Interaction of a pair coherent state with a three-level Λ-type atom and generation of a modified Bessel–Gaussian state with a vortex structure

Tang Hui-Qin(唐慧琴)[a], Li Shao-Xin(李绍新)[b], Tang Ying(唐 英)[a], Zheng Xiao-Juan(郑小娟)[a], and Zhu Kai-Cheng(朱开成)[a]†

[a] School of Physical Science and Technology, Central South University, Changsha 410083, China
[b] Physical Staff Room, Guangdong Medical College, Dongguan 523808, China

(Received 8 April 2012; revised manuscript received 27 June 2012)

The evolution of a system state is derived based on the nonresonant interaction of a three-level "Λ" type atom with two cavity modes at a pair coherent state and two classic fields, and a cavity field state is analyzed in detail under conditional detecting. It is found that the quantized modified Bessel–Gaussian states as well as the superposition states consisting of the quantized vortex states with different weighted coefficients may be prepared through carefully preparing an initial atomic state and appropriately adjusting the interaction time. The scheme provides an additional choice to realize the two-mode quantized vortex state within the context of cavity quantum electrodynamics (QED).

Keywords: quantized vortex state, three-level Λ-type atom, cavity quantum electrodynamics (QED)

PACS: 03.67.Mn, 42.50.Dv **DOI:** 10.1088/1674-1056/22/2/020310

1. Introduction

Entanglement between two quantum systems is recognized nowadays as a key ingredient for testing quantum mechanics versus local hidden-variable theory.[1] Entangled quantum state as a valuable resource has been applied to quantum information processing.[2–4] Among the important two-mode entangled states, a pair coherent state[5,6] and a quantized vortex state[7] have received a great deal of attention. These states bear strong nonclassical properties and are highly entangled.[8,9] For instance, the pair coherent state extends the candidates of states that could provide continuous variable realizations of the Einstein–Podolsky–Rosen (EPR) paradox,[10] and so on.[11,12] In addition the experimental schemes to generate pair coherent states[13–15] or quantized vortex states[16–19] with currently available technology were proposed. For example, Tara and Agarwal[10] have shown that a two-mode state through linear transformation belonging to $SU(2)$ group may lead to a vortex state under special conditions. Very recently, the schemes to produce a generalized quantum elliptic vortex with the help of a beam splitter or a dual channel directional coupler to a two-mode squeezed coherent state[17] and to generate a two-mode quantized vortex state through a squeezed vacuum by subtracting a photon were proposed.[16] Moreover, we[18–21] also suggested that a quantum modified Bessel–Gaussian vortex state or Bessel–Gaussian state may be prepared with a pair coherent state through a parametric frequency conversion process.

In fact, there has been much interest in the evolution of the pair coherent state interacting with the two-level[22] and three-level[23] Jaynes–Cummings model atoms. In this research, we present an alternative method to prepare a two-mode quantized vortex state of a cavity field within the context of cavity quantum electrodynamics (QED). Based on the nonresonant interaction of a three-level Λ-type atom with two cavity modes and two classic fields,[24] we can obtain the quantized modified Bessel–Gaussian states as well as the superposition state consisting of the quantized vortex states with different weighted coefficients. The rest of the present paper is organized as follows. In Section 2, the state evolution of a pair coherent state interacting with a three-level Λ-type atom is derived. Then, in Section 3 we investigate the vortex structure associated with the cavity field state in the quadrature space. Finally, a summary is presented in Section 4.

2. Atomic model and evolution of pair coherent states

In this paper we aim at how to prepare newly quantized field states with vortices through the interaction of a pair coherent state with an atom placed in a cavity. Here the system consists of a three-level atom in Λ configuration placed inside a two-mode field cavity. Although the system has been considered before in Ref. [25], for the sake of completeness, some main results and steps are still presented here. The level structure of the atom is depicted in Fig. 1, where the two atomic transitions $|c\rangle \leftrightarrow |e\rangle$ and $|c\rangle \leftrightarrow |g\rangle$ interact with the two cavity modes with the same detuning Δ but with different coupling constants g_1 and g_2, respectively. The two atomic transitions $|c\rangle \leftrightarrow |e\rangle$ and $|c\rangle \leftrightarrow |g\rangle$ are also driven by two classical fields

†Corresponding author. E-mail: KCZhu058@csu.edu.cn

© 2013 Chinese Physical Society and IOP Publishing Ltd

with detuning Δ', and Ω_1 and Ω_2 are the Rabi frequencies of the two classical fields. The Hamiltonian describing the system can be expressed as

$$\begin{aligned} H &= \omega_e |e\rangle\langle e| + \omega_c |e\rangle\langle e| + \omega_1 a_1^\dagger a_1 + \omega_2 a_2^\dagger a_2 \\ &+ g_1 \left(a_1^\dagger |e\rangle\langle c| + a_1 |c\rangle\langle e| \right) + g_2 \left(a_2^\dagger |g\rangle\langle c| + a_2 |c\rangle\langle g| \right) \\ &+ \Omega_1 \left(\mathrm{e}^{-\mathrm{i}(\omega_c - \omega_e - \Delta')t} |c\rangle\langle e| + \mathrm{H.C.} \right) \\ &+ \Omega_2 \left(\mathrm{e}^{-\mathrm{i}(\omega_c - \Delta')t} |c\rangle\langle g| + \mathrm{H.C.} \right), \end{aligned} \tag{1}$$

where $a_i^\dagger$ and a_i are the creation and annihilation operators for the cavity fields of frequencies ω_i ($i = 1, 2$), while ω_c and ω_e are the Bohr frequencies associated with the two atomic transitions $|e\rangle \leftrightarrow |g\rangle$ and $|c\rangle \leftrightarrow |g\rangle$, respectively.

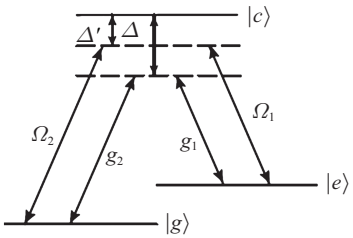


Fig. 1. Schematic diagram of a three-level Λ-type atom interacting with two cavity modes and two classic fields with detunings Δ and Δ', respectively.

Under the large detuning conditions

$$\left(\frac{\Omega_1}{\Delta'}, \frac{\Omega_2}{\Delta'}, \frac{g_1}{\Delta}, \frac{g_2}{\Delta} \right) \ll 1 \tag{2}$$

and after adiabatically eliminating the excited level $|c\rangle$, the effective Hamiltonian may be expressed as[25]

$$H_{\mathrm{eff}} = -\omega_{\mathrm{eff}} \left(a_1^\dagger a_2 \sigma^\dagger + a_1 a_2^\dagger \sigma \right) - \Omega_{\mathrm{eff}} \left(\sigma^\dagger + \sigma \right), \tag{3}$$

where $g_{\mathrm{eff}} = g_1 g_2/\Delta$, $\Omega_{\mathrm{eff}} = \Omega_1 \Omega_2/\Delta$, $\sigma^\dagger = |e\rangle\langle g|$ and $\sigma = |g\rangle\langle e|$ are raising and lowering atomic operators, respectively. In Eq. (3) we have assumed that the Stark shifts can be corrected by retuning the laser frequencies.[26]

In the strong driving regime $\Omega_{\mathrm{eff}} \gg g_{\mathrm{eff}}$, we choose $H_{\mathrm{eff}}^0 = -\Omega_{\mathrm{eff}} \left(\sigma^\dagger + \sigma \right)$ and $H_{\mathrm{eff}}^1 = -\omega_{\mathrm{eff}} \left(a_1^\dagger a_2 \sigma^\dagger + a_1 a_2^\dagger \sigma \right)$. By performing the unitary transformation $U = \exp\left(-\mathrm{i} H_{\mathrm{eff}}^0 t\right)$ on H_{eff}^1, in which we neglect the terms that oscillate with high frequencies, the Hamiltonian reads

$$H_{\mathrm{eff}}^{\mathrm{int}} = -\frac{1}{2} \omega_{\mathrm{eff}} \left(a_1^\dagger a_2 + a_1 a_2^\dagger \right) \left(\sigma^\dagger + \sigma \right). \tag{4}$$

