Quantum correlation dynamics of three non-coupled two-level atoms in different reservoirs*

Wang Xiao-Yun(王小云)†, Ding Bang-Fu(丁邦福)‡, and Zhao He-Ping(赵鹤平)

College of Physics and Mechanical & Electrical Engineering, Jishou University, Jishou 416000, *China*

(Received 4 April 2012; revised manuscript received 6 July 2012)

Time evolution dynamics of three non-coupled two-level atoms independently interacting with their reservoirs is solved exactly by considering a damping Lorentzian spectral density. For three atoms initially prepared in Greenberger–Horne–Zeilinger-type state, quantum correlation dynamics in a Markovian reservoir is compared with that in a non-Markovian reservoir. By increasing detuning quantity in the non-Markovian reservoir, three-atom correlation dynamics measured by negative eigenvalue presents a trapping phenomenon which provides long-time quantum entanglement. Then we compare the correlation dynamics of three atoms with that of two atoms, measured by quantum entanglement and quantum discord for an initial robuster-entangled type state. The result further confirms that quantum discord is indeed different from quantum entanglement in identifying quantum correlation of many bodies.

Keywords: quantum correlation dynamics, Lorentzian spectral density, Markovian and non-Markovian reservoirs

PACS: 03.67.Mn, 03.65.Ud, 03.65.Yz **DOI:** 10.1088/1674-1056/22/2/020309

1. Introduction

Quantum correlation measured by quantum entanglement[1] or quantum discord[2] has been an active area of research under different channels, such as Markovian and non-Markovian channels.[3–10] Entanglement is one of the most essential features in quantum mechanics, which is applied to many fields such as quantum computation,[11] teleportation,[12] cryptography,[13] and universal quantum computation.[14] For pure or mixed quantum states, quantum discord and quantum entanglement are two methods of identifying quantum correlation. Entanglement is zero for some quantum states, but discord is non-zero which can be used to perform some useful quantum tasks.[15] Quantum entanglement measured by concurrence[16] or negative eigenvalue[17] refers to separation of states while discord may be all non-classical correlation and contain it[18] in certain cases. Their relation is complicated in the general case.[19,20] Recently, a unified classification of classical and non-classical correlation was proposed and applied to multipartite systems such as quantum dissonance.[21]

Quantum correlation qualification has been applied to many specific models,[8–10] for example, two quantum systems (atoms, qubits, and pseudospin particles) are placed in a common reservoir or two independent reservoirs. It generates many new phenomena: entanglement sudden death (ESD) and birth (ESB) or quantum discord reducing asymptotically in different conditions.[8] The regime of system–reservoir interaction is often treated as Markovian dynamics[22,23] and a non-Markovian regime is also dealt with.[24] Most previous work concentrates on the evolution of correlation dynamics for a bipartite system, which lacks understanding of how information is transferred in many systems especially multi-system correlation dynamics in different reservoirs. Hence, recent research mainly discusses multi-system correlation dynamics in a resonance condition.[25–28] However, multi-system correlation dynamics itself in a non-resonance condition is complicated and well-accepted measures of correlation are still missing. Due to the interaction between systems and reservoirs called decoherence, an important problem is correlation control in quantum information processing (QIP). As a result, the application of quantum correlation is twofold. Our studies not only provide the information about how correlation dynamics evolves with time but also propose the way for it to be manipulated in practical application.

In this work, we first present nonperturbative dynamics of three non-coupled two-level atoms interacting with three separate reservoirs by considering the damping spectral density distribution. The quantum correlation dynamics of three atoms and that of two atoms are analyzed. The results show that three-atom correlation dynamics displays a trapping phenomenon by changing the detuning of an initial Greenberger–Horne–Zeilinger (GHZ) type state. For an initial robuster-entangled (W) type state, correlation dynamics of three atoms and that of two atoms, measured by negativity, concurrence, and quantum discord, are compared with each other. There-

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11104113 and 11264011) and the Natural Science Foundation of Hunan Province, China (Grant Nos. 09JJ6011 and 11JJ6007).

†Corresponding author. E-mail: wxyyun@163.com

‡Corresponding author. E-mail: dbf1982@126.com

© 2013 Chinese Physical Society and IOP Publishing Ltd

fore, long-time quantum correlation of three atoms is obtained, which may be used in quantum information theory and quantum communication. We may further conclude that the relation of quantum entanglement and quantum discord is complicated in qualifying many-body quantum correlation.

2. Model and its solutions

We consider three non-coupled two-level atoms interacting with three separate structure reservoirs modeled as bosons at zero temperature, which is shown in Fig. 1. Under dipole and rotating wave approximate (RWA), the Hamiltonian of the total system is written as[29]

$$\begin{aligned} H &= \sum_{j=A,B,C} H^j \\ &= \sum_{j=A,B,C} \Big[\frac{\omega_0}{2}\sigma_z^j + \sum_k \omega_k^j b_k^{j\dagger} b_k^j + \sigma_+^j \otimes \sum_k g_k^j b_k^j \\ &\quad + \sigma_-^i \otimes \sum_k g_k^{j*} b_k^{j\dagger} \Big]. \end{aligned} \tag{1}$$

For simplicity, the transition frequency of three atoms is assumed to be the same and denoted by ω_0. The σ_z^j is the Pauli operator of the j-th atom. Three reservoirs are characterized by proper frequencies ω_k^j, $b_k^{j\dagger}$ (b_k^j) is the creation (destruction) operator of the k-th mode of the j-th reservoir, g_k^j (g_k^{j*}) is the coupling constant, and σ_+^j (σ_-^j) is the raising (lowering) operator of the j-th atom.

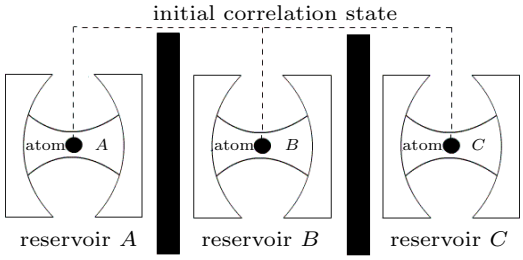


Fig. 1. Schematic diagram of three atoms in three separate reservoirs under the present study. There is neither interaction between the three atoms nor communication between the three reservoirs.

In order to obtain exact solutions of three-atom dynamics, we first solve dynamics of an atom coupled to a reservoir. The state vector of the composite system is assumed to be

$$\begin{aligned} |\psi^j(t)\rangle &= a_0^j|0^j\rangle\otimes|\mathbf{0}^j\rangle + a_1^j(t)\,\mathrm{e}^{-\mathrm{i}\omega_0 t}|1^j\rangle\otimes|\mathbf{0}^j\rangle \\ &\quad + \sum_k a_k^j(t)\,\mathrm{e}^{-\mathrm{i}\omega_k^j t}|0^j\rangle\otimes|\mathbf{1}_k^j\rangle, \end{aligned} \tag{2}$$

where exponential terms consider phase in the Schrödinger picture; a_0^j, $a_1^j(t)$, $a_k^j(t)$ are probability amplitudes corresponding to product states; $|1^j\rangle$ ($|0^j\rangle$) is the excited (ground) state of the j-th atom; $|\mathbf{0}^j\rangle$ denotes vacuum state; $|\mathbf{1}_k^j\rangle$ is the state with one photon in the k-th mode of the j-th reservoir; amplitude value a_0^j is constant since $H^j|0^j\rangle\otimes|\mathbf{0}^j\rangle = 0$. Substituting Eq. (2) into the Schrödinger equation, we can obtain a set of differential equations of $a_1^j(t)$ and $a_k^j(t)$. Assuming that $a_k^j(0) = 0$, integrating the equation about $a_k^j(t)$ and inserting this result into the equation about $a_1^j(t)$, an integral-differential equation about $a_1^j(t)$ is obtained. Here a correlation function[29] is defined as $f^j(t-t') = \int \mathrm{d}\omega^j J^j(\omega^j)\,\mathrm{e}^{\mathrm{i}(\omega_0-\omega^j)(t-t')}$ with spectral density J^j (ω^j) of the j-th reservoir. We further introduce time evolving factor $G^j(t)$ such that $a_1^j(t) = G^j(t)a_1^j(0)$ with $G^j(0) = 1$, which satisfies

$$\frac{\mathrm{d}G^j(t)}{\mathrm{d}t} = -\int_0^t \mathrm{d}t' f^j(t-t')G^j(t').$$

Therefore, the reduced dynamics of a single two-level atom can be represented as

$$\rho^j(t) = \begin{bmatrix} \rho_{11}^j(0)u^j & \rho_{10}^j(0)v^j \\ \rho_{01}^j(0)v^{j*} & \rho_{00}^j(0)+\rho_{11}^j(0)(1-u^j) \end{bmatrix}. \tag{3}$$

The coefficients $u^j = |G^j(t)|^2$ and $v^j = G^j(t)$ are called time evolution function, which is determined by concrete conditions, such as initial conditions or the form of reservoir.

For three atoms denoted by A,B,C are placed in three separate reservoirs (Fig. 1). We exploit the Kraus operator-sum representation[30,31] and choose three-atom standard computational basis

$$\left\{ \begin{array}{l} |1\rangle = |0^A0^B0^C\rangle, |2\rangle = |0^A0^B1^C\rangle, |3\rangle = |0^A1^B0^C\rangle, |4\rangle = |1^A0^B0^C\rangle \\ |5\rangle = |1^A1^B0^C\rangle, |6\rangle = |1^A0^B1^C\rangle, |7\rangle = |0^A1^B1^C\rangle, |8\rangle = |1^A1^B1^C\rangle \end{array} \right\}.$$

According to Refs. [30] and [31], we can obtain the diagonal elements of the three-atom reduced density matrix as follows:

$$\begin{aligned} \rho_{11}(t) &= \rho_{11}(0)+\rho_{22}(0)(1-u^C)+\rho_{33}(0)(1-u^B)+\rho_{44}(0)(1-u^A)(0)(1-u^B)(1-u^C)+\rho_{66}(0)(1-u^A)(1-u^C) \\ &\quad +\rho_{77}(0)(1-u^A)(1-u^B)+\rho_{88}(0)(1-u^A)(1-u^B)(1-u^C), \\ \rho_{22}(t) &= \rho_{22}(0)u^C+\rho_{55}(0)(1-u^B)u^C+\rho_{66}(0)(1-u^A)u^C+\rho_{88}(0)(1-u^A)(1-u^B)u^C, \\ \rho_{33}(t) &= \rho_{33}(0)u^B+\rho_{55}(0)u^B(1-u^C)+\rho_{77}(0)(1-u^A)u^B+\rho_{88}(0)(1-u^A)u^B(1-u^C), \\ \rho_{44}(t) &= \rho_{44}(0)u^A+\rho_{66}(0)u^A(1-u^C)+\rho_{77}(0)u^A(1-u^B)+\rho_{88}(0)u^A(1-u^B)(1-u^C), \\ \rho_{55}(t) &= \rho_{55}(0)u^Bu^C+\rho_{88}(0)(1-u^A)u^Bu^C, \quad \rho_{66}(t) = \rho_{66}(0)u^Au^C+\rho_{88}(0)u^A(1-u^B)u^C, \\ \rho_{77}(t) &= \rho_{77}(0)u^Au^B+\rho_{88}(0)u^Au^B(1-u^C), \quad \rho_{88}(t) = \rho_{88}(0)u^Au^Bu^C. \end{aligned} \tag{4}$$

The off-diagonal elements are given below

$$\begin{aligned}
\rho_{12}(t) &= \rho_{12}(0)v^{C*} + \rho_{35}(0)(1-u^B)v^{C*} \\
&\quad + \rho_{46}(0)(1-u^A)v^{C*} + \rho_{78}(0)(1-u^A)(1-u^B)v^{C*}, \\
\rho_{13}(t) &= \rho_{13}(0)v^{B*} + \rho_{25}(0)v^{B*}(1-u^C) \\
&\quad + \rho_{68}(0)(1-u^A)v^{B*}(1-u^C), \\
\rho_{14}(t) &= \rho_{14}(0)v^{A*} + \rho_{26}(0)v^{A*}(1-u^C) \\
&\quad + \rho_{37}(0)v^{A*}(1-u^B) + \rho_{58}(0)v^{A*}(1-u^B)(1-u^C), \\
\rho_{15}(t) &= \rho_{15}(0)v^{B*}v^{C*} + \rho_{48}(0)(1-u^A)v^{B*}v^{C*}, \\
\rho_{16}(t) &= \rho_{16}(0)v^{A*}v^{C*} + \rho_{38}(0)v^{A*}(1-u^B)v^{C*}, \\
\rho_{17}(t) &= \rho_{17}(0)v^{A*}v^{B*} + \rho_{28}(0)v^{A*}v^{B*}(1-u^C), \\
\rho_{18}(t) &= \rho_{18}(0)v^{A*}v^{B*}v^{C*}, \\
\rho_{23}(t) &= \rho_{23}(0)v^{B*}v^{C} + \rho_{67}(0)(1-u^A)v^{B*}v^{C}, \\
\rho_{24}(t) &= \rho_{24}(0)v^{A*}v^{C} + \rho_{57}(0)v^{A*}(1-u^B)v^{C}, \\
\rho_{25}(t) &= \rho_{25}(0)v^{B*}u^{C} + \rho_{68}(0)(1-u^A)v^{B*}u^{C}, \\
\rho_{26}(t) &= \rho_{26}(0)v^{A*}u^{C} + \rho_{58}(0)v^{A*}(1-u^B)u^{C}, \\
\rho_{27}(t) &= \rho_{27}(0)v^{A*}v^{B*}v^{C}, \quad \rho_{28}(t) = \rho_{28}(0)v^{A*}v^{B*}u^{C}, \\
\rho_{34}(t) &= \rho_{34}(0)v^{A*}v^{B} + \rho_{56}(0)v^{A*}v^{B}(1-u^C), \\
\rho_{35}(t) &= \rho_{35}(0)u^{B}v^{C*} + \rho_{78}(0)(1-u^A)u^{B}v^{C*}, \\
\rho_{36}(t) &= \rho_{36}(0)v^{A*}v^{B}v^{C*}, \\
\rho_{37}(t) &= \rho_{37}(0)v^{A*}u^{B} + \rho_{58}(0)v^{A*}u^{B}(1-u^C), \\
\rho_{38}(t) &= \rho_{38}(0)v^{A*}u^{B}v^{C*}, \\
\rho_{45}(t) &= \rho_{38}(0)v^{A}v^{B*}v^{C*}, \\
\rho_{46}(t) &= \rho_{46}(0)u^{A}v^{C*} + \rho_{78}(0)u^{A}(1-u^B)v^{C*}, \\
\rho_{47}(t) &= \rho_{47}(0)u^{A}v^{B*} + \rho_{68}(0)u^{A}v^{B*}(1-u^C), \\
\rho_{48}(t) &= \rho_{48}(0)u^{A}v^{B*}v^{C*}, \quad \rho_{56}(t) = \rho_{56}(0)v^{A*}v^{B}u^{C}, \\
\rho_{57}(t) &= \rho_{57}(0)v^{A*}u^{B}v^{C}, \quad \rho_{58}(t) = \rho_{58}(0)v^{A*}u^{B}u^{C}, \\
\rho_{67}(t) &= \rho_{67}(0)u^{A}v^{B*}v^{C}, \quad \rho_{68}(t) = \rho_{68}(0)u^{A}v^{B*}u^{C}, \\
\rho_{78}(t) &= \rho_{78}(0)u^{A}u^{B}v^{C*}. \qquad (5)
\end{aligned}$$

$\rho_{lm}(0)$ $(l,m = 1,2,\ldots,8)$ is the initial condition of the three atoms and $\rho_{ml}(t) = \rho^*_{lm}(t)$.

3. Quantum entanglement and quantum discord

For a pair of atoms denoted by A and B, their quantum correlation is measured by quantum entanglement such as quantum concurrence and negative eigenvalue, or quantum discord. For a kind of X structure density matrix of two atoms, these quantum correlation identifications have obvious expressions. We choose two-atom standard basis as $\{|1\rangle = |0^A0^B\rangle, |2\rangle = |0^A1^B\rangle, |3\rangle = |1^A0^B\rangle, |4\rangle = |1^A1^B\rangle\}$. The X type density matrix is

$$\rho(t) = \begin{pmatrix} a & 0 & 0 & w \\ 0 & b & z & 0 \\ 0 & z^* & c & 0 \\ w^* & 0 & 0 & d \end{pmatrix}, \qquad (6)$$

where $a = \rho_{11}(t)$, $b = \rho_{22}(t)$, $c = \rho_{33}(t)$, $d = \rho_{44}(t)$, and $a+b+c+d = 1$ which means that the sum of probability is 1. The quantum entanglement measured by quantum concurrence has a simple analytic form, i.e., concurrence $(t) = 2\max\{0, |z| - \sqrt{ad}, |w| - \sqrt{bc}\}$.[16]

For quantum discord,[2] it is difficult to obtain its analytical expression. However, for Eq. (6), quantum discord also has a simple form[18] and the values of quantum discord between measuring atoms A and B are taken to be the same. The negativity degree of two atoms is defined as the negative value of partial transposition of the density matrix with respect to another atom. For example, partially transposing atom B, the density matrix of two atoms like Eq. (6) becomes

$$\rho(t) = \begin{pmatrix} a & 0 & 0 & z \\ 0 & b & w & 0 \\ 0 & w^* & c & 0 \\ z^* & 0 & 0 & d \end{pmatrix}. \qquad (7)$$

Solving all the eigenvalues of Eq. (7), we only take the negative eigenvalues. For three atoms, the method is the same and the detailed introduction can be found in Refs. [23] and [24].

4. Discussion and results

We take the detuning Lorentzian spectral density which is the central frequency of the reservoir against atomic transition frequency ω_0 detuned by a quantity Δ^j. In this case the spectral density is[29]

$$J^j(\omega) = \frac{\gamma_0(\lambda^j)^2}{2\pi} \frac{1}{(\omega_0 - \Delta^j - \omega^j)^2 + (\lambda^j)^2}.$$

The parameter λ^j represents the spectral coupling width of the j-th reservoir, which is reversely proportional to correlation time τ_B^j of the j-th reservoir. The changing time scale τ_R of the three-atom state is given by $\tau_R = \gamma_0^{-1}$. The correlation function becomes $f^j(t-t') = \frac{1}{2}\gamma_0\lambda^j \exp\{-(\lambda^j - \mathrm{i}\Delta^j)(t-t')\}$. Making use of the result in Section 2, the time evolution factor becomes

$$\begin{aligned}
G^j(t) &= \exp\left\{-\frac{(\lambda^j - i\Delta^j)}{2}t\right\} \\
&\quad \times \left[\mathrm{ch}\left(\frac{\Omega^j t}{2}\right) + \frac{\lambda^j - \mathrm{i}\Delta^j}{\Omega^j}\,\mathrm{sh}\left(\frac{\Omega^j t}{2}\right)\right], \qquad (8)
\end{aligned}$$

where $\Omega^j = \sqrt{(\lambda^j - \mathrm{i}\Delta^j)^2 - 2\gamma_0\lambda^j}$. Simply assuming three reservoirs to be the same, that is, $\lambda^j = \lambda$ and $\Delta^j = \Delta$, the time evolution functions of three atoms are

$$\begin{aligned}
u^j &= u = |G^j(t)|^2 = |G(t)|^2, \\
v^j &= v = G^j(t) = G(t). \qquad (9)
\end{aligned}$$

4.1. Initial GHZ-type state

We study quantum correlation dynamics of three atoms in the initial GHZ-type state[26,32]

$$\rho(0) = \frac{1-p}{8} I_{8\times 8} + p|\Psi_{\mathrm{GHZ}}\rangle\langle\Psi_{\mathrm{GHZ}}|, \qquad (10)$$

where

$$|\Psi_{\rm GHZ}\rangle = (|1^A1^B1^C\rangle + |0^A0^B0^C\rangle)/\sqrt{2}$$

is GHZ state, p is the purity of three atoms with $p \in [0,1]$, and $I_{8\times8}$ is the eight-dimensional (8D) identity operator. Three-atom dynamics changing with time is obtained by Eqs. (4) and (5). We take the negative eigenvalue to measure three-atom quantum correlation dynamics. From Section 3, we have a non-zero tripartite negative eigenvalue which is equal to

$$N_{ABC} = \max\left\{0, -\frac{\rho_{44}(t)+\rho_{55}(t)-\sqrt{[\rho_{44}(t)-\rho_{55}(t)]^2+4|\rho_{18}(t)|^2}}{2}\right\}. \tag{11}$$

The coefficients in Eq. (11) can be easily calculated as

$$\begin{aligned}
\rho_{44}(t) &= \frac{1-p}{8}u + \frac{1-p}{4}u(1-u) + \frac{1+3p}{8}u(1-u)^2,\\
\rho_{55}(t) &= \frac{1-p}{8}u^2 + \frac{1+3p}{8}u^2(1-u),\\
\rho_{18}(t) &= \frac{p}{2}v^{*3}.
\end{aligned} \tag{12}$$

For a weak coupling regime, the relaxation time is larger than the correlation time corresponding to $2\tau_R > \tau_B^j$ with $\tau_B^j = \tau_B$. The behavior of time evolution factor $G(t)$ is of a Markovian exponential decay basically, which is determined by parameter γ_0. In the strong coupling regime, the relaxation time is smaller than the correlation time corresponding to $2\tau_R < \tau_B$ and a non-Markovian memory effect becomes very important.[8]

According to these conclusions, three-atom quantum correlation dynamics identified by the negative eigenvalue N_{ABC} is displayed in Fig. 2. The strong and weak coupling regimes correspond to non-Markovian $\lambda = 0.1\gamma_0$ and Markovian reservoirs $\lambda = 10\gamma_0$ under initial GHZ state with $p = 0.98$, respectively. The detuning quantity Δ is 8λ. From Fig. 2, quantum correlation values of three atoms gradually decrease with time going by in non-Markovian and Markovian reservoirs. They tend to zero finally, because of decoherence which leads to quantum correlation information dissipated into the external environment. In the non-Markovian reservoir, the negative eigenvalue N_{ABC} shows oscillatory behavior due to three atoms strongly interacting and rapidly exchanging information with their respective reservoirs. In Markovian reservoirs, the negative eigenvalue N_{ABC} always decreases with time parameter $\gamma_0 t$ increasing. Thus each of the reservoirs has a dissipative effect. Therefore, we need a method to control the decoherence problem and acquire the long-time quantum correlation of three atoms.

When the detuning quantity Δ changes, three-atom quantum correlation dynamics varies with time (Fig. 3) for the initial GHZ state. As shown in Fig. 3, three-atom correlation dynamics still shows oscillatory behavior in a non-Markovian reservoir, and it decays to zero in a very long time. When the detuning quantity Δ is small which means near-resonant interaction between three atoms and their reservoirs (solid line in Fig. 3), the negative eigenvalue N_{ABC} shows phenomena of ESD and ESB. The larger the quantity Δ becomes, the more slowly the negativity degree of the three atoms deceases. For example, the negativity degree is almost constant in $\Delta = 20\lambda$, thus it is usually called correlation trapping phenomenon.[33] We can adjust the detuning quantity to control the quantum correlation time of three atoms, which may be useful in QIP.

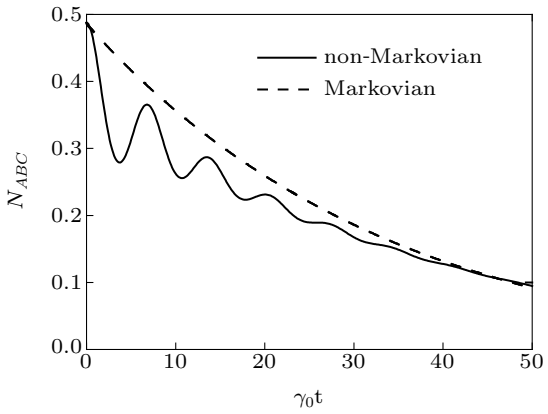


Fig. 2. Evolutions of correlation dynamics of three atoms with time under non-Markovian reservoir $\lambda = 0.1\gamma_0$ and Markovian reservoirs $\lambda = 10\gamma_0$ for initial type state with $p = 0.98$. The detuning quantity Δ is 8λ.

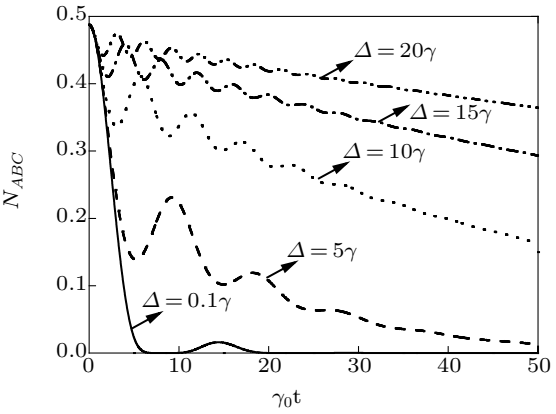


Fig. 3. Negativity dynamics of three atoms changes with time under the non-Markovian reservoirs with $\lambda = 0.1\gamma_0$ in different detuning quantity cases. The initial state of three atoms is the GHZ-type state with $p = 0.98$.

4.2. Initial W-type states

The initial W-type state[28] can be expressed as

$$\rho(0) = \frac{1-p}{8}I_{8\times8} + p|\Psi_{\rm W}\rangle\langle\Psi_{\rm W}|, \tag{13}$$

where W-type state is $(|1^A0^B0^C\rangle + |0^A1^B0^C\rangle + |0^A0^B1^C\rangle)/\sqrt{3}$. The non-zero elements of reduced density matrix are

$$\rho_{11}(t) = \frac{1-p}{8} + \frac{3+5p}{8}(1-u) + \frac{3(1-p)}{8}(1-u)^2,$$

$$\begin{aligned}
\rho_{22}(t) &= \rho_{33}(t)=\rho_{44}(t)\\
&= \frac{3+5p}{24}u+\frac{1-p}{4}(1-u)u+\frac{1-p}{8}(1-u)^2u,\\
\rho_{55}(t) &= \rho_{66}(t)=\rho_{77}(t)=\frac{1-p}{8}u^2+\frac{1-p}{8}(1-u)u^2,\\
\rho_{88}(t) &= \frac{1-p}{8}u^3,\\
\rho_{23}(t) &= \rho_{24}(t)=\rho_{34}(t)=\rho_{32}(t)=\rho_{42}(t)\\
&= \rho_{43}(t)=\frac{p}{3}|v|^2. \qquad (14)
\end{aligned}$$

Unlike GHZ-type state, W-type state has a high degree of bipartite correlation. Hence we compare quantum correlation dynamics of tripartite ABC and that of bipartite AB for initial W-type state. The analytical expression of tripartite quantum correlation N_{ABC} is

$$N_{ABC}=\max\left\{0,-[\rho_{22}(t)-\rho_{23}(t)],\ -\frac{\rho_{11}(t)+\rho_{66}(t)-\sqrt{[\rho_{11}(t)-\rho_{66}(t)]^2+8\rho_{24}(t)}}{2}\right\}. \qquad (15)$$

For the bipartite AB quantum correlation, the reduced density matrix of two atoms AB tracing atom C is X structure density matrix like Eq. (6). All coefficients are given by

$$\begin{aligned}
a &= \rho_{11}(t)+\rho_{22}(t),\ b=\rho_{33}(t)+\rho_{55}(t),\\
c &= \rho_{44}(t)+\rho_{66}(t),\ d=\rho_{77}(t)+\rho_{88}(t),\\
w &= 0,\ z=\rho_{43}(t). \qquad (16)
\end{aligned}$$

The bipartite quantum correlation that is measured by the negativity degree N_{AB}, the quantum concurrence and the quantum discord can be given as

$$\begin{aligned}
N_{AB} &= \max\left\{0,-[a+d-\sqrt{(a-d)^2+4|z|^2}]/2\right\},\\
C_{AB} &= 2\max[0,|z|-\sqrt{ad}],\ D_{AB}=\min[D_{1AB},D_{2AB}], \qquad (17)
\end{aligned}$$

where $D_{1AB}(\rho)=S(\rho^B)-S(\rho^{AB})-M$ and $D_{2AB}(\rho)=S(\rho^B)-S(\rho^{AB})-N$. The von Neumann entropies $S(\rho^B)$ and $S(\rho^{AB})$ are

$$\begin{aligned}
S(\rho^B) &= -(a+c)\log_2(a+c)-(b+d)\log_2(b+d),\\
S(\rho^{AB}) &= \sum_{l=1}^{4}\varepsilon_l\log_2(\varepsilon_l), \qquad (18)
\end{aligned}$$

where ε_l is the eigenvalues of reduced density matrix $\rho_{AB}(t)$. The two quantities M and N are defined in Ref. [18]. Due to $b=c$, the expression of quantum discord measuring atom A is equal to the one measuring atom B.

The quantum correlation dynamics of three atoms and that of two atoms are displayed in strong non-Markovian reservoirs (Fig. 4). All the quantum correlation identifications can show oscillatory behavior due to the non-Markovianity, which means that information returns to three atoms from three reservoirs. At initial $t=0$, three-atom quantum correlation N_{ABC} is almost equal to correlation value of two atoms A and B. Thus the initial W-type state is a high entanglement state. With time going by, two-atom quantum discord D_{AB} is basically equal to quantum concurrence C_{AB} expected for the range of initial time. However, the two-atom negativity degree N_{AB} is completely different from quantum discord D_{AB} and quantum concurrence C_{AB}. Therefore, two-atom quantum correlation measured by negative eigenvalue, quantum concurrence or quantum discord are different and their relations are complicated.

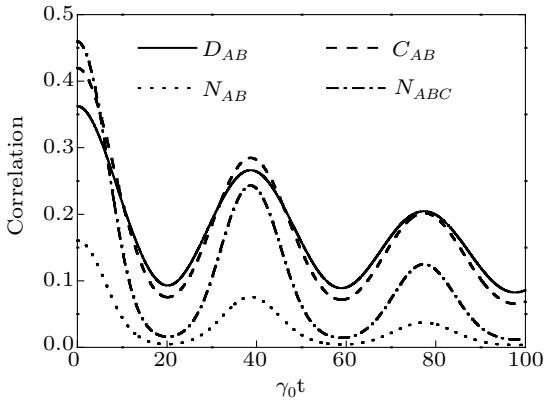


Fig. 4. Evolutions of dynamics of quantum discord D_{AB}, concurrence C_{AB}, and negativity N_{AB} for two atoms AB and the dynamics of negativity N_{ABC} for three atoms ABC evolve with time in the non-Markovian reservoir $\lambda=0.1\gamma_0$ and $\Delta=8\lambda$. The initial state is W-type state with $p=0.98$.

Finally, we compare three-atom quantum correlation evolving with purity p (Fig. 5) in non-Markovian reservoir $\lambda=0.01\gamma_0$ with that in Markovian reservoir $\lambda=10\gamma_0$ with $\Delta=0.01\lambda$. Three-atom quantum correlation is completely dead in $p<0.3$ in any reservoir. The phenomena of ESD and ESB take place for $0.3<p<1.0$ in the strong interaction regime.

In strong dissipative reservoirs, three-atom quantum correlation dynamics decays to zero rapidly and has no phenomena of ESD and ESB. It testifies that the non-Markovian reservoir has a memory effect and the Markovian reservoir has no memory function. According to Fig. 5, we also find that the W state has indeed a high entanglement feature for large purity p.

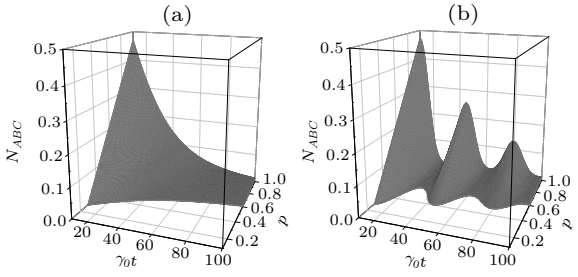


Fig. 5. Evolutions of three-atom correlation dynamics degree N_{ABC} with purity p and time in Markovian reservoir $\lambda=10\gamma_0$ (a) and non-Markovian reservoir $\lambda=0.01\gamma_0$ (b) with $\Delta=0.1\lambda$. The initial state is W-type state.

5. Conclusions

The dynamics of quantum correlations measured by the quantum discord, the dynamics of quantum concurrence, and the negative eigenvalue for three or two atoms are analyzed. Three-atom correlation dynamic behaviors are studied under two initial states: initial GHZ and W-type states. For the first initial state, quantum correlation dynamics measured by negative eigenvalues in a non-Markovian reservoir displays oscillatory behavior, but it always decreases in Markovian reservoirs. It is also found that three-atom negativity degree shows correlation trapping phenomenon by increasing detuning quantity. For the second initial case, we give analytical forms of three- and two-atom quantum correlation. Then quantum correlation dynamics of three atoms and that of two atoms are compared in non-Markovian reservoirs. Finally, quantum correlation dynamics of three atoms changing with purity p is also studied. Three-atom quantum correlation dynamics is more complicated than two-atom correlation dynamics. The relation between quantum discord, and quantum concurrence may not be a simple relationship.[8–10,34] The results further confirm that the quantum correlation identification, such as negative eigenvalue, quantum discord, and quantum concurrence is applicable to theoretical and experiment models.

References

[1] Zhou F X, Zhang L D, Qi Y H, Nim Y P, Zhang J T and Gong S Q 2011 *Chin. Phys.* B **20** 080302
[2] Ollivier H and Zurek W H 2002 *Phys. Rev. Lett.* **88** 017901
[3] Yu T and Eberly J H 2004 *Phys. Rev. Lett.* **93** 140404
[4] Ji Y H and Hu J J 2010 *Chin. Phys.* B **19** 060304
[5] Luo X Q, Wang D L, Zhang Z Q, Ding J W and Liu W M 2011 *Phys. Rev.* A **84** 033803
[6] Tong Q J, An J H, Luo H G and Oh C H 2010 *J. Phys.* B: *At. Mol. Opt. Phys.* **43** 155501
[7] Zheng Y P, Tang N, Wang G Y and Zeng H S 2011 *Chin. Phys.* B **20** 110301
[8] Huang L Y and Fang M F 2010 *Chin. Phys.* B **19** 090318
[9] Li J Q and Liang J Q 2010 *Phys. Lett.* A **374** 1975
[10] Xu G F and Tong D M 2011 *Chin. Phys. Lett.* **28** 060305
[11] Nielsen M and Chuang I 2000 *Quantum Information and Computation* (Cambridge: Cambridge University Press)
[12] Yan W and Zhang W J 2007 *Chin. Phys.* **16** 2584
[13] Gao X, Sun S H and Liang L M 2009 *Chin. Phys. Lett.* **26** 100307
[14] DiVincenzo D P 2000 *Fort. Phys.* **48** 9
[15] Hao X, Pan T, Sha J Q and Zhu S Q 2011 *Commun. Theor. Phys.* **55** 41
[16] Yu T and Eberly J H 2007 *Quantum Inf. Comput.* **7** 459
[17] Su H Y, Deng D L and Chen J L 2010 *Commun. Theor. Phys.* **54** 663
[18] Ding B F, Wang X Y and Zhao H P 2011 *Chin. Phys.* B **20** 100302
[19] Luo S L 2008 *Phys. Rev.* A **77** 042303
[20] Li J G, Zou J, Xu B M and Shao B 2011 *Chin. Phys. Lett.* **28** 090301
[21] Modi K, Paterek T, Son W, Vedral V and Williamson M 2010 *Phys. Rev. Lett.* **104** 080501
[22] An J H, Wang S J and Luo H G 2007 *Physica* A: *Statistical Mechanics and Its Applications* **382** 753
[23] Benatti F and Nagy A 2011 *Ann. Phys.* **326** 740
[24] Li Y L and Fang M F 2011 *Chin. Phys.* B **20** 100312
[25] Man Z X, Xia Y J and An N B 2010 *New. J. Phys.* **12** 033020
[26] Shan C J, Liu J B, Chen T, Liu T K, Huang Y X and Li H 2010 *Chin. Phys. Lett.* **27** 100301
[27] An N B 2010 *Phys. Rev.* A **84** 022329
[28] Altintas F and Eryigit R 2010 *Phys. Lett.* A **374** 4283
[29] Breuer H P and Petruccione F 2002 *The Theory of Open Quantum Systems* (Oxford: Oxford University Press)
[30] Bellomo B, Franco R Lo and Compagno G 2007 *Phys. Rev. Lett.* **99** 160502
[31] Bellomo B, Franco R Lo and Compagno G 2008 *Phys. Rev.* A **77** 032342
[32] Sabin C and Garcia-Alcaine G 2008 *Eur. Phys. J.* D **48** 435
[33] McMahon D 2007 *Quantum Computing Explained* (New York: Wiley and Sons)
[34] Peng J S and Li G X 1996 *Introduction to Modern Quantum Optics* (Beijing: Science Press) (in Chinese)