Optimization of fin geometry in heat convection with entransy theory*

Cheng Xue-Tao(程雪涛)[a], Zhang Qin-Zhao(张勤昭)[b], Xu Xiang-Hua(徐向华)[a], and Liang Xin-Gang(梁新刚)[a]†

[a] Key Laboratory for Thermal Science and Power Engineering of Ministry of Education, School of Aerospace, Tsinghua University, Beijing 100084, China
[b] Institute of Nuclear and New Energy Technology, Tsinghua University, Beijing 100084, China

(Received 20 June 2012; revised manuscript received 19 July 2012)

The entransy theory developed in recent years is used to optimize the aspect ratio of a plate fin in heat convection. Based on a two-dimensional model, the theoretical analysis shows that the minimum thermal resistance defined with the concept of entransy dissipation corresponds to the maximum heat transfer rate when the temperature of the heating surface is fixed. On the other hand, when the heat flux of the heating surface is fixed, the minimum thermal resistance corresponds to the minimum average temperature of the heating surface. The entropy optimization is also given for the heat transfer processes. It is observed that the minimum entropy generation, the minimum entropy generation number, and the minimum revised entropy generation number do not always correspond to the best heat transfer performance. In addition, the influence factors on the optimized aspect ratio of the plate fin are also discussed. The optimized ratio decreases with the enhancement of heat convection, while it increases with fin thermal conductivity increasing.

Keywords: entransy dissipation, thermal resistance, heat convection optimization, fin efficiency

PACS: 05.70.Ln, 44.27.+g, 44.05.+e **DOI:** 10.1088/1674-1056/22/2/020503

1. Introduction

Analyses and enhancement of heat transfer have become important research topics due to the worldwide energy problem since the middle of last century because nearly 80% of the total energy consumption is related to heat transfer.[1–5] Many studies have been conducted to improve or analyze the convective heat transfer because it has broad applications in various engineering areas.[3,4,6,7]

For the enhancement of convective heat transfer, researchers have developed some passive techniques, such as mixing the main flow and/or the flow in the wall region by using a rough surface, inserts, etc., reducing the flow boundary layer thickness by using offset strip fins, jet impingement, etc., creating the rotating and/or the secondary flow by using swirl flow device, duct rotation, and so on.[6,7]

On the other hand, the optimization of convective heat transfer is also an important topic. For instance, increasing the fluid velocity or turbulence intensity can enhance the convection heat transfer rate. However, the power consumption is always limited when considering the cost. Therefore, the flow field of heat convection should be optimized under the limited power consumption to increase the heat transfer rate as much as possible.

For heat convection enhancement and optimization, some theories have been developed in the last two decades, such as the thermodynamic optimization,[8] the field synergy principle (FSP),[6,7] and the entransy theory.[9] From the viewpoint of thermodynamics, Bejan[8] developed the expressions of entropy generation (EG) for heat and fluid flows. The best heat transfer process is related to the minimum EG.[8] Bejan[8] analyzed the performance of heat exchangers and thermal systems. As the objective of this optimization is to minimize the total EG, it is called thermodynamic optimization. However, Bejan[8] also noted that there is an EG paradox in analyzing the balanced counter flow heat exchangers. The heat exchanger effectiveness ε does not always increase with EG number decreasing. Instead, it decreases with EG number reducing for $\varepsilon \in [0, 0.5]$. Shah and Skiepko[10] analyzed the relationship between the heat exchanger effectiveness and EG for 18 kinds of heat exchangers. The effectiveness was found to be maximum, intermediate or minimum at the maximum EG.

Guo *et al.*[9] introduced a new physical parameter, entransy, which describes the heat transfer ability. For one object whose internal energy is U and temperature is T, its entransy is[9]

$$G = \frac{1}{2}UT. \tag{1}$$

The entransy decrease principle,[11] the principles of the extremum entransy dissipation (EED) and the minimum entransy-dissipation-based thermal resistance were developed successively.[9,12] Cheng *et al.*[5] derived the microscopic expression of entransy for a monatomic ideal gas system, related the entransy to the microstate number, and discussed the microscopic physical meaning of entransy. The principles based on the concept of entransy have been used to optimize heat conduction,[9,13,14] heat convection,[1,9] thermal radiation,[15,16] heat exchangers,[17,18] and thermal networks.[19] The research has shown that the entransy optimization can lead to smaller average temperature difference or

*Project supported by the National Natural Science Foundation of China (Grant No. 51106082) and the Tsinghua University Initiative Scientific Research Program.

†Corresponding author. E-mail: liangxg@tsinghua.edu.cn

© 2013 Chinese Physical Society and IOP Publishing Ltd

higher heat transfer rate than that from the minimum principle of EG,[15,17–19] and no paradox is observed like the EG paradox when entransy is used to analyze heat exchangers.[17,20]

For the FSP, Yuan and Chen[1] found that the heat convection optimization results based on the entransy theory and the FSP are the same for fixed dissipation rate, with temperature boundary condition fixed. However, for the fixed heat flux boundary condition, only the optimized flow based on the entransy theory could be used to optimize the heat transfer process, whereas the optimization based on the FSP makes no sense.[1]

Fins are widely used in heat exchangers to enhance the convective heat transfer rate. Researchers have been paying attention to optimizing the parameters of fins to enhance heat convection as much as possible through theoretical analyses, numerical simulations, and experiments.[21–24] However, there are few reports on the application of the entransy theory for the parameter optimization of the fin in heat convection. Therefore, it is worth making an investigation of the entransy theory on the optimization of the parameters of a fin.

2. Entransy dissipation and entransy balance equation

The entransy theory indicates that entransy dissipation (ED) is produced as heat is transferred from a high temperature body to a low temperature one.[9,11] It can be expressed as

$$G_{\mathrm{dis}} = Q(T_{\mathrm{high}} - T_{\mathrm{low}}), \tag{2}$$

where Q is the heat transfer rate, T_{high} is the high temperature, and T_{low} is the low temperature. The principles of the EED indicate that the maximum entransy dissipation corresponds to the maximum heat transfer rate, with heat transfer temperature difference fixed, while the minimum ED corresponds to the minimum heat transfer temperature difference, with heat transfer rate fixed.[6] When the principles of the EED are used to optimize heat transfer processes, ED should be calculated first. For steady heat conduction or steady heat convection without any heat source, the ED can be calculated as follows.

For the steady heat conduction without heat sources, the entransy balance equation is[9]

$$\int_V \lambda(\nabla T)^2 \mathrm{d}V = -\int_A (\boldsymbol{q}T)\cdot\boldsymbol{n}\mathrm{d}A, \tag{3}$$

where λ is the thermal conductivity, T is the temperature, $\boldsymbol{q}$ is the heat flux vector, V is the control volume, and A is the surface of the volume. The left part in Eq. (3) is the ED in the volume, while the right part is the entransy flow that comes into the volume. The ED due to the steady heat conduction without heat sources can be calculated with this equation.

For the steady heat convection without heat sources, the entransy balance equation for incompressible fluid is[9]

$$\int_V \lambda(\nabla T)^2 \mathrm{d}V = -\int_A \left(\frac{1}{2}\rho c T^2\right)\boldsymbol{u}\cdot\boldsymbol{n}\mathrm{d}A - \int_A (\boldsymbol{q}T)\cdot\boldsymbol{n}\mathrm{d}A + \int_V (\Phi T)\mathrm{d}V, \tag{4}$$

where ρ is the density of the fluid, c is the specific heat capacity, Φ is the viscous dissipation and $\boldsymbol{u}$ is the velocity vector. This is the entransy balance equation for the steady heat convection without heat sources. The left part in Eq. (4) is the ED produced in the control volume, the first term on the right-hand side is the entransy flow that is brought into the volume by the fluid, the second term is the entransy flow due to heat exchange on the volume boundary, and the third is the entransy generated from the viscous dissipation. When the dynamic viscosity of the fluid is small, there is

$$\int_V \lambda(\nabla T)^2 \mathrm{d}V = -\int_A \left(\frac{1}{2}\rho c T^2\right)\boldsymbol{u}\cdot\boldsymbol{n}\mathrm{d}A - \int_A (\boldsymbol{q}T)\cdot\boldsymbol{n}\mathrm{d}A. \tag{5}$$

The entransy balance equation of conduction can be obtained by dropping the first term on the right-hand side. Based on the above equation, the heat convection with a fin can be analyzed and optimized.

3. Two-dimensional analyses

We consider a two-dimensional (2D) model with a plate fin to show the relation between heat transfer and ED for simplicity. As there are many fins in a heat exchanger, here we consider only one of them as shown in Fig. 1. A repeating heat transfer element is composed of a plate fin and the solid base of the fins. The total length of the element is L, while the thickness of the solid base is d. The height and the width of the fin are H and W, respectively. Assume that the heat transfer coefficient between the fluid and the element is a constant, h, and the temperature of the fluid is also considered as a constant, T_{f}. As the heat transfer elements are arrayed periodically, the left and the right boundaries of the solid base are treated as adiabatic. The weight of the fin should be limited under the consideration of cost of materials. This means that the area of the fin is limited in Fig. 1. Then, we assume that the area of the fin is a constant,

$$A_{\mathrm{F}} = HW = \text{constant}. \tag{6}$$

Under this constraint condition, the fin aspect ratio $r\,(=H/W)$ should be optimized to improve the heat transfer performance of the element.

For this problem, two cases are considered. In the first case, the temperature of the heating surface (the lower boundary of the base part), T_{h}, is fixed and the maximum heat transfer rate is the object to be optimized. The ED of the heat transfer process is composed of two parts. One part is the ED due to the heat convection between the solid base and the fluid. According to Eq. (2), it is expressed as

$$G_{\mathrm{dis-1}} = \int_{A_{\mathrm{f-s}}} \boldsymbol{q}_{A_{\mathrm{f-s}}}(T_{A_{\mathrm{f-s}}} - T_{\mathrm{f}})\cdot\boldsymbol{n}\mathrm{d}A, \tag{7}$$

where $A_{\mathrm{f-s}}$ is the heat transfer area between the element and the fluid, $\boldsymbol{q}_{A_{\mathrm{f-s}}}$ is the heat flux, and $T_{A_{\mathrm{f-s}}}$ is the temperature of

the solid surface. The other part is the ED in the heat conduction,

$$\begin{aligned} G_{\text{dis–2}} &= -\int_A \boldsymbol{q}_A T_A \cdot \boldsymbol{n}\,\mathrm{d}A \\ &= -\Big(\int_{A_{\text{f–s}}} \boldsymbol{q}_{A_{\text{f–s}}} T_{A_{\text{f–s}}} \cdot \boldsymbol{n}\,\mathrm{d}A + \int_{A_{\text{h}}} \boldsymbol{q}_{A_{\text{h}}} T_{\text{h}} \cdot \boldsymbol{n}\,\mathrm{d}A \\ &\quad + \int_{A_{\text{L}}} \boldsymbol{q}_{A_{\text{L}}} T_{A_{\text{L}}} \cdot \boldsymbol{n}\,\mathrm{d}A + \int_{A_{\text{R}}} \boldsymbol{q}_{A_{\text{R}}} T_{A_{\text{R}}} \cdot \boldsymbol{n}\,\mathrm{d}A\Big), \end{aligned} \tag{8}$$

where A_{h}, A_{L}, and A_{R} are the areas of the base part, the left boundary, and the right boundary, while $\boldsymbol{q}_{A_{\text{h}}}$, $\boldsymbol{q}_{A_{\text{L}}}$, and $\boldsymbol{q}_{A_{\text{R}}}$ are the heat fluxes of the heating surface, the left boundary, and the right boundary, respectively. As the left and the right boundaries are adiabatic, their heat fluxes are both 0. Therefore,

$$\begin{aligned} G_{\text{dis–2}} &= -\Big(\int_{A_{\text{f–s}}} \boldsymbol{q}_{A_{\text{f–s}}} T_{A_{\text{f–s}}} \cdot \boldsymbol{n}\,\mathrm{d}A + \int_{A_{\text{h}}} \boldsymbol{q}_{A_{\text{h}}} T_{\text{h}} \cdot \boldsymbol{n}\,\mathrm{d}A\Big) \\ &= -\Big(\int_{A_{\text{f–s}}} \boldsymbol{q}_{A_{\text{f–s}}} T_{A_{\text{f–s}}} \cdot \boldsymbol{n}\,\mathrm{d}A + T_{\text{h}} \int_{A_{\text{h}}} \boldsymbol{q}_{A_{\text{h}}} \cdot \boldsymbol{n}\,\mathrm{d}A\Big). \end{aligned} \tag{9}$$

Combining Eqs. (7) and (9), the ED of the heat transfer process is

$$\begin{aligned} G_{\text{dis}} &= G_{\text{dis–1}} + G_{\text{dis–2}} \\ &= \int_{A_{\text{f–s}}} \boldsymbol{q}_{A_{\text{f–s}}} (T_{A_{\text{f–s}}} - T_{\text{f}}) \cdot \boldsymbol{n}\,\mathrm{d}A \\ &\quad -\Big(\int_{A_{\text{f–s}}} \boldsymbol{q}_{A_{\text{f–s}}} T_{A_{\text{f–s}}} \cdot \boldsymbol{n}\,\mathrm{d}A + T_{\text{h}} \int_{A_{\text{h}}} \boldsymbol{q}_{A_{\text{h}}} \cdot \boldsymbol{n}\,\mathrm{d}A\Big) \\ &= -T_{\text{f}} \int_{A_{\text{f–s}}} \boldsymbol{q}_{A_{\text{f–s}}} \cdot \boldsymbol{n}\,\mathrm{d}A - T_{\text{h}} \int_{A_{\text{h}}} \boldsymbol{q}_{A_{\text{h}}} \cdot \boldsymbol{n}\,\mathrm{d}A. \end{aligned} \tag{10}$$

Considering the conservation of energy, there must be

$$Q = \int_{A_{\text{f–s}}} \boldsymbol{q}_{A_{\text{f–s}}} \cdot \boldsymbol{n}\,\mathrm{d}A = -\int_{A_{\text{h}}} \boldsymbol{q}_{A_{\text{h}}} \cdot \boldsymbol{n}\,\mathrm{d}A, \tag{11}$$

where Q is the total heat rate transferred from the heat surface to the fluid. Then, we can obtain

$$G_{\text{dis}} = Q(T_{\text{h}} - T_{\text{f}}). \tag{12}$$

As T_{h} and T_{f} are both known, the maximum ED corresponds to the maximum heat transfer rate. The principles of the EED can be used to optimize the heat transfer process shown in Fig. 1 when the temperature of the wall is given.

In the second case, the heat flux of the heating surface, q_{h}, is fixed and the minimum average temperature is the object to be optimized. For this case, equations (7) and (8) are also sound. Considering that q_{h} is given and the left and the right boundaries are adiabatic, we have

$$\begin{aligned} G_{\text{dis–2}} &= -\Big(\int_{A_{\text{f–s}}} \boldsymbol{q}_{A_{\text{f–s}}} T_{A_{\text{f–s}}} \cdot \boldsymbol{n}\,\mathrm{d}A + \int_{A_{\text{h}}} \boldsymbol{q}_{A_{\text{h}}} T_{\text{h}} \cdot \boldsymbol{n}\,\mathrm{d}A \\ &\quad + \int_{A_{\text{L}}} \boldsymbol{q}_{A_{\text{L}}} T_{A_{\text{L}}} \cdot \boldsymbol{n}\,\mathrm{d}A + \int_{A_{\text{R}}} \boldsymbol{q}_{A_{\text{R}}} T_{A_{\text{R}}} \cdot \boldsymbol{n}\,\mathrm{d}A\Big) \\ &= -\Big(\int_{A_{\text{f–s}}} \boldsymbol{q}_{A_{\text{f–s}}} T_{A_{\text{f–s}}} \cdot \boldsymbol{n}\,\mathrm{d}A - q_{\text{h}} \int_{A_{\text{h}}} T_{\text{h}}\,\mathrm{d}A\Big). \end{aligned} \tag{13}$$

The ED of the system is

$$\begin{aligned} G_{\text{dis}} &= G_{\text{dis–1}} + G_{\text{dis–2}} \\ &= \int_{A_{\text{f–s}}} \boldsymbol{q}_{A_{\text{f–s}}} (T_{A_{\text{f–s}}} - T_{\text{f}}) \cdot \boldsymbol{n}\,\mathrm{d}A \\ &\quad -\Big(\int_{A_{\text{f–s}}} \boldsymbol{q}_{A_{\text{f–s}}} T_{A_{\text{f–s}}} \cdot \boldsymbol{n}\,\mathrm{d}A + q_{\text{h}} \int_{A_{\text{h}}} T_{\text{h}}\,\mathrm{d}A\Big) \\ &= -T_{\text{f}} \int_{A_{\text{f–s}}} \boldsymbol{q}_{A_{\text{f–s}}} \cdot \boldsymbol{n}\,\mathrm{d}A + q_{\text{h}} \int_{A_{\text{h}}} T_{\text{h}}\,\mathrm{d}A. \end{aligned} \tag{14}$$

The average temperature of the heating surface could be defined as

$$T_{\text{av}} = \int_{A_{\text{h}}} T_{\text{h}}\,\mathrm{d}A / A_{\text{h}}. \tag{15}$$

According to the definition of Eq. (15) and energy conservation, we have

$$G_{\text{dis}} = -T_{\text{f}} \int_{A_{\text{f–s}}} \boldsymbol{q}_{A_{\text{f–s}}} \cdot \boldsymbol{n}\,\mathrm{d}A + q_{\text{h}} A_{\text{h}} T_{\text{av}} = Q(T_{\text{av}} - T_{\text{f}}). \tag{16}$$

As q_{h} is known, Q is a constant here. Hence, the minimum ED corresponds to the minimum average temperature of the base bottom. Therefore, the principle of the EED can also be used to optimize the heat transfer process in Fig. 1 when the heat flux of the base bottom is given.

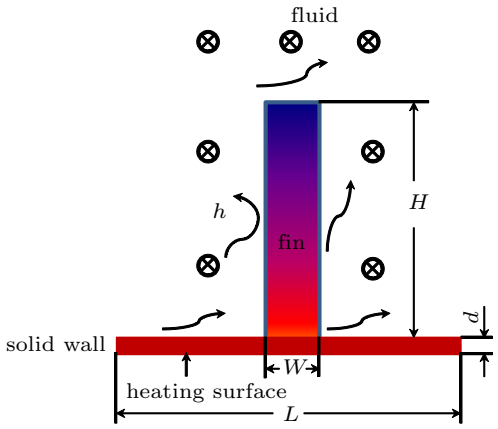


Fig. 1. (color online) Schematic plot of a 2D model.

If we define the thermal resistance, according to entransy dissipation, as[9]

$$R = G_{\text{dis}} / Q^2, \tag{17}$$

we have

$$R_T = (T_{\text{h}} - T_{\text{f}})/Q, \tag{18}$$

$$R_q = (T_{\text{av}} - T_{\text{f}})/Q, \tag{19}$$

for fixed temperature and fixed heat flux of the base bottom, respectively. The EEDs correspond to the minimum thermal resistance. Therefore, the principle of the minimum thermal resistance is also appropriate for optimizing the heat transfer process in Fig. 1.

Now, we come to discuss numerical examples. Assume that the fin is made of an aluminum alloy whose thermal conductivity is 130 W/(m·K), the solid base is made of stainless steel whose thermal conductivity is 15 W/(m·K), $T_{\text{f}} = 300$ K, $h = 1500$ W/m^2, $A_{\text{F}} = 4 \times 10^{-6}$ m^2, $d = 1 \times 10^{-3}$ m and $L = 5 \times 10^{-3}$ m. For different values of r $(= H/W)$, solving the energy conservation equation will give us the temperature distribution. The total heat transfer rate can be calcu-

lated by Eq. (11) when T_h is given (T_h is 320 K in the calculation). On the other hand, the average temperature of the heating surface can be obtained by Eq. (15) when q_h is given ($q_h = 5\times10^4$ W/m^2).

For the case with a fixed base bottom temperature, the heat transfer rate, the ED and the thermal resistance can be calculated under different ratios between H and W. The results are shown in Fig. 2. The ED and the heat transfer rate both reach their maximum value when r is 8.0. At the same time, the thermal resistance reaches its minimum value. On the other hand, for the case with a fixed base bottom heat flux, the numerical results in Fig. 3 demonstrate that the ED, the thermal resistance, and the average temperature of the base bottom all reach their minimum value when r is 8.0. The numerical results of the above cases have verified the results of the theoretical analyses, which are shown in Eqs. (12), (16), (18), and (19), indicating that the entransy theory can be used to optimize the aspect parameters of a fin in heat convection.

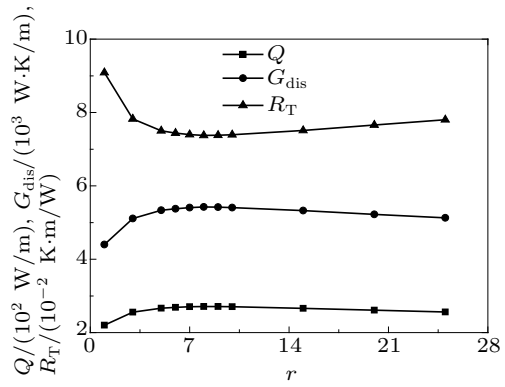


Fig. 2. Variations of average temperature of heating surface, entransy dissipation, and the thermal resistance with fin aspect ratio for fixed T_h.

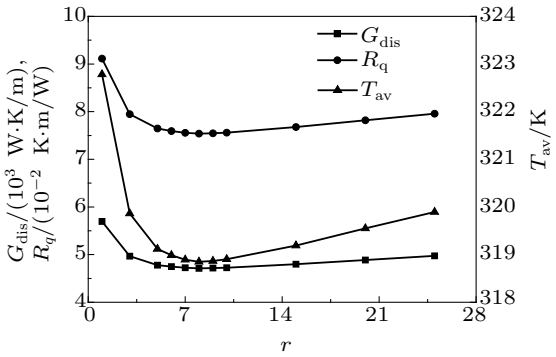


Fig. 3. Variations of average temperature of the heating surface, entransy dissipation, and the thermal resistance with fin aspect ratio for fixed q_h.

On the other hand, the minimum principle of EG is also calculated in this heat transfer process.[25] The entropy equation is[26]

$$\mathrm{d}S = \mathrm{d}S_f + \delta S_g, \tag{20}$$

where $\mathrm{d}S$ is the entropy change, $\mathrm{d}S_f$ is the entropy flow, and δS_g is the EG rate. As the heat transfer process is steady, $\mathrm{d}S$ is zero. The EG in the heat conduction region can be expressed as

$$\begin{aligned} S_{g-1} &= \int_V \delta S_g = -\int_V \mathrm{d}S_f \\ &= \int_V \nabla\cdot(\boldsymbol{q}/T)\,\mathrm{d}V = \int_A \boldsymbol{q}_A/T_A\cdot\boldsymbol{n}\,\mathrm{d}A. \end{aligned} \tag{21}$$

As the left and the right boundaries are adiabatic, Eq. (21) reduces to

$$S_{g-1} = \int_{A_{f-s}} \boldsymbol{q}_{A_{f-s}}/T_{A_{f-s}}\cdot\boldsymbol{n}\,\mathrm{d}A + \int_{A_h} \boldsymbol{q}_h/T_h\cdot\boldsymbol{n}\,\mathrm{d}A. \tag{22}$$

The EG due to the heat convection between the fluid and the solid base is

$$S_{g-2} = -\int_{A_{f-s}} \boldsymbol{q}_{A_{f-s}}(1/T_{A_{f-s}} - 1/T_f)\cdot\boldsymbol{n}\,\mathrm{d}A. \tag{23}$$

Therefore, the total EG in the heat transfer process is

$$\begin{aligned} S_g &= S_{g-1} + S_{g-2} \\ &= \int_{A_{f-s}} \boldsymbol{q}_{A_{f-s}}/T_{A_{f-s}}\cdot\boldsymbol{n}\,\mathrm{d}A + \int_{A_h} \boldsymbol{q}_h/T_h\cdot\boldsymbol{n}\,\mathrm{d}A \\ &\quad - \int_{A_{f-s}} \boldsymbol{q}_{A_{f-s}}(1/T_{A_{f-s}} - 1/T_f)\cdot\boldsymbol{n}\,\mathrm{d}A \\ &= \int_{A_{f-s}} \boldsymbol{q}_A\cdot\boldsymbol{n}\,\mathrm{d}A/T_f + \int_{A_h} \boldsymbol{q}_h/T_h\cdot\boldsymbol{n}\,\mathrm{d}A \\ &= Q/T_f + \int_{A_h} \boldsymbol{q}_h/T_h\cdot\boldsymbol{n}\,\mathrm{d}A. \end{aligned} \tag{24}$$

When the temperature of the heating surface is fixed, Eq. (24) reduces to

$$\begin{aligned} S_{g-T} &= Q/T_f + \int_{A_h} \boldsymbol{q}_h/T_h\cdot\boldsymbol{n}\,\mathrm{d}A \\ &= Q/T_f + \int_{A_h} \boldsymbol{q}_h\cdot\boldsymbol{n}\,\mathrm{d}A/T_h = Q(1/T_f - 1/T_h). \end{aligned} \tag{25}$$

For the case with fixed heat flux of the heating surface, there is

$$\begin{aligned} S_{g-q} &= Q/T_f + \int_{A_h} \boldsymbol{q}_h/T_h\cdot\boldsymbol{n}\,\mathrm{d}A = Q/T_f - q_h\int_{A_h}(1/T_h)\,\mathrm{d}A \\ &= Q\Big[1/T_f - \int_{A_h}(1/T_h)\,\mathrm{d}A/A_h\Big]. \end{aligned} \tag{26}$$

The EG of the heat transfer processes of the above two cases can be calculated according to Eqs. (25) and (26). The results are shown in Fig. 4. The comparison between the results shown in Figs. 2 and 4 indicate that the maximum heat transfer rate does not correspond to the minimum EG but the maximum EG when T_h is fixed. When q_h is fixed, the minimum EG corresponds to the minimum average temperature of the heating surface as can been seen from Figs. 3 and 4. Hence, the minimum EG does not always correspond to the best heat transfer process. This is because EG is a physical quantity that is used to analyze the irreversibility of heat-work conversion. Less EG means less loss of the ability of doing work. However, in heat transfer, what we are concerned with is not the loss of the ability to do work, but the rate of heat transfer. Therefore, the minimum principle of EG is not always appropriate to heat transfer optimization if only heat transfer is of concern.[27] On the other hand, the EED and the minimum

thermal resistance are derived from the heat transfer equation and closely related to the heat transfer performance, so the entransy theory is appropriate to heat transfer optimization.

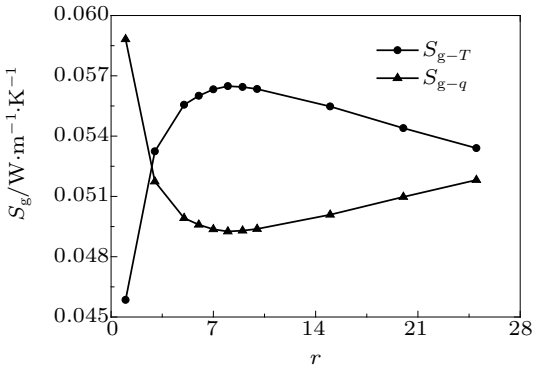


Fig. 4. Variations of entropy generation with fin aspect ratio for fixed T_h and q_h.

Furthermore, the dimensionless EG has also been used to analyze heat transfer processes, such as the EG number[8] and the revised EG number.[28] For the problem shown in Fig. 1, the definitions of EG number and revised EG number are

$$N_S = S_g/C_f, \tag{27}$$

$$N_{RS} = T_f S_g/Q, \tag{28}$$

where C_f is the heat flow capacity rate of the fluid. For the numerical results as shown in Figs. 2–4, the heat capacity flow rate is fixed, so the variation tendency of the EG number should be the same as the entropy generation. Therefore, the maximum EG number corresponds to the maximum heat transfer rate, with temperature of the heating surface fixed, while the minimum EG number corresponds to the minimum average temperature of the heating surface, with heat transfer rate fixed. The minimum EG number does not always correspond to the best performance of the heat transfer process.

Considering Eqs. (25) and (26), the revised EG number is

$$N_{RS} = T_f(1/T_f - 1/T_h) = 1 - T_f/T_h = \text{const}, \tag{29}$$

with temperature of the heating surface fixed, while it is

$$\begin{aligned} N_{RS} &= T_f\left[1/T_f - \int_{A_h} (1/T_h)\,\mathrm{d}A/A_h\right] \\ &= 1 - T_f\int_{A_h} (1/T_h)\,\mathrm{d}A/A_h, \end{aligned} \tag{30}$$

with heat flux of the heating surface fixed. It can be found that the revised EG number is constant for the case where the temperature of the heating surface is fixed, while it has the same variation tendency with the EG when the heat flux of the heating surface is fixed. Therefore, the minimum revised EG number does not always correspond to the best performance of the heat transfer processes, either.

The influencing factors of the optimized ratio between H and W, such as the heat transfer coefficient between, h, and the solid material of the fin, are investigated. The results are shown in Figs. 5 and 6. Figure 5 elucidates that the optimized r decreases as h increases when q_h is fixed. On the other hand, the optimized r increases with fin thermal conductivity increasing when q_h is fixed as depicted in Fig. 6. The results for fixed T_h are similar to those in Figs. 5 and 6.

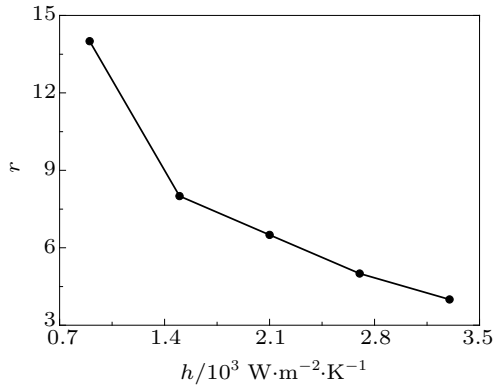


Fig. 5. Variation of optimized r with h for fixed q_h.

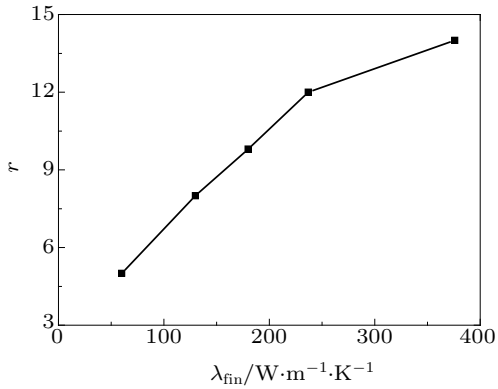


Fig. 6. Variation of optimized r with fin thermal conductivity for fixed q_h.

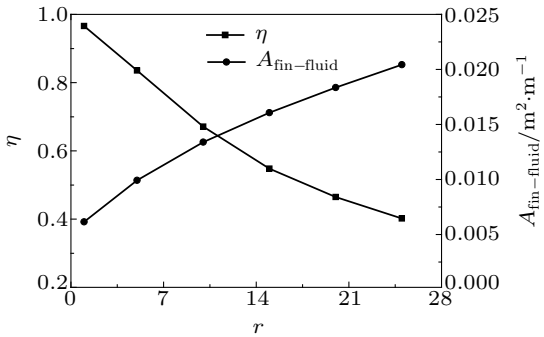


Fig. 7. Influences of r on fin surface area and fin efficiency when h is 1500 W/m^2 and the thermal conductivity of the fin is 130 W/(m·K).

The aspect ratio, r, affects the heat transfer area between the fin and the fluid, and the fin efficiency, η. As shown in Fig. 7 where $h = 1500$ W/m^2 and the fin thermal conductivity is 130 W/(m·K), the heat transfer area increases as r rises, which is beneficial to the heat transfer performance. At the same time, the fin efficiency decreases, and this goes against the improvement of the heat transfer performance. Therefore, there is a proper r that will make a balance between the two parameters and the best heat transfer performance when h and the fin thermal conductivity are given. As h increases, the heat

transfer between the fin and the fluid is enhanced but the original balance of the two parameters is destroyed. The fin temperature gradient becomes larger along the fin height direction and the fin efficiency decreases. So, the increase of r leads to a lowered fin efficiency and a poor heat transfer process. The optimized r decreases with h increasing as shown in Fig. 5. On the other hand, as the fin thermal conductivity increases, the fin resistance reduces and the fin efficiency increases. Increasing r will lead to a larger heat transfer area between the fin and the fluid. Then the heat transfer performance will be improved and the optimized r increases in Fig. 6.

4. Conclusions

The entransy theory is used to analyze the geometric parameters of a plate fin in heat convection in this paper. Based on the 2D model, the theoretical analysis shows that the minimum thermal resistance defined with the concept of ED corresponds to the maximum heat transfer rate when the temperature of the heating surface is fixed. On the other hand, when the heat flux of the base bottom is fixed, the minimum thermal resistance corresponds to the minimum average temperature of the base bottom. These relations are verified by the numerical examples.

The entropy generation of the heat transfer processes is also discussed. It is found that the minimum entropy generation, the minimum entropy generation number, and the minimum revised entropy generation number do not always correspond to the best performance of the heat transfer processes.

The influence factors on the optimized fin aspect ratio are discussed. It is found that the optimized ratio decreases as the heat transfer coefficient between the solid and the fluid increases. When the thermal conductivity of the fin increases, the numerical results show that the optimized ratio increases.

References

[1] Yuan F and Chen Q 2011 *Energ.* **36** 5476
[2] Liu X B and Guo Z Y 2009 *Acta Phys. Sin.* **58** 4766 (in Chinese)
[3] Cheng R J and Ge H X 2010 *Chin. Phys.* B **19** 090201
[4] Salem A M and Fathy R 2012 *Chin. Phys.* B **21** 054701
[5] Cheng X T, Liang X G and Xu X H 2011 *Acta Phys. Sin.* **60** 060512 (in Chinese)
[6] Guo Z Y, Li D Y and Wang B X 1998 *Int. J. Heat Mass Transfer* **41** 2221
[7] Guo Z Y 2001 *Chin. Sci. Bull.* **46** 596
[8] Bejan A 1998 *Advanced Engineering Thermodynamics* (New York: Wiley)
[9] Guo Z Y, Zhu H Y and Liang X G 2007 *Int. J. Heat Mass Transfer* **50** 2545
[10] Shah R K and Skiepko T 2004 *J. Heat Transfer* **126** 994
[11] Cheng X T, Liang X G and Guo Z Y 2011 *Chin. Sci. Bull.* **56** 847
[12] Zhu H Y, Chen Z J and Guo Z Y 2007 *Prog. Nat. Sci.* **17** 1692 (in Chinese)
[13] Xiao Q H, Chen L G and Sun F R 2011 *Chin. Sci. Bull.* **56** 102
[14] Cheng X T, Xu X H and Liang X G 2009 *Sci. China* E: *Tech. Sci.* **52** 2937
[15] Cheng X T and Liang X G 2011 *Int. J. Heat Mass Transfer* **54** 269
[16] Cheng X T, Xu X H and Liang X G 2011 *Sci. China* E: *Tech. Sci.* **54** 2446
[17] Guo Z Y, Liu X B, Tao W Q and Guo Z Y 2010 *Int. J. Heat Mass Transfer* **53** 2877
[18] Guo J F, Xu M T and Chen L 2011 *Chin. Sci. Bull.* **56** 2071
[19] Cheng X T, Xu X H and Liang X G 2011 *Sci. China: Tech. Sci.* **54** 964
[20] Qian X D and Li Z X 2011 *Int. J. Therm. Sci.* **50** 608
[21] Hsiao K L 2010 *J. Therm. Sci.* **19** 337
[22] Oyakawa K, Didarul I M and Yaga M 2006 *J. Therm. Sci.* **15** 145
[23] Kim M H and Bullard C W 2002 *Int. J. Refrigeration* **25** 390
[24] Mertol A 1993 *ASME J. Electronic Packaging* **115** 440
[25] Poulikakos D and Bejan A 1982 *J. Heat Transfer* **104** 616
[26] Zeng D L, Ao Y, Zhang X M and Liu C 2002 *Engineering Thermodynamics* (Beijing: High Eduction Press) (in Chinese)
[27] Cheng X G, Meng J A and Guo Z Y 2005 *J. Eng. Thermophys.* **26** 1034 (in Chinese)
[28] Hesselgreaves J E 2000 *Int. J. Heat Mass Transfer* **43** 4189