Immediately, it can be recognized that the field Hamiltonian part $-\omega_{\mathrm{eff}}(a_1^\dagger a_2 + a_1 a_2^\dagger)/2$ is the generator of the $SU(2)$ coherent state.[27] Here, we are interested in using the Hamiltonian of Eq. (4) to convert a pair coherent state of the two cavity modes into a state with a vortex structure. For this purpose we consider the case that the atom state is initially prepared in a superposition atomic state $|\Psi_A(0)\rangle = \cos\delta_A |g\rangle + \sin\delta_A |e\rangle$ with δ_A being a real parameter, while the two cavity fields are both in a pair coherent state $\left|\Psi_f(0)\right\rangle = N_n \int_0^{2\pi} \mathrm{d}\theta\, \mathrm{e}^{\mathrm{i}n\theta} \left|\alpha\, \mathrm{e}^{\mathrm{i}\theta}, \alpha\, \mathrm{e}^{-\mathrm{i}\theta}\right\rangle$, where n is an integer $N_n^2 = \exp\left(\alpha^2\right)/(4\pi^2 \mathrm{I}_n\left(\alpha^2\right))$ is the normalized constant while α is related to the field intensity. Thus the initial state of the system is

$$\begin{aligned} |\Psi(0)\rangle &= |\Psi_A(0)\rangle \otimes \left|\Psi_f(0)\right\rangle \\ &= N_n \left(\cos\delta_A |g\rangle + \sin\delta_A |e\rangle\right) \otimes \int_0^{2\pi} \mathrm{d}\theta \\ &\times \mathrm{e}^{\mathrm{i}n\theta} \left|\alpha\, \mathrm{e}^{\mathrm{i}\theta}, \alpha\, \mathrm{e}^{-\mathrm{i}\theta}\right\rangle. \end{aligned} \tag{5}$$

On the basis of $|\pm\rangle = (|g\rangle \pm |e\rangle)/\sqrt{2}$, which are the eigenstates of $\sigma^\dagger + \sigma$ with eigenvalues ± 1, initial state (5) of the system can be rewritten as

$$\begin{aligned} |\Psi(0)\rangle &= \frac{1}{\sqrt{2}} N_n \int_0^{2\pi} \mathrm{d}\theta\, \mathrm{e}^{\mathrm{i}n\theta} \left[A_+ \left|+, \alpha\, \mathrm{e}^{\mathrm{i}\theta}, \alpha\, \mathrm{e}^{-\mathrm{i}\theta}\right\rangle \right. \\ &\left. + A_- \left|-, \alpha\, \mathrm{e}^{\mathrm{i}\theta}, \alpha\, \mathrm{e}^{-\mathrm{i}\theta}\right\rangle \right], \end{aligned} \tag{6}$$

where $A_\pm = \cos\delta_A \pm \sin\delta_A$. The time evolution of the whole system state is given by

$$\begin{aligned} |\Psi(t)\rangle &= \exp\left(-\mathrm{i} H_{\mathrm{eff}}^{\mathrm{int}} t\right) |\Psi(0)\rangle \\ &= \frac{1}{\sqrt{2}} N_n A_+ \exp\left[\mathrm{i}\tau\left(K_+ + K_-\right)\right] \int_0^{2\pi} \mathrm{d}\theta \\ &+ \mathrm{e}^{\mathrm{i}n\theta} \left|+, \alpha\, \mathrm{e}^{\mathrm{i}\theta}, \alpha\, \mathrm{e}^{-\mathrm{i}\theta}\right\rangle \\ &+ \frac{1}{\sqrt{2}} N_n A_- \exp\left[-\mathrm{i}\tau\left(K_+ + K_-\right)\right] \int_0^{2\pi} \mathrm{d}\theta \\ &+ \mathrm{e}^{\mathrm{i}n\theta} \left|+, \alpha\, \mathrm{e}^{\mathrm{i}\theta}, \alpha\, \mathrm{e}^{-\mathrm{i}\theta}\right\rangle, \end{aligned} \tag{7}$$

