Optimal entanglement concentration for three-photon W states with parity check measurement*

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(Received 9 August 2012; revised manuscript received 24 September 2012)

We present an efficient entanglement concentration protocol (ECP) for the less-entangled W state with some identical conventional polarized single photons. In the protocol, two of the parties say Alice and Charlie should perform the parity check measurements and they can ultimately obtain the maximally entangled W state with a certain success probability. Otherwise, they can obtain another less-entangled W state, which can be reconcentrated into the maximally entangled W state. By iterating this ECP, a high success probability can be achieved. This ECP may be an optimal one and it is useful in current quantum information processing.

Keywords: quantum communication, entanglement, entanglement concentration

PACS: 03.67.Dd, 03.67.Hk, 03.65.Ud **DOI:** 10.1088/1674-1056/22/2/020307

1. Introduction

Entanglement is an essential role in quantum information processing.[1] Quantum teleportation,[2] quantum dense coding,[3] quantum key distribution protocols (QKD),[4] quantum secret sharing,[5–7] quantum secure direct communication,[8–18] and other quantum information processing[19–25] all need to set up the entanglement channel. However, in applications, a quantum channel which is used to distribute the photon pairs, is usually noisy. The noisy channel will degrade the entanglement, and make the maximally entangled state become a mixed state or a pure less-entangled state. Thus, in practical applications, it is of vital importance to ensure that the communication parties, say Alice and Bob, can share the maximally entangled states in such noisy channels. Entanglement purification and concentration are two powerful tools which are proposed for that purpose.

Entanglement purification can extract some high-fidelity entangled systems from a less-entangled ensemble in a mixed state by local operations and classical communications (LOCC).[26] In 1996, Bennett *et al.* proposed the first entanglement purification protocol, which was based on the controlled-not (CNOT) gate.[26–28] Entanglement concentration, which will be explained here, is used to distill some maximally entangled states from an ensemble in a less-entangled pure state.[29–38] In 1996, Bennett *et al.* proposed an original entanglement concentration protocol (ECP), which is called the Schmidt projection method.[29] In 1999, Bose *et al.* proposed an ECP based on the quantum swapping and it was developed by Shi *et al.* later.[30,31] In 2001, Yamamoto *et al.* and Zhao *et al.* proposed two similar ECPs independently, which are based on the linear optics.[32,33] Recently, ECPs based on the cross-Kerr nonlinearity were proposed.[34,38] In 2011, Wang *et al.* proposed an efficient ECP based on the quantum dot in an optical cavity.[37] The ECPs described above in optical systems are all focused on the two-particle Bell-state. All of them are used to convert a less-entangled state $\alpha|H\rangle|H\rangle+\beta|V\rangle|V\rangle$ into a maximally entangled state $(|H\rangle|H\rangle+|V\rangle|V\rangle)/\sqrt{2}$. Here $|H\rangle$ and $|V\rangle$ represent the horizontal and the vertical polarizations of photons, respectively. Usually, this kind of ECPs can be extended to concentrate the multipartite less-entangled GHZ state $\alpha|H\rangle|H\rangle|H\rangle+\beta|V\rangle|V\rangle|V\rangle$ into the maximally entangled GHZ state $(|H\rangle|H\rangle|H\rangle+|V\rangle|V\rangle|V\rangle)/\sqrt{2}$. Unfortunately, they cannot concentrate the less-entangled W state $\alpha|V\rangle|H\rangle|H\rangle+\beta|H\rangle|V\rangle|H\rangle+\gamma|H\rangle|H\rangle|V\rangle$ into a maximally entangled W state $(|V\rangle|H\rangle|H\rangle+|H\rangle|V\rangle|H\rangle+|H\rangle|H\rangle|V\rangle)/\sqrt{3}$, for the W state cannot be converted into a GHZ state by LOCC. In 2003, Cao and Yang proposed an ECP for W class state with the help of the joint unitary transformation.[39] In 2007, Zhang *et al.* proposed an ECP for the W state based on the Bell-state measurement.[40] However, the joint unitary transformation and the Bell-state measurement are difficult to realize under present experimental conditions. In 2010, an ECP for a

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11104159 and 61271238), the Scientific Research Foundation of Nanjing University of Posts and Telecommunications (Grant No. NY211008), the Open Research Fund Program of the State Key Laboratory of Low-Dimensional Quantum Physics, Tsinghua University, the Open Research Fund Program of National Laboratory of Solid State Microstructures, Nanjing University, (Grant Nos. M25020 and M25022), the University Natural Science Research Foundation of Jiangsu Province (Grant No. 11KJA510002), the Open Research Fund of the Key Laboratory of Broadband Wireless Communication and Sensor Network Technology, Nanjing University of Posts and Telecommunications, Ministry of Education, China, and the Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions.

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special W state $\alpha|H\rangle|H\rangle|V\rangle+\beta(|H\rangle|V\rangle|H\rangle+|V\rangle|H\rangle|H\rangle)$ was proposed.[41] Yildiz proposed an optimal distillation of three-qubit asymmetric W states of the form $|001\rangle/\sqrt{2}+|010\rangle/2+|100\rangle/2$.[42] In 2012, an ECP for W state based on the cross-Kerr nonlinearity was proposed.[43] In this protocol, in order to reach higher success probability, this protocol needs the giant π phase shift of the coherent state, which is hard to realize in current technology. Moreover, it requires two single-photon sources to prepare different special polarized single photons.

In this paper, we describe an optimal ECP which is focused on an arbitrary three-photon less-entangled W state, inspired by the previous work of Ref. [38]. We can concentrate an arbitrary less-entangled W state $\alpha|V\rangle|H\rangle|H\rangle+\beta|H\rangle|V\rangle|H\rangle+\gamma|H\rangle|H\rangle|V\rangle$ into a maximally entangled W state $(|V\rangle|H\rangle|H\rangle+|H\rangle|V\rangle|H\rangle+|H\rangle|H\rangle|V\rangle)/\sqrt{3}$, with the help of two identical conventional single photons of the form $(|H\rangle+|V\rangle)/\sqrt{2}$. Compared with the other ECPs for the W state, our ECP has several advantages. First, without the joint unitary transformation, it can concentrate an arbitrary less-entangled W state, while conventional ECPs usually focus on some special W states. Second, it only requires one copy of the less-entangled state and the conventional single photons, while other ECPs need two copies of the less-entangled states or some special polarized single photons. Third, the cross-Kerr nonlinearity can work on the weak area, and it does not require the coherent state to pick up a large phase shift, which makes this protocol more feasible in current technology. Fourth, this ECP can be repeated to get a higher success probability.

This paper is organized as follows. In Section 2, we first describe the basic element of this ECP, the parity check measurement (PCM) constructed by the cross-Kerr nonlinearity. In Section 3, we explain our ECP. In Section 4, we calculate the entanglement transformation efficiency for the ECP. Finally, in Section 5, we present a discussion and summary.

2. Parity check measurement

Before we start this ECP, we first introduce the PCM constructed by the cross-Kerr nonlinearity, which is the key element of our ECP. The cross-Kerr nonlinearity has been widely studied in current quantum information processing, such as constructing the controlled-not (CNOT) gates,[44] making a Bell-state measurement,[45,46] performing the entanglement purification and concentration,[34–36,47] and so on.[48–54] From Fig. 1, the Hamiltonian of a cross-Kerr nonlinearity is[44,45]

$$H=\hbar\chi a_s^{\dagger}a_s a_p^{\dagger}a_p. \tag{1}$$

Here χ is the coupling strength of the nonlinearity. $a_s^{\dagger}$ and $a_p^{\dagger}$ are the creation operators, while a_s and a_p are the destruction operators. As shown in Fig. 1, we consider two single photon states $|\psi_1\rangle=c_1|H\rangle+d_1|V\rangle$ and $|\psi_2\rangle=c_2|H\rangle+d_2|V\rangle$ coupled with a coherent state $|\alpha\rangle$, where $|c_1|^2+|d_1|^2=1$, and $|c_2|^2+|d_2|^2=1$. The state $|\psi_1\rangle$ is in the spatial mode a_1 and $|\psi_2\rangle$ is in the spatial mode a_2. The whole system can be written as

$$\begin{aligned}|\psi_1\rangle\otimes|\psi_2\rangle\otimes|\alpha\rangle &= (c_1|H\rangle+d_1|V\rangle)\otimes(c_2|H\rangle+d_2|V\rangle)\otimes|\alpha\rangle\\ &= (c_1c_2|H\rangle|H\rangle+c_1d_2|H\rangle|V\rangle+ d_1c_2|V\rangle|H\rangle\\ &\quad + d_1d_2|V\rangle|V\rangle)\otimes|\alpha\rangle\\ &\rightarrow (c_1c_2|H\rangle|H\rangle+ d_1d_2|V\rangle|V\rangle)|\alpha\rangle\\ &\quad + (c_1d_2|H\rangle|V\rangle|\alpha\,\mathrm{e}^{-2\mathrm{i}\theta}\rangle\\ &\quad + d_1c_2|V\rangle|H\rangle|\alpha\,\mathrm{e}^{2\mathrm{i}\theta}\rangle).\end{aligned} \tag{2}$$

From Eq. (2), it is obvious that the even parity states $|H\rangle|H\rangle$ and $|V\rangle|V\rangle$ lead the coherent state to pick up no phase shift. The odd parity state $|H\rangle|V\rangle$ leads the coherent state to pick up -2θ phase shift and $|V\rangle|H\rangle$ leads it to pick up the 2θ phase shift. With an X quadrature measurement in which the states $|\alpha\,\mathrm{e}^{\pm\mathrm{i}2\theta}\rangle$ cannot be distinguished, one can distinguish the even parity state from the odd parity state according to the different phase shifts of the coherent state.[44] Therefore, the setup shown in Fig. 2 can achieve the function of a PCM.

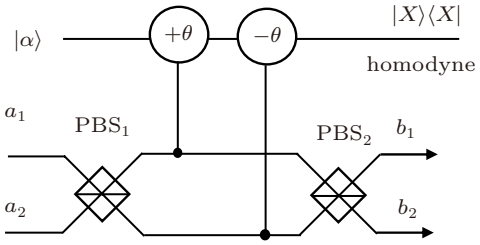


Fig. 1. The principle of a parity check measurement (PCM) constructed by cross-Kerr nonlinearity, the same as that in Ref. [54]. PBSs represent the polarizing beam splitters which transmit horizontal polarization $|H\rangle$ and reflect the vertical polarization $|V\rangle$. $\pm\theta$ denote the different phase shifts of the coherent state $|\alpha\rangle$ when there is a photon passing through the media. $|X\rangle\langle X|$ represents an X quadrature measurement described in Ref. [44].

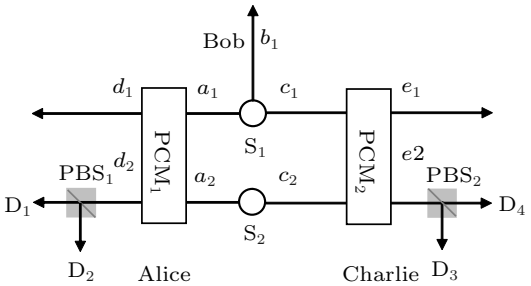


Fig. 2. Schematic drawing of this ECP with two conventional single photons of the form $\frac{1}{\sqrt{2}}(|H\rangle+|V\rangle)$. S$_1$ is the less-entangled W state source and S$_2$ is the single photon source. PCM1 and PCM2 are the same setup of parity check measurement as described in Fig. 1.

3. ECP with the PCM

With the PCM shown in Fig. 1, the principle of our ECP for a less-entangled W state is shown in Fig. 2. Three photons

emitted from the source S_1 are sent to Alice, Bob, and Charlie in the spatial modes a_1, b_1 and c_1, respectively, which can create the less-entangled three-photon W state as

$$\begin{aligned}|\Phi\rangle_{a_1b_1c_1} &= \alpha|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1}+\beta|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1}\\ &\quad+\gamma|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1},\end{aligned}\tag{3}$$

with $|\alpha|^2+|\beta|^2+|\gamma|^2=1$. Then the single-photon source S_2 emits two single photons to Alice and Charlie in the spatial mode a_2 and c_2 respectively of the same form

$$|\Phi\rangle_{a_2}=|\Phi\rangle_{c_2}=\frac{1}{\sqrt{2}}(|H\rangle+|V\rangle).\tag{4}$$

In this way, the five-photon system can be written as

$$\begin{aligned}|\Psi\rangle &= |\Phi\rangle_{a_1b_1c_1}\otimes|\Phi\rangle_{a_2}\otimes|\Phi\rangle_{c_2}\\ &= (\alpha|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1}+\beta|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1}+\gamma|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1})\otimes\frac{1}{\sqrt{2}}(|H\rangle_{a_2}+|V\rangle_{a_2})\otimes\frac{1}{\sqrt{2}}(|H\rangle_{c_2}+|V\rangle_{c_2})\\ &= \alpha|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1}|V\rangle_{a_2}|H\rangle_{c_2}+\beta|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1}|H\rangle_{a_2}|H\rangle_{c_2}+\gamma|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1})|H\rangle_{a_2}|V\rangle_{c_2}\\ &\quad+\alpha|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1}|H\rangle_{a_2}|V\rangle_{c_2}+\beta|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1}|V\rangle_{a_2}|V\rangle_{c_2}+\gamma|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1})|V\rangle_{a_2}|H\rangle_{c_2}\\ &\quad+\alpha|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1}|H\rangle_{a_2}|H\rangle_{c_2}+\beta|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1}|V\rangle_{a_2}|H\rangle_{c_2}+\gamma|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1})|V\rangle_{a_2}|V\rangle_{c_2}\\ &\quad+\alpha|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1}|V\rangle_{a_2}|V\rangle_{c_2}+\beta|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1}|H\rangle_{a_2}|V\rangle_{c_2}+\gamma|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1})|H\rangle_{a_2}|H\rangle_{c_2}.\end{aligned}\tag{5}$$

Then Alice and Charlie both make their two photons pass through the PCM gates respectively to make a parity check measurement. If they both pick up the even parity states, the state of Eq. (5) will collapse to

$$|\Psi'\rangle=\alpha|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1}|V\rangle_{a_2}|H\rangle_{c_2}+\beta|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1}|H\rangle_{a_2}|H\rangle_{c_2}+\gamma|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1})|H\rangle_{a_2}|V\rangle_{c_2},\tag{6}$$

with a probability of 1/4. Certainly, there are another three cases when Alice picks up the even parity states but Charlie picks up the odd parity states, Alice picks up the odd parity states but Charlie picks up the even parity state, and both Alice and Charlie pick up the odd parity states. The other three cases each have a probability of 1/4. If Alice and Bob both pick up the odd parity states, equation (5) will collapse to

$$|\Psi''\rangle=\alpha|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1}|H\rangle_{a_2}|V\rangle_{c_2}+\beta|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1}|V\rangle_{a_2}|V\rangle_{c_2}+\gamma|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1})|V\rangle_{a_2}|H\rangle_{c_2}.\tag{7}$$

In this case, Alice or Bob only needs to perform a bit-flip operation $\sigma_x=|H\rangle\langle V|+|V\rangle\langle H|$ on one of his or her photons to change it into both even parity states in Eq. (6). Then we can rewrite the state $|\Psi'\rangle$ under the two orthogonal bases

$$|\varphi_1\rangle_a=\frac{\alpha}{\sqrt{\alpha^2+\beta^2}}|H\rangle+\frