A fractional order hyperchaotic system derived from a Liu system and its circuit realization*

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In this paper we propose a novel four-dimensional fractional order hyperchaotic system derived from a Liu system. Electronics workbench (EWB) and Matlab simulations show the dynamical behavior of the proposed four-dimensional fractional order hyperchaotic system. Finally, after separately using EWB and Matlab, an electronic circuit is designed to realize the novel four-dimensional fractional order hyperchaotic system and the experimental circuit results are obtained which are identical to software simulations.

Keywords: fractional order hyperchaotic system, numerical simulation, circuit experiment

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1. Introduction

During the last century, Lorenz[1] found the first strange attractor, which was called the Lorenz attractor in numerical experiment. Since then, chaotic phenomena have been studied and developed in more and more fields. These theories cover the intrinsic quality of chaos, the basic characteristic of chaos.[2–5] Since the discovery of the first hyperchaotic Rössler system, many new hyperchaotic systems have been developed and studied. Hyperchaos is more complex than chaos and plays a significant role in nonlinear sciences. The study of chaos and hyperchaos has received great attention in the past decades, especially the synchronization and control.

Fractional order calculus is a kind of calculus with an arbitrary order. In fact, it has the same history as the integer order calculus. However, it has become attractively important during the latest decades because of the discovery of its application in engineering.[6–8]

In the present study, a new four-dimensional fractional order hyperchaotic system is designed. EWB and Matlab simulations show the dynamical behavior of the proposed four-dimensional fractional order hyperchaotic system. Finally, after separately using EWB and Matlab, an electronic circuit is designed to realize the novel four-dimensional fractional order hyperchaotic system, and the experimental circuit results are obtained which are identical to software simulations.

2. Novel four-dimensional fractional order hyperchaotic system

Before we discuss the fractional-order system, a four-dimensional integral-order hyperchaotic Liu system[4] is given by

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = a(y-x)+w, \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = bx - xz + 0.5w, \\ \dfrac{\mathrm{d}z}{\mathrm{d}t} = -cz + 4x^2 - w, \\ \dfrac{\mathrm{d}w}{\mathrm{d}t} = -dy - w, \end{cases} \tag{1}$$

$$a = 10, \quad b = 40, \quad c = 2.5, \quad d = 10/15.$$

We use Matlab to analyze the system and find that the above system has three equilibrium points, they are $O = (0,0,0,0)$, $E_1 = (4.89, 5.24, 39.64, -3.49)$, and $E_2 = (3.62, 39.64, -5.07, -5.43)$.

For equilibrium point $O = (0,0,0,0)$, we make the system linearized, and define the Jacobian matrix as

$$\begin{bmatrix} -10 & 10 & 0 & 1 \\ 40-z & 0 & -x & 0.5 \\ 8x & 0 & -2.5 & -1 \\ 0 & -2/3 & 0 & -1 \end{bmatrix},$$

then we can gain its eigenvalues as follows:

$$\lambda_i = (-2.5000, -25.6366, 15.5639, -0.9573), \quad i = 1,2,3,4.$$

We can observe that there is a positive eigenvalue. For the other two equilibrium points, we can also gain the eigenvalues of the Jacobian matrix as follows: $\lambda_i = (-17.5943,\ 2.5226 + 10.0947\mathrm{i},\ 2.5226 - 10.0947\mathrm{i},\ -0.9509)$, $i = 1,2,3,4$, and $\lambda_i = (-17.8657,\ 2.6429 + 10.3590\mathrm{i},\ 2.6429 - 10.3590\mathrm{i},\ -0.9202)$, $i = 1,2,3,4$.

We can observe that there is a pair of complex conjugate eigenvalues with positive real parts among the four eigenvalues of either of the equilibrium points. So we observe that

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all the equilibrium points are unstable. What is more, by calculation four Lyapunov exponents are obtained; two positive Lyapunov exponents are $LE_1 = 1.4595$ and $LE_2 = 0.0225$, two negative Lyapunov exponents are $LE_3 = -14.0117$ and $LE_4 = -0.9602$. According to chaos theory, two positive Lyapunov exponents imply that the dynamics of the system is hyperchaotic. The hyperchaotic strange attractors of system (1) are shown in Fig. 1.

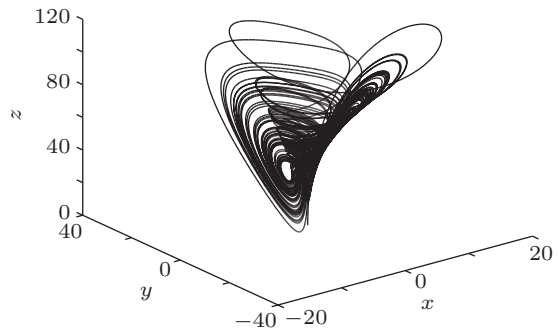


Fig. 1. Three-dimensional strange attractors of the system.

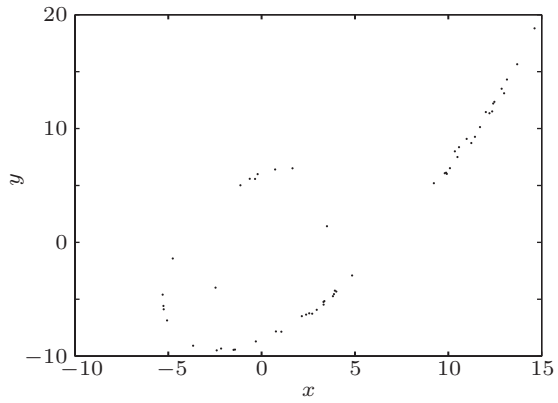


Fig. 2. Poincarè map of x–y plane.

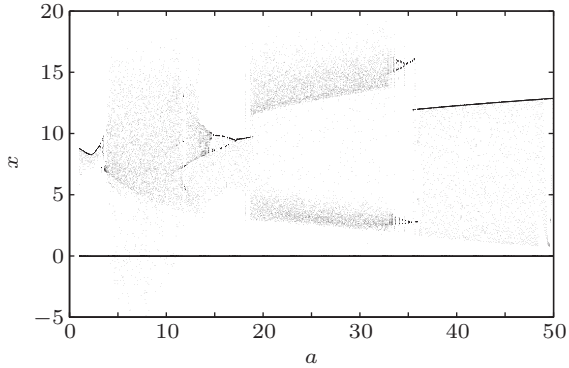


Fig. 3. Bifurcation diagram versus parameter a.

Poincarè mapping is an intuitive method to determine whether a system is chaotic. It can be seen that the Poincarè mappings are formed by these points in confusion as shown in Fig. 2. The bifurcation diagram of x with the value of a increasing is given in Fig. 3. It shows abundant and complex dynamical behaviors.

According to the above descriptions, we modify the differential operators in Eq. (1) into the ones corresponding to a fractional-order α. Thus equation (1) is converted into

$$\begin{cases} \dfrac{\mathrm{d}^{0.9}x}{\mathrm{d}t} = 10(y-x)+w, \\ \dfrac{\mathrm{d}^{0.9}y}{\mathrm{d}t} = 40x - xz + 0.5w, \\ \dfrac{\mathrm{d}^{0.9}z}{\mathrm{d}t} = -2.5z + 4x^2 - w, \\ \dfrac{\mathrm{d}^{0.9}w}{\mathrm{d}t} = -\mathrm{d}y - w, \end{cases} \tag{2}$$

where $\alpha = 0.9$ is the fractional order satisfying $0 < \alpha < 1$.

Now we use Laplace transformation to transform the four-dimensional fractional-order hyperchaotic system into a twelve-dimensional integer-order hyperchaotic system. The following Laplace transformations are used

$$\begin{aligned} &s^{0.9}X(s) = 10(Y(s)-X(s))+W(s), \\ &s^{0.9}Y(s) = 40X(s) - X(s)*Z(s) + 0.5W(s), \\ &s^{0.9}Z(s) = -2.5Z(s) + 4X(s)*X(s) - W(s), \\ &s^{0.9}W(s) = -dY(s) - W(s). \end{aligned}$$

In Ref. [9], an effective algorithm is developed to approximate the fractional-order transfer functions. From Table 11 in Ref. [10], we can obtain an approximation of $1/s^{0.9}$ with an error of about 2 dB as follows:

$$\begin{aligned} \frac{1}{s^{0.9}} &\approx \frac{2.2675(s+1.292)(s+215.4)}{(s+0.01292)(s+2.154)(s+359.4)} \\ &= \frac{2.2675(s^2+216.692s+278.2968)}{s^3+361.567s^2+778.819s+10} \\ &= \frac{g(s^2+ks+l)}{s^3+ms^2+ns+p}, \\ g &= 2.2675, k = 216.692, l = 278.2968, \\ m &= 316.567, n = 778.819, p = 10. \end{aligned}$$

Let the variables

$$\begin{aligned} &x_1 = x,\ x_2 = x_1',\ x_3 = x_1^{(2)}, \\ &x_4 = y,\ x_5 = x_4',\ x_6 = x_4^{(2)}, \\ &x_7 = z,\ x_8 = x_7',\ x_9 = x_7^{(2)}, \\ &x_{10} = w,\ x_{11} = x_{10}',\ x_{12} = x_{10}^{(2)}, \end{aligned}$$

then we will have

$$\begin{aligned} x_3' &= 10g(x_6 + kx_5 + lx_4) - [(10g+m)x_3 + (10gk+n)x_2 \\ &\quad + (10gl+p)x_1] + g(x_{12} + kx_{11} + lx_{10}), \\ x_6' &= 40g(x_3 + kx_2 + lx_1) - (mx_6 + nx_5 + px_4) \\ &\quad + 0.5g(x_{12} + kx_{11} + lx_{10}) - g[(x_3x_7 + 2x_2x_8 + x_1x_9) \\ &\quad + k(x_2x_7 + x_1x_8) + lx_1x_7], \end{aligned}$$

$$
\begin{aligned}
x'_9 &= -[(2.5g+m)x_9+(2.5gk+n)x_8+(2.5gl+p)x_7] \\
&\quad +4g[(2x_2^2+2x_1x_3)+2kx_1x_2+lx_1^2] \\
&\quad -g(x_{12}+kx_{11}+lx_{10}), \\
x'_{12} &= -\mathrm{d}g(x_6+kx_5+lx_4)-[(g+m)x_{12} \\
&\quad +(gk+n)x_{11}+(gl+p)x_{10}]. \qquad (3)
\end{aligned}
$$

We utilize these approximations to realize the fractional-order hyperchaotic system by using Matlab. Figure 4 shows the fractional-order hyperchaotic phase diagram with $a = 10$, $b = 40$, $c = 2.5$, $d = 10/15$.

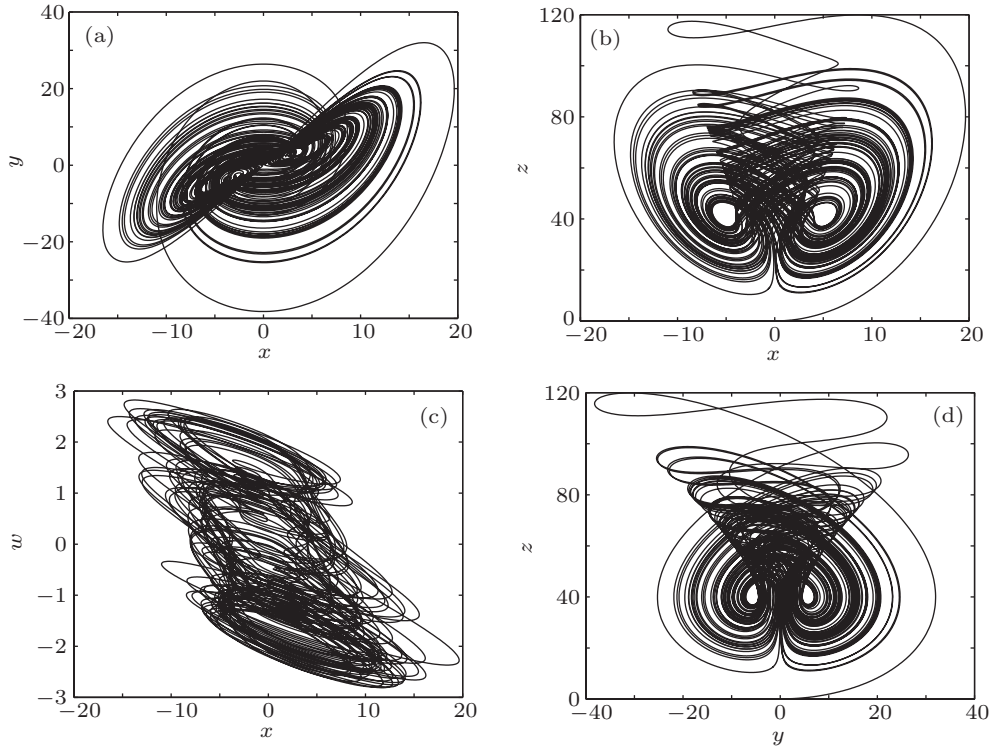


Fig. 4. Strange attractors in fractional-order phase planes. (a) x–y phase plane, (b) x–z phase plane, (c) x–w phase plane, (d) y–z phase plane.

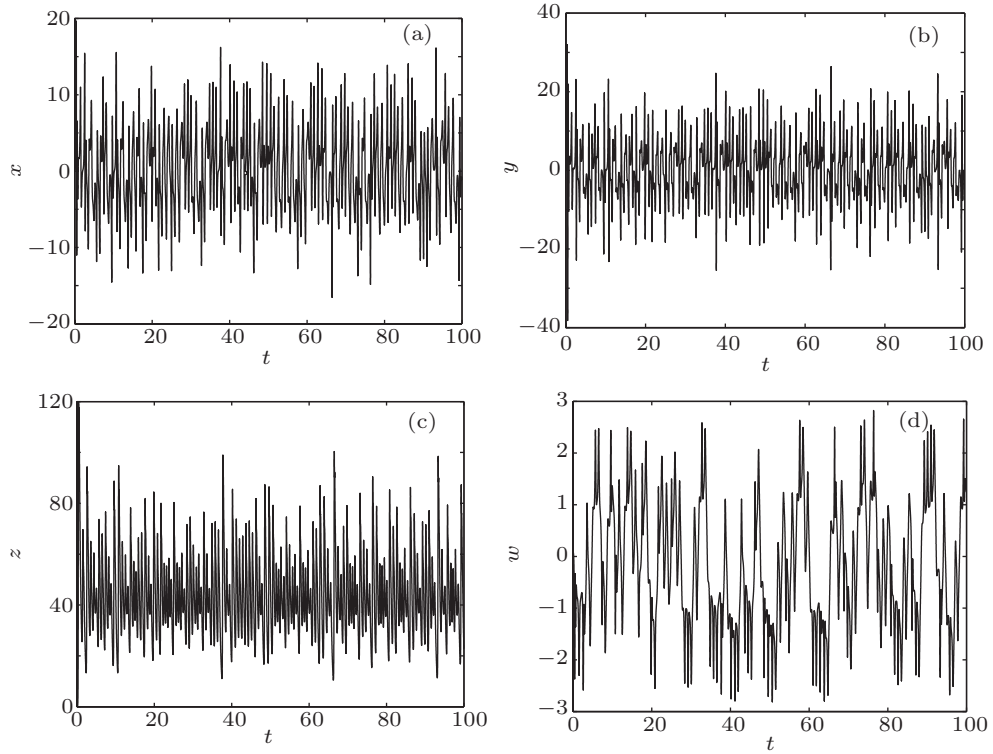


Fig. 5. Waveforms of the fractional-order system. (a) $x(t)$ waveform of the system, (b) $y(t)$ waveform of the system, (c) $z(t)$ waveform of the system, (d) $w(t)$ waveform of the system.

We transform the fractional-order differential equations into integer-order differential equations which have 12 variables similarly. By considering it to be a new twelve-dimensional vector $(x,x',x'',y,y',y'',z,z',z'',w,w',w'')$, a new four-dimensional vector $\boldsymbol{x} = (x,y,z,w)$ can be obtained. Thus we can obtain the phase plane view and $\boldsymbol{x}(t)$. By using fast Fourier transform (FFT), the variable frequency view is obtained finally. The waveforms in time domain are shown in Fig. 5 and the spectra of the nonlinear system are shown in Fig. 6. These figures clearly show that the fractional-order hyperchaotic system exhibits chaotic behaviors.

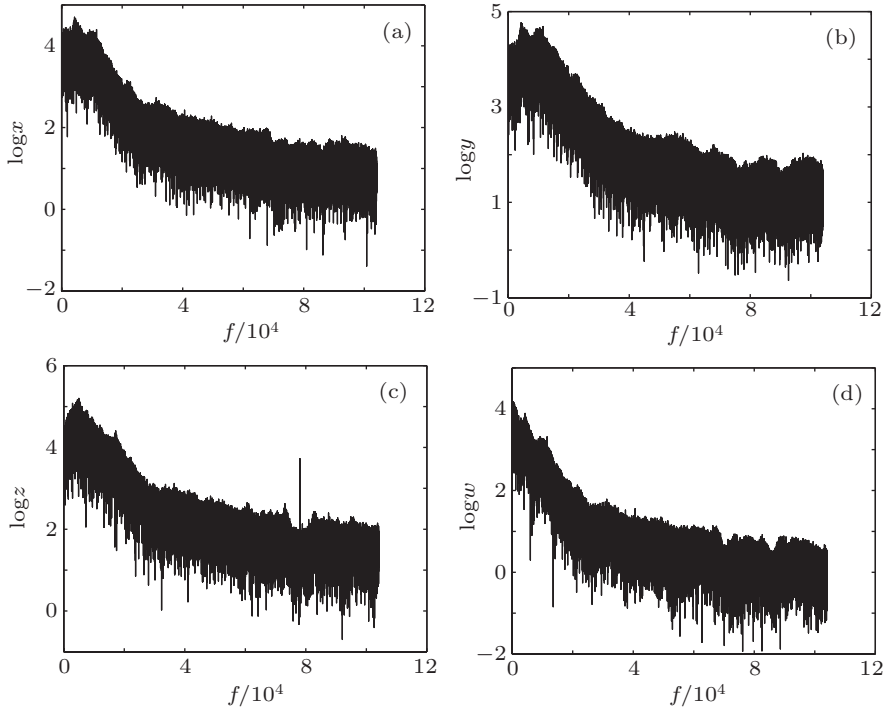


Fig. 6. Spectra of the fractional-order system. (a) spectrum $|x|$ in the system, (b) spectrum $|y|$ in the system, (c) spectrum $|z|$ in the system, (d) spectrum $|w|$ in the system.

3. Circuit experiment of the fractional-order hyperchaotic system

In this section, by using the EWB an electronic oscillation circuit is designed to realize the fractional order hyperchaotic system (2).

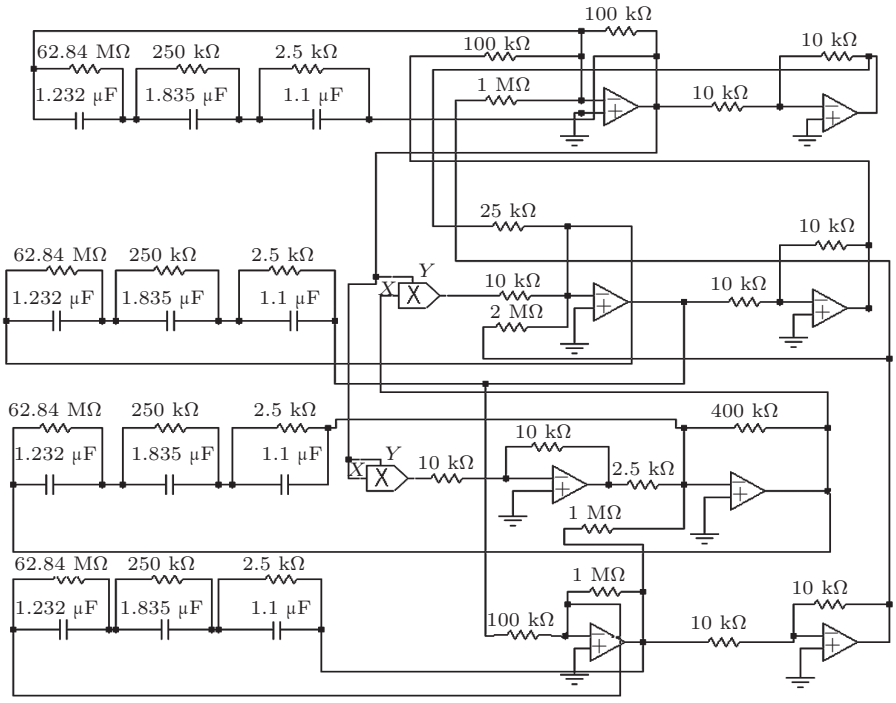


Fig. 7. Circuit of the novel four-dimensional fractional-order hyperchaotic system.

The designed circuit of the novel four-dimensional fractional-order hyperchaotic system is shown in Fig. 7, by using a linear resistor, linear electrical capacity, LM741 and AD633. The LM741 is used to operate addition and subtraction, while the AD633 is used to achieve the nonlinear unit in the four-dimensional fractional-order hyperchaotic system. In order to avoid the saturational distortion of the LM741, we make the voltage to be one tenth of the value that is a normal value in such a system.

The above circuit is designed to achieve the novel four-dimensional fractional-order hyperchaotic system. The experimental circuit results are obtained and shown below in the following figures, which are identical to the software simulations.

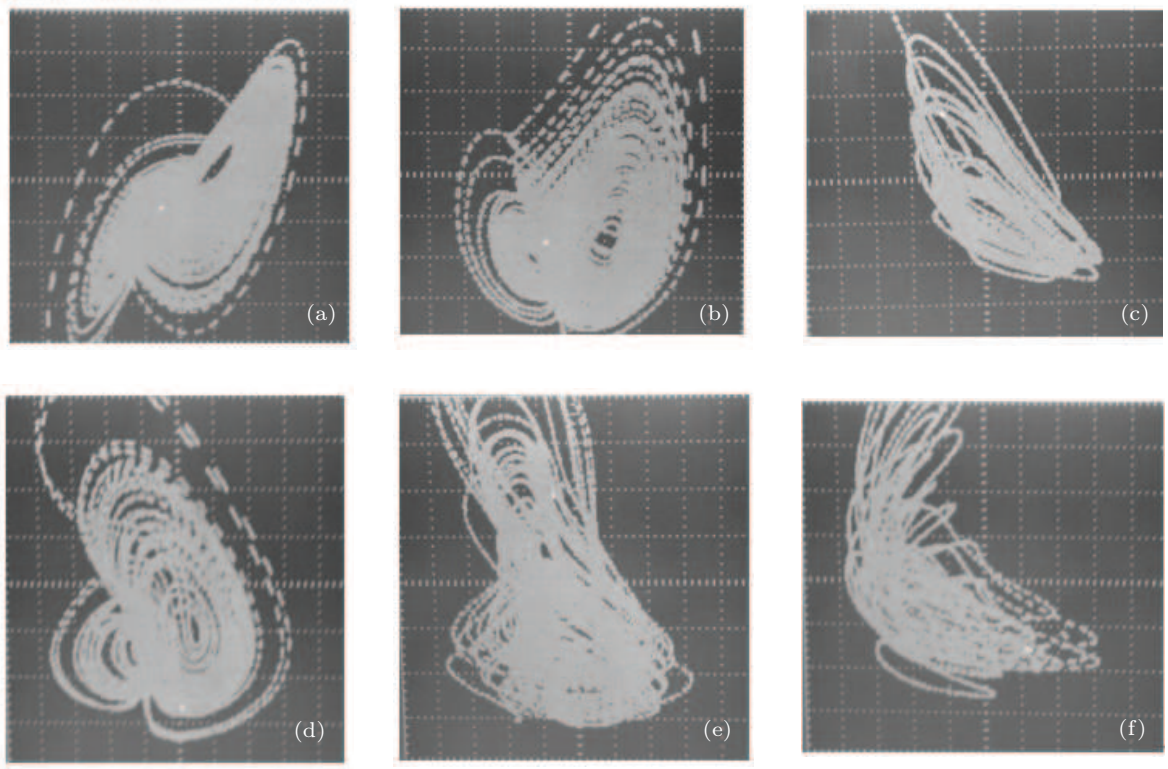


Fig. 8. (color online) Waveforms of circuit experiment: (a) x–y phase plane, (b) x–z phase plane, (c) x–w phase plane, (d) y–z phase plane, (e) y–w phase plane, (f) z–w phase plane.

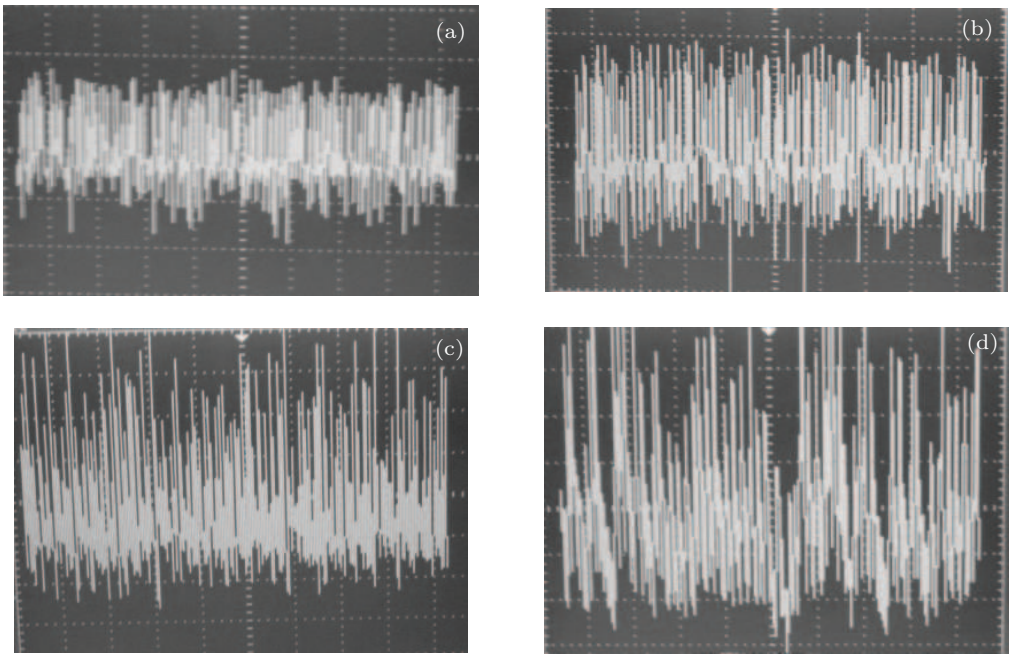


Fig. 9. Time evolutions of circuit experiment: (a) $x(t)$ waveform of the system, (b) $y(t)$ waveform of the system, (c) $z(t)$ waveform of the system, (d) $w(t)$ waveform of the system.

The experimental phase portraits of the fractional-order hyperchaotic system are shown in Fig. 8. It can be observed that the experimental circuit results are identical to the numerical simulations in the corresponding phase planes. The waveforms in the time domain are shown in Fig. 9. The experimental results of the circuit are shown to be in agreement with the numerical simulations.

4. Conclusion

In this paper, we proposed a new four-dimensional fractional-order hyperchaotic system that has abundant and complex dynamical behaviors. We converted it into a twelve-dimensional integer-ordered hyperchaotic system. The dynamical behaviors of the proposed four-dimensional fractional-order hyperchaotic system are shown through EWB and Matlab simulations. Finally, an electronic circuit is designed to realize the novel four-dimensional fractional-ordered hyperchaotic system, and the experimental circuit results, which are identical to numerical simulations, are obtained. It is expected that more and more detailed analyses of the theory and investigations on simulation related to the fractional-ordered hyperchaotic system will be achieved in the future.

References

[1] Lorenz E N 1963 *J. Atmos. Sci.* **20** 130
[2] Liu C X 2007 *Acta Phys. Sin.* **56** 6865 (in Chinese)
[3] Liu C X 2007 *Chin. Phys.* **16** 3279
[4] Zhong G Q and Tang W K S 2002 *Int. J. Bifurc. Chaos* **12** 1423
[5] Liu C X, Liu T, Liu L and Liu K 2004 *Chaos, Solitons & Fractals* **22** 1031
[6] Diethelm K and Ford N J 2002 *Mathematical Analysis and Applications* **265** 229
[7] Hasler M, Dedien H and Kennedy M P 1993 *Secure Communication via Chua's Circuit* (Proc. NOLTA'93 Workshops: Hawaii) pp. 87–92
[8] Wang F Q and Liu C X 2006 *Chin. Phys.* **15** 963
[9] Silva M F, Machado J A T and Lopes A M 2004 *Nonlinear Dynamics* **38** 417
[10] Ahmad W M and Sprott J C 2003 *Chaos, Solitons & Fractals* **16** 339