where $K_+ = a_1^\dagger a_2$, $K_- = a_1 a_2^\dagger$, and $\tau = g_{\mathrm{eff}} t/2$ is a dimensionless interaction time. These operators satisfy the $SU(2)$ commutation relations, i.e., $[K_+, K_-] = -2K_0$ and $[K_0, K_\pm] = \pm K_\pm$, with $K_0 = (a_1^\dagger a_1 - a_2^\dagger a_2)/2$. Thus we can use the $SU(2)$ Lie algebra[27] to expand the unitary evolution operator $\exp\left[\pm\mathrm{i}\tau\left(K_+ + K_-\right)\right]$ into

$$\begin{aligned} &\exp\left[\pm\mathrm{i}\tau\left(K_+ + K_-\right)\right] \\ &= \exp\left(\pm x_+ K_+\right) \exp\left(K_0 \ln x_0\right) \exp\left(\pm x_- K_-\right), \end{aligned} \tag{8}$$

in which

$$x_0 = \left[\cosh\left(\mathrm{i}\tau\right)\right]^{-2} \quad \text{and} \quad x_+ = x_- = \tanh\left(\mathrm{i}\tau\right). \tag{9}$$

According to Eq. (7) we can conveniently derive the evolution of the system as

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}} A_+ |+\rangle \left|\psi_f(t)\right\rangle + \frac{1}{\sqrt{2}} A_- |-\rangle \left|\psi_f^{\oplus}(t)\right\rangle, \tag{10}$$

where

$$\left|\psi_f(t)\right\rangle = N_n \int_0^{2\pi} \mathrm{d}\theta\, \mathrm{e}^{\mathrm{i}n\theta} \left|\alpha_{t,\theta}, \beta_{t,\theta}\right\rangle,$$

$$\left|\psi_f^{\oplus}(t)\right\rangle = N_n \int_0^{2\pi} \mathrm{d}\theta\, \mathrm{e}^{\mathrm{i}n\theta} \left|\alpha_{t,\theta}^*, \beta_{t,\theta}^*\right\rangle, \tag{11}$$

$$\alpha_{t,\theta} = \alpha\left(\mathrm{e}^{\mathrm{i}\theta}\cos\tau + \mathrm{i}\,\mathrm{e}^{-\mathrm{i}\theta}\sin\tau\right),$$

and

$$\beta_{t,\theta} = \alpha\left(\mathrm{e}^{-\mathrm{i}\theta}\cos\tau + \mathrm{i}\,\mathrm{e}^{\mathrm{i}\theta}\sin\tau\right). \tag{12}$$

Or, we change the basis back to atomic states

$$\begin{aligned}|\Psi(t)\rangle = \frac{1}{2}\Big\{&|g\rangle\left[A_+\left|\psi_f(t)\right\rangle + A_-\left|\psi_f^{\oplus}(t)\right\rangle\right] \\ &+ |e\rangle\left[A_+\left|\psi_f(t)\right\rangle - A_-\left|\psi_f^{\oplus}(t)\right\rangle\right]\Big\}.\end{aligned} \tag{13}$$

When the atom comes out of the two-mode cavity, we can use level-selective ionizing counters to detect the atomic state. If the internal state of the atom is detected to be state $|g\rangle$ or $|e\rangle$, from Eq. (13) it is easy to know that the two-mode cavity field state evolves into

$$|\Psi_{\mathrm{F}}(t)\rangle = M\left[A_+\left|\psi_f(t)\right\rangle \pm A_+\left|\psi_f^{\oplus}(t)\right\rangle\right], \tag{14}$$

where for our discussion, M is an unimportant parameter related to the normalization factor. In the next section we will discuss the wavefunction feature of field state (13) in the quadrature space.

3. Wavefunction of cavity field state in the quadrature space and generation of the quantized vortex state

First, we evaluate the wavefunction expression $\Psi_{\mathrm{F}}(x,y,t) = \langle x,y/\Psi_{\mathrm{F}}(t)\rangle$ of the two-mode cavity field, where $|x,y\rangle$ is the eigenvector of $\left(a^{\dagger}+a\right)/\sqrt{2}$ and $\left(b^{\dagger}+b\right)/\sqrt{2}$ with eigenvalues x and y. Based on the relation

$$\langle x|\alpha\rangle = \frac{1}{\pi^{1/4}}\exp\left(-\frac{x^2}{2} + \sqrt{2}\alpha x - \frac{\alpha^2}{2} - \frac{|\alpha|^2}{2}\right), \tag{15}$$

the state $\Psi_{\mathrm{F}}(x,y,t)$ becomes

$$\Psi_{\mathrm{F}}(x,y,t) = A_+\psi(x,y,t) \pm A_-\psi^{\oplus}(x,y,t) \tag{16}$$

where

$$\begin{aligned}\psi(x,y,t) &= \langle x,y|\psi_f(t)\rangle \\ &= \frac{N_n}{\sqrt{\pi}}\int_0^{2\pi}\mathrm{d}\theta\,\mathrm{e}^{\mathrm{i}n\theta}\langle x,y|\alpha_{t,\theta},\beta_{t,\theta}\rangle \\ &= \frac{N_n}{\sqrt{\pi}}\exp\left[-\frac{x^2+y^2}{2} - \alpha^2(1+\mathrm{i}\sin 2\tau)\right] \\ &\quad\times\int_0^{2\pi}\mathrm{d}\theta\,\mathrm{e}^{\mathrm{i}n\theta}\exp\left(-\alpha^2\cos 2\tau\cos 2\theta\right) \\ &\quad\times\exp\left\{\sqrt{2}\alpha\left[x\left(\mathrm{e}^{\mathrm{i}\theta}\cos\tau + \mathrm{i}\,\mathrm{e}^{-\mathrm{i}\theta}\sin\tau\right)\right.\right. \\ &\quad\left.\left.+ y\left(\mathrm{e}^{-\mathrm{i}\theta}\cos\tau + \mathrm{i}\,\mathrm{e}^{\mathrm{i}\theta}\sin\tau\right)\right]\right\}.\end{aligned} \tag{17}$$

By virtue of the generating function of the modified Bessel function $\mathrm{I}_n(x)$,

$$\begin{aligned}\exp\left(a\mathrm{e}^{\mathrm{i}\theta} + b\mathrm{e}^{-\mathrm{i}\theta}\right) &= \exp\left[\sqrt{ab}\left(\sqrt{\frac{a}{b}}\mathrm{e}^{\mathrm{i}\theta} + \sqrt{\frac{b}{a}}\mathrm{e}^{-\mathrm{i}\theta}\right)\right] \\ &= \sum_{n=-\infty}^{\infty}\mathrm{I}_n\left(2\sqrt{ab}\right)\left(\sqrt{\frac{a}{b}}\mathrm{e}^{\mathrm{i}\theta}\right)^n,\end{aligned} \tag{18}$$

one finally arrives at

$$\begin{aligned}\psi(x,y,t) &= 2\sqrt{\pi}N_n\exp\left[-\frac{x^2+y^2}{2} - \alpha^2(1+\mathrm{i}\sin 2\tau)\right] \\ &\quad\times\sum_{p=-\infty}^{\infty}\mathrm{I}_{n+2p}\left(2\sqrt{2}\alpha\sqrt{X_+Y_+}\right) \\ &\quad\times\mathrm{I}_p\left(-\alpha^2\cos 2\tau\right)\left(Y_+/X_+\right)^{p+n/2},\end{aligned} \tag{19}$$

where $X_{\pm} = x\cos\tau \pm \mathrm{i}y\sin\tau$ and $Y_{\pm} = y\cos\tau \pm \mathrm{i}x\sin\tau$. Similarly, one also obtains

$$\begin{aligned}\psi^{\oplus}(x,y,t) &= \left\langle x,y|\psi_f^{\oplus}(t)\right\rangle \\ &= \frac{N_n}{\sqrt{\pi}}\int_0^{2\pi}\mathrm{d}\theta\,\mathrm{e}^{\mathrm{i}n\theta}\langle x,y|\alpha_{t,\theta}^*,\beta_{t,\theta}^*\rangle \\ &= 2\sqrt{\pi}N_n\exp\left[-\frac{x^2+y^2}{2} - \alpha^2(1-\mathrm{i}\sin 2\tau)\right] \\ &\quad\times\sum_{p=-\infty}^{\infty}\mathrm{I}_{n-2p}\left(2\sqrt{2}\alpha\sqrt{X_-Y_-}\right) \\ &\quad\times\mathrm{I}_p\left(-\alpha^2\cos 2\tau\right)\left(X_-/Y_-\right)^{p+n/2}.\end{aligned} \tag{20}$$

In fact, depending on the initial atomic state preparation and the interaction time selection, we can obtain a few different final field states from Eq. (16). For example, we can obtain the individual state $\psi(x,y,t)$ or $\psi^{\oplus}(x,y,t)$ as well as a few superposition states consisting of the two states with different weighted coefficients. In particular, when the initial atomic state is carefully prepared and the dimensionless interaction time is appropriately adjusted so that $\tau = -\delta_A = \pi/4$ equations (11) and (20) respectively turn out to be a single two-mode state

$$\begin{aligned}\left|\Psi_f\left(\frac{\pi}{4}\right)\right\rangle &= \left|\psi_f^{\oplus}\left(\frac{\pi}{4}\right)\right\rangle \\ &= N_n\int_0^{2\pi}\mathrm{d}\theta\,\mathrm{e}^{\mathrm{i}n\theta}\left|\frac{\alpha}{\sqrt{2}}\left(\mathrm{e}^{-\mathrm{i}\theta} + \mathrm{i}\,\mathrm{e}^{\mathrm{i}\theta}\right),\right. \\ &\quad\left.\frac{\alpha}{\sqrt{2}}\left(\mathrm{e}^{\mathrm{i}\theta} + \mathrm{i}\,\mathrm{e}^{-\mathrm{i}\theta}\right)\right\rangle \\ &= N_n\int_0^{2\pi}\mathrm{d}\theta\,\mathrm{e}^{\mathrm{i}n\theta}\left|\beta\cos\left(\theta + \frac{\mathrm{i}}{4}\pi\right),\right. \\ &\quad\left.\beta\sin\left(\theta + \frac{i}{4}\pi\right)\right\rangle,\end{aligned} \tag{21}$$

$$\begin{aligned}\psi^{\oplus}(x,y,t) &= 2\sqrt{\pi}N_n\exp\left[-\frac{x^2+y^2}{2} - \alpha^2(1-\mathrm{i})\right] \\ &\quad\times\mathrm{I}_n\left(2\alpha\sqrt{(x-\mathrm{i}y)(y-\mathrm{i}x)}\right)\left(\frac{y-\mathrm{i}x}{x-\mathrm{i}y}\right)^{n/2} \\ &= 2\sqrt{\pi}(-\mathrm{i})^{n/2}N_n\exp\left[-\frac{x^2+y^2}{2} - \alpha^2(1-\mathrm{i})\right]\end{aligned}$$

$$\times \mathrm{I}_n\left(\beta\sqrt{2\left(x^2+y^2\right)}\right)\mathrm{e}^{\mathrm{i}n\varphi}, \tag{22}$$

where $\varphi = a\tan(y/x)$ and $\beta = \sqrt{2}\alpha\exp(-\mathrm{i}\pi/4)$. As is well known, equation (22) just represents a standard modified Bessel–Gaussian wavepacket carrying vortex with a topological charge index n, which represents a new kind of non-Gaussian states.[18–21]

In addition, we also investigate the vortex structure embedded in the wavefunction of the cavity field when τ is not exactly equal to $\pi/4$. The results are shown in Fig. 2 for $n=2$ and $\alpha = 1.5$. We can see that with the deviation of τ increasing from $\pi/4$, $|\psi^{\oplus}(x,y,t)|^2$ loses the perfect ring structure and finally evolves into a two-peak structure for $\tau = 0$ or $\pi/2$. This is reasonable because at $\tau = 0$ or $\pi/2$ the cavity field state is a pair coherent state, and $\psi(x,y,t)$ has similar properties.

Finally, it is pointed out that the states $|\psi_f(t)\rangle$ and $|\psi_f^{\oplus}(t)\rangle$ are different because

$$\begin{aligned} D &= \left|\left\langle\psi_f^{\oplus}|\psi_f\right\rangle\right|^2 \\ &= \left|N_n^2\int_0^{2\pi}\mathrm{d}\theta\,\mathrm{d}\theta'\,\mathrm{e}^{\mathrm{i}n(\theta-\theta')}\left\langle\alpha_{t,\theta'}^*,\beta_{t,\theta'}^*|\alpha_{t,\theta},\beta_{t,\theta}\right\rangle\right|^2 \\ &= \left|\mathrm{e}^{-\alpha^2}\mathrm{I}_0\left(2\alpha^2\cos 2\tau\right)\mathrm{I}_n\left(2\mathrm{i}\alpha^2\sin 2\tau\right)/\mathrm{I}_n\left(\alpha^2\right)\right|^2. \end{aligned} \tag{23}$$

Obviously, $\tau = 0$ corresponding to $D = 0$ means that the two states are orthonormal to each other and wholly different.

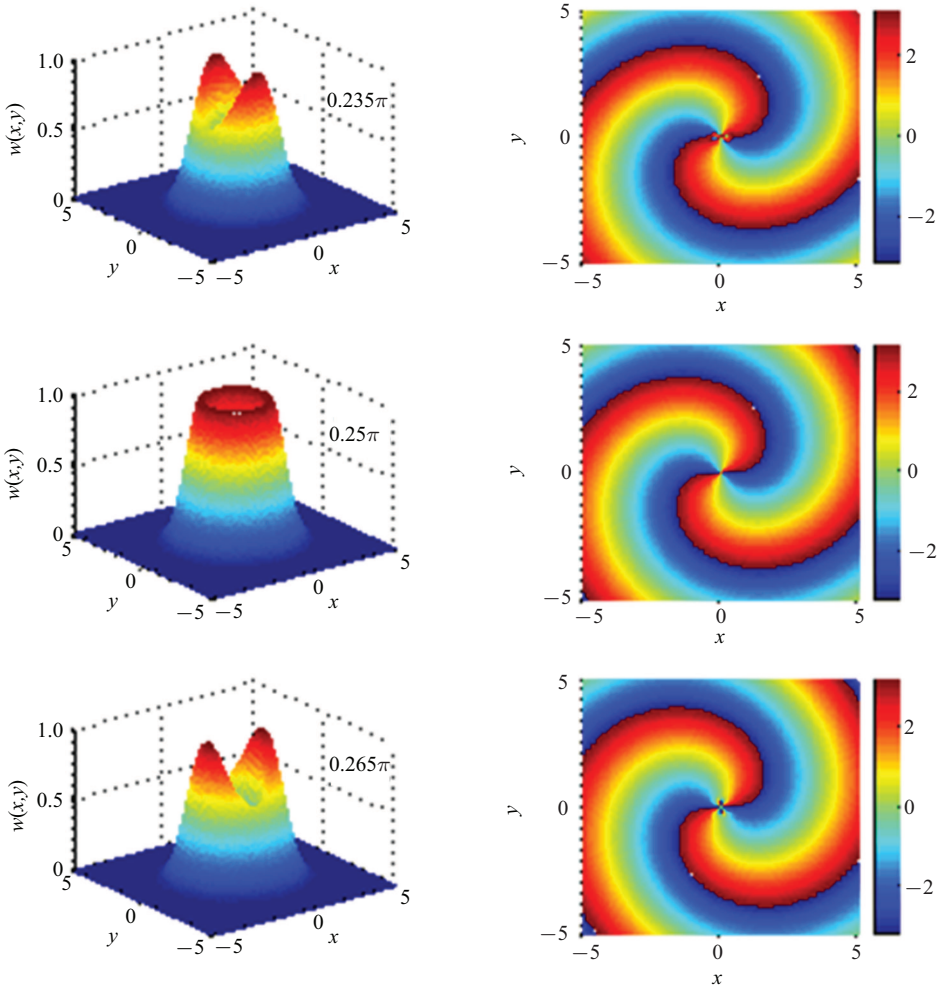


Fig. 2. (color online) Probability $|\psi^{\oplus}(x,y,t)|^2$ (right) and the phase distribution (left) associated with the wavefunction $\psi^{\oplus}(x,y,t)$ for $n=2$, $\alpha=1$ but $\tau = 0.235\pi$ (upper), 0.25π (middle), and 0.265π (bottom).

4. Conclusions

In the present paper, with the help of the nonresonant interaction of a three-level Λ-type atom with two cavity modes at a pair coherent state and two classic fields, the evolution of the system state is derived. Under conditional detection, the cavity field state is analyzed in detail. It is found that the quantized modified Bessel–Gaussian states as well as the superposition states consisting of the quantized vortex states with different weighted coefficients may be prepared through carefully preparing the initial atomic state and appropriately adjusting the interaction time. In fact, the preparation of two-mode quantized modified Bessel–Gaussian vortex states has

been designed in Refs. [18]–[21], in which such states can be obtained with a pair coherent state through a parametric frequency conversion process, so that the scheme here proposed provides an additional choice to realize the two-mode quantized modified Bessel–Gaussian vortex state within the context of cavity QED.

References

[1] Bell J S 1965 *Physics* **1** 195
[2] Ekert A and Jozsa R 1996 *Rev. Mod. Phys.* **68** 733
[3] Knill E, Laflamme R and Milburn G J 2001 *Nature* **46** 409
[4] Braunstein S L and von Loock P 2005 *Rev. Mod. Phys.* **77** 513
[5] Agarwal G S 1986 *Phys. Rev. Lett.* **57** 827
[6] Agarwal G S 1988 *J. Opt. Soc. Am.* B **5** 1940
[7] Agarwal G S, Puri R R and Singh R P 1997 *Phys. Rev.* A **56** 4207
[8] Agarwal G S and Biswas A 2005 *J. Opt.* B: *Quantum Semiclass. Opt.* **7** 350
[9] Agarwal G S and Banerji J 2006 *J Phys.* A: *Math. Gen.* **39** 11503
[10] Tara K and Agarwal G S 1994 *Phys. Rev.* A **50** 2870
[11] Meng X G, Wang J S, Liang B L and Li H Q 2008 *Chin. Phys.* B **17** 1791
[12] Meng X G, Wang J S and Liang B L 2011 *Chin. Phys.* B **20** 014204
[13] Zheng S B 2002 *Chin. J. Phys.* **40** 36
[14] Zheng S B 2001 *J. Opt.* B: *Quantum Semiclass. Opt.* **3** 298
[15] Gerry C C, Mimih J and Birrittella R 2011 *Phys. Rev.* A **84** 023810
[16] Agarwal G S 2011 *New J. Phys.* **13** 073008
[17] Bandyopadhyay A and Singh R P 2011 *Opt. Commun.* **284** 256
[18] Zhu K C, Li S X, Zheng X J and Tang H Q 2012 *J. Opt. Soc. Am.* B **29** 1179
[19] Zhu K C, Li S X, Zheng X J and Zhou Y P 2012 *J. Mod. Opt.* **59** 873
[20] Zhu K C, Li S X, Tang Y, Zheng X J and Tang H Q 2012 *Chin. Phys.* B **21** 084204
[21] Zhu K C, Li S X, Zheng X J and Tang H Q 2012 *Acta Phys. Sin.* **61** 194206 (in Chinese)
[22] Gou S C 1990 *J. Mod. Opt.* **37** 1469
[23] D'Souza R and Puri R R 1994 *J. Opt. Soc. Am.* B **11** 1108
[24] Mu Q X, Ma Y H and Zhou L 2007 *arXiv*: 0711.4186v1 [quant-ph]
[25] Lougovski P, Solano E and Walther H 2005 *Phys. Rev.* A **71** 013811
[26] Biswas A and Agarwal G S 2004 *Phys. Rev.* A **69** 062306
[27] Ban M 1993 *J. Opt. Soc. Am.* B **10** 1347