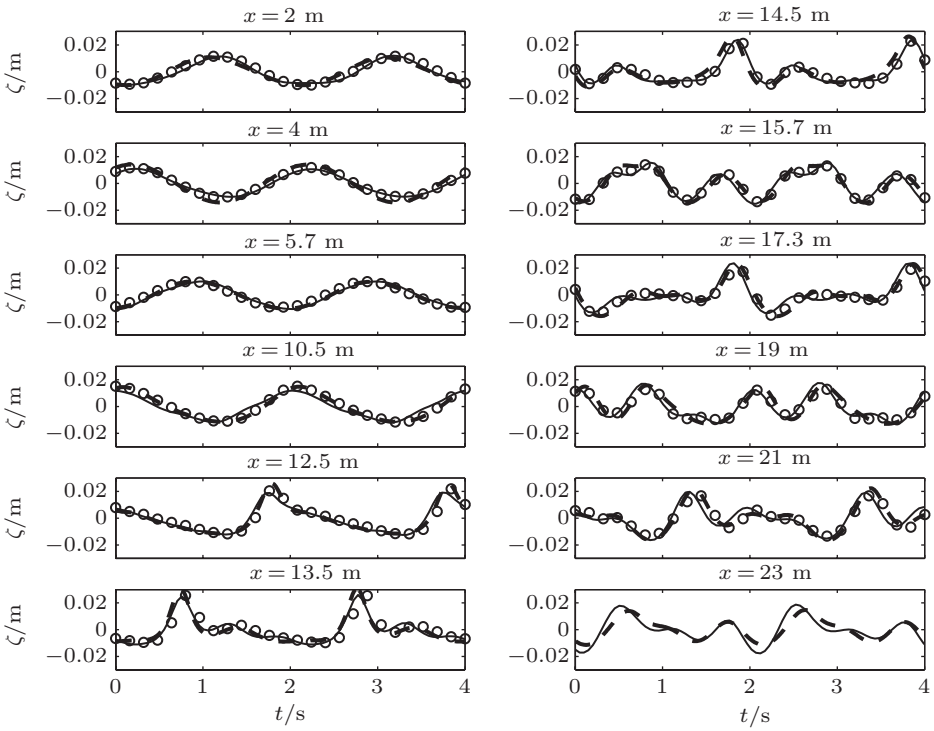


Fig. 3. Comparisons between time series of wave elevation and those in case a of Luth *et al.*'s (1994) experimental data at several gauge locations. WBE result is denoted with dashed line, BEM result is represented with solid line, and data are indicated with circles.

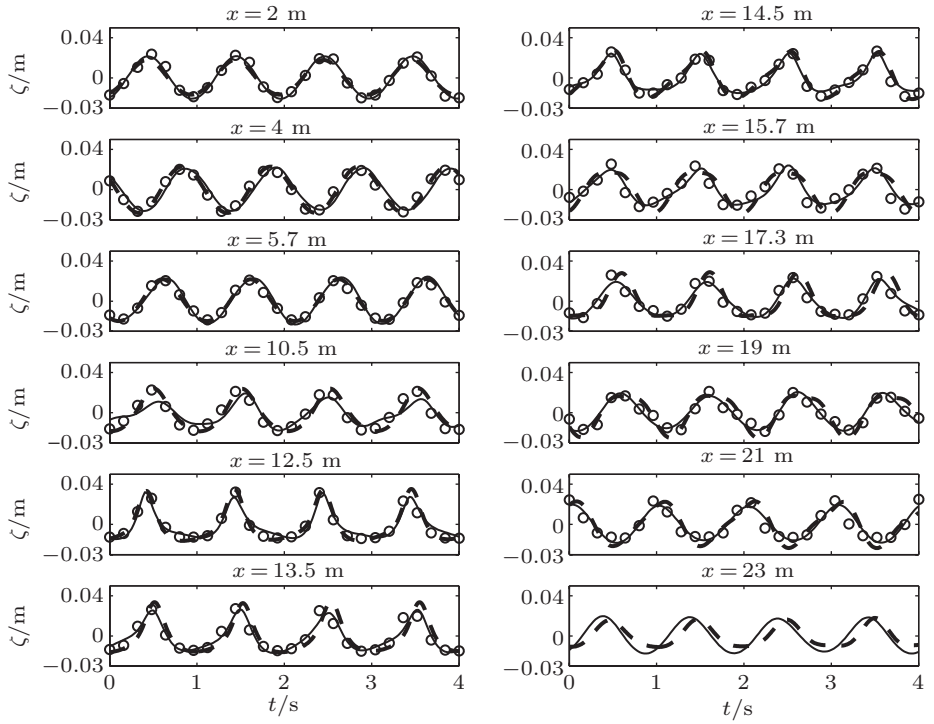


Fig. 4. Comparisons between time series of wave elevation with those in case c of Luth *et al.*'s (1994) experimental data at several gauge locations. NBE result is denoted with dashed line, BEM result is represented with solid line, and data are indicated with circles.

For case c, figures 4 and 5 show the comparisons with laboratory data of NBE, WBE and BEM models. For this case, the higher challenge than case a is confronted by NBE and WBE, since the incident wave condition covers $\varepsilon = 0.051$ and $\mu = 1.69$,

much higher than those in case a. The number of free waves behind the submerged bar stays the same as that of the incident wave, which means that the energy transformation from the leading wave to higher harmonics is much lower than that in case a. Results support that NBE and WBE do not lose more accuracy than BEM, if only the free surface displacement is concerned.

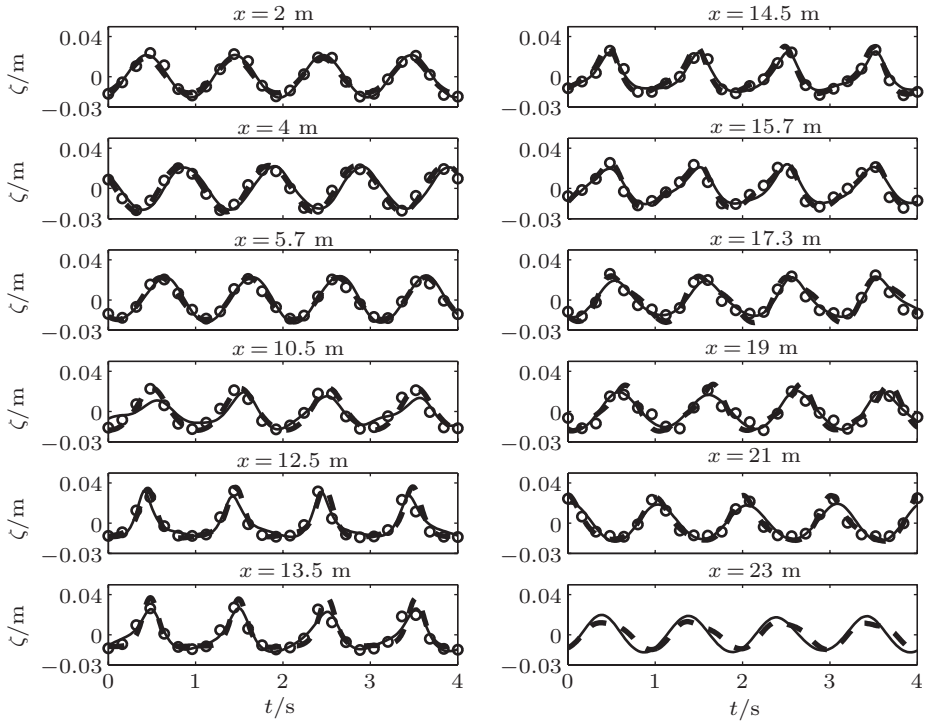


Fig. 5. Comparisons between time series of wave elevation with those in case c of Luth *et al.*'s (1994) experimental data at several gauge locations. WBE result is denoted with dashed line, BEM result is represented with solid line, and data are indicated with circles.

Besides the qualitative comparisons of the surface displacement, we also discuss the so-called agreement index (denoted by d) which was proposed by Wilmott.[28] The formula of d is

$$d_i = 1 - \frac{\sum_{j=n_1}^{n_2} [y(j) - y_{\mathrm{d}}(j)]^2}{\sum_{j=n_1}^{n_2} [|y(j) - \bar{y}_{\mathrm{d}}| + |y_{\mathrm{d}}(j) - \bar{y}_{\mathrm{d}}|]}, \qquad (1)$$

where y_{d} is the measured datum to be compared with the calculated one, $y(j)$ are the predicted values from the model, $\bar{y}_{\mathrm{d}}$ is the mean value of $y_{\mathrm{d}}(j)$, and n_1 and n_2 cover a full wave period in the time series.

Table 1. Indexes of agreement d for Luth *et al.*'s (1994) experiment.

Gauge/m	Case a			Case c		
	NBE	WBE	BEM	NBE	WBE	BEM
2.0	0.981	0.981	0.990	0.981	0.980	0.994
4.0	0.980	0.981	0.995	0.991	0.991	0.951
10.5	0.998	0.998	0.962	0.958	0.967	0.916
12.5	0.861	0.870	0.970	0.990	0.959	0.976
13.5	0.922	0.921	0.947	0.899	0.935	0.968
14.5	0.951	0.952	0.990	0.985	0.979	0.982
15.7	0.981	0.990	0.981	0.890	0.981	0.954
17.3	0.976	0.988	0.954	0.910	0.963	0.973
19.0	0.949	0.970	0.969	0.946	0.914	0.969
21.0	0.982	0.980	0.925	0.909	0.992	0.959

Table 1 shows the indexes of agreement d, defined by Eq. (1) of the models NBE, WBE, and BEM, for case a and case c of the Luth *et al.* (1994) experiments for all gauges except 5.7 m and 23 m. The results confirm that the best performance is obtained from the BEM model, because the mean d is 0.966, higher than 0.952 for the NBE and 0.965 for the WBE. Although the NBE has much the same dispersion relationship as the WBE, since it was not capable of generating higher harmonics when the wave is shoaling, the overall accuracy becomes less than that of the WBE.

4. Numerical results on the submerged bar with steep slope

In this section, we make comparisons between the numerical models and the experiment of Ohyama *et al.*[22] A schematic plot of the wave flume with gauge locations are shown in Fig. 6. We consider three cases of Ohyama *et al.*[22] with wave amplitude 0.025 m for all the cases, but 1.341 s (case 2), 2.012 s (case 4), 2.683 s (case 6) for wave period respectively.

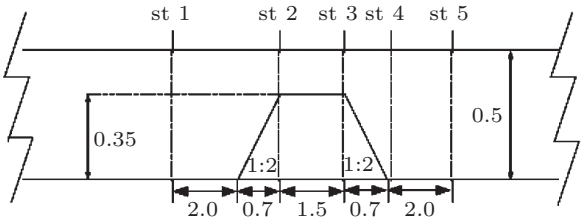


Fig. 6. Schematic plot of wave tank of Ohyama *et al*. (1994). All dimensions are in meters. Station is abbreviated to st in the figure.

We assume the NBE and WBE to have $\Delta x = 0.05$ m and $\Delta t = 0.01$ s for case 2, $\Delta x = 0.1$ m and $\Delta t = 0.02$ s for cases 4 and 6. We also keep 25 nodes per wave length for the BEM. The BEM is unstable until additional smoothing is added. Therefore, we perform the smoothing five times per wave period with a five-point formula on the wave profile within the horizontal section of the submerged bar. Figures 7 and 8 show the comparisons between the laboratory data and numerical results. We can see that the time series of wave elevations are in agreement with laboratory data. The BEM result shows less wave height, for which one of the reasons is the energy dissipation of artificial numerical smoothing. Compared with the BEM, the NBE and WBE give higher harmonics at stations 3 and 5 of case 2, which accord better with the experimental results. However, this higher harmonics produces negative effects on stations 3 and 5 of case 6.

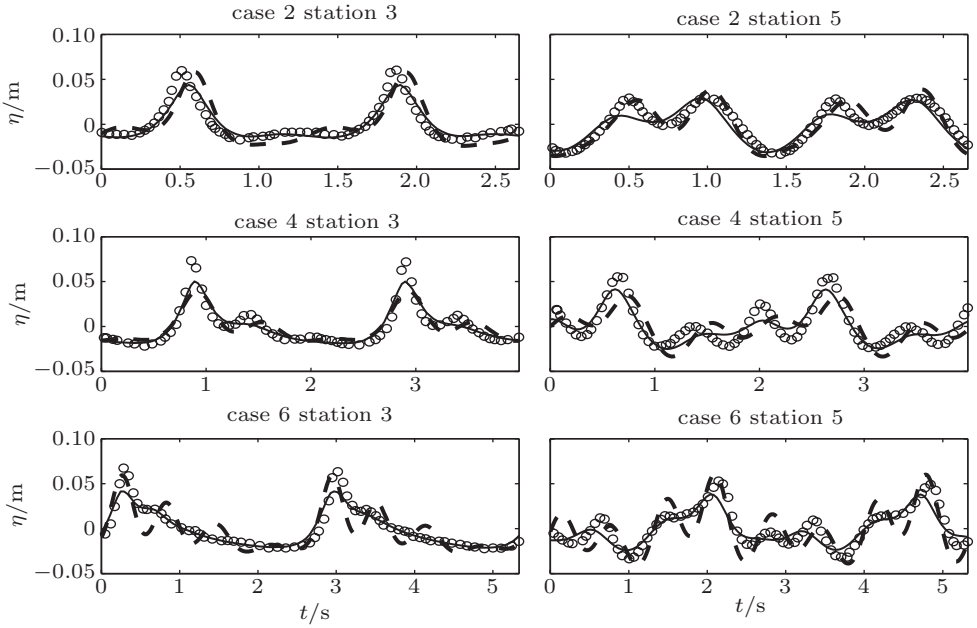


Fig. 7. Comparisons between time series of wave elevation and Ohyama *et al*.'s (1994) experimental data. NBE result is denoted with dashed line, BEM result is represented with solid line, and data are indicated with circles.

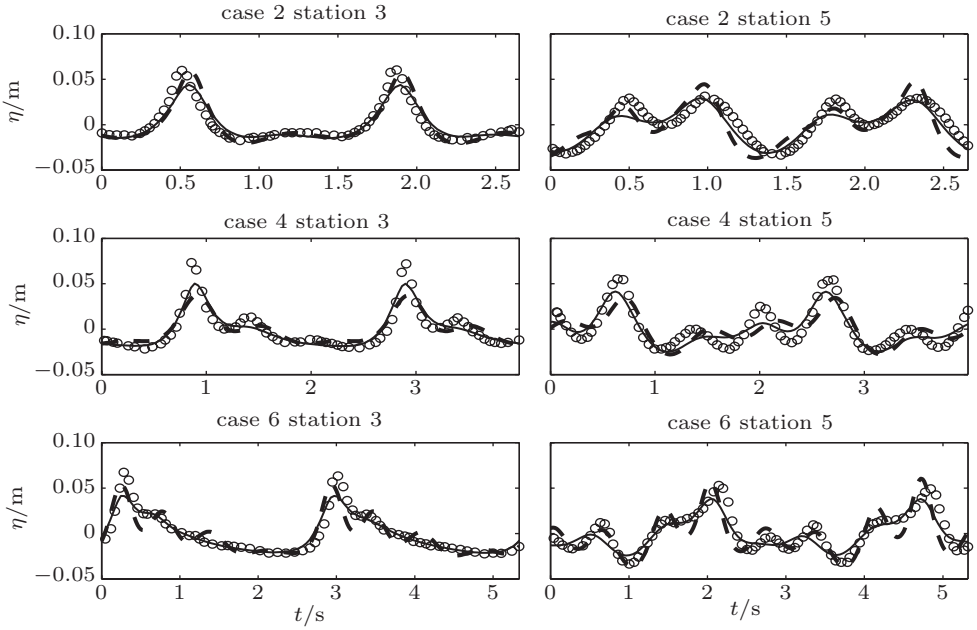


Fig. 8. Comparisons between time series of wave elevation and Ohyama *et al*.'s (1994) experimental data. WBE result is denoted with dashed line, BEM result is represented with solid line, and data are indicated with circles.

Table 2 shows the indexes of agreement d for Ohyama *et al.*'s[22] experiment. Results support cases 4 ($\mu = 0.769$) and 6 ($\mu = 0.555$), and fully nonlinear assumption causes the agreement index of WBE to be higher than that of NBE. Agreement indexes of BEM are all greater than 0.9, which means that the BEM performance is stable for these cases, because of its different theoretical background.

Table 2. Indexes of agreement d for Ohyama *et al.*'s (1994) experiment.

Station	Case 2			Case 4			Case 6		
	NBE	WBE	BEM	NBE	WBE	BEM	NBE	WBE	BEM
3	0.898	0.964	0.963	0.930	0.926	0.969	0.914	0.933	0.971
5	0.958	0.891	0.944	0.804	0.861	0.941	0.894	0.910	0.933

5. Discussion

We simulate the wave transformation over a submerged bar. The results of a gentle slope, given by Luth *et al.*,[21] are significantly better than those of a steep slope, presented by Ohyama *et al.*[22] There are at least two reasons. Firstly, the gentle slope produces moderate energy transfer when the wave is shoaling, and so generates relatively small energy of higher harmonics. Secondly, the gentle slope needs neither numerical smoothing in the model simulation, nor artificial dissipation in the model simulation.

Compared with the WBE, the drawback of the NBE is significant for case c of Luth *et al.*[21] and also for cases 4 and 6 of Ohyama *et al.*[22] due to its low nonlinearity limitation. Although the BEM model also needs the artificial smoothing process for the Ohyama *et al.*'s[22] experiment, its advantage of theoretical capability in nonlinearity and dispersion is distinguishable from these simulations.

After adding some breaking criteria and breaking dissipation formula, FUNWAVE is also used for the nearshore wave breaking simulations.[29] However, as He *et al.*[30] argued, the general breaking criteria in the BE model requires more studies. The present consistence on the free surface displacement confirms that the coupling of the BEM with the BE is feasible for the waves passing over a submerged bar. Although the coupling system seems to be unnecessary for the present non-breaking waves, it is an efficient tool for the wave breaking case. We leave the coupling work to future studies.

6. Conclusion

We tested two types of numerical wave models of simulating the regular wave transform over a submerged bar. The first type is based on the Boussinesq equation, which includes Nwogu's[7] extended Boussinesq equation and Wei *et al.*'s[10] fully nonlinear Boussinesq equation. The second type uses the boundary element method to solve the fully nonlinear potential flow. Experimental data from Luth *et al.*[21] and Ohyama *et al.*[22] are used to calibrate the model performance.

Three numerical models all capture the wave transformation well, and the cases with gentle slopes of submerged bar show better consistence on the free surface displacement. The relative advantages of Wei *et al.*'s [10] model to Nwogu's[7] are distinguishable from the tests. The numerical wave model based on the boundary element method shows more stable performance on the wave transformation simulation, and benefits by its different theoretical background compared with the Boussinesq equations.

The consistence on the free surface displacement in the present studies confirms that coupling of the boundary element method with the Boussinesq equation is feasible in nearshore wave modeling. This is expected to solve the general difficulty in breaking criteria of the Boussinesq equation for nearshore breaking wave simulation.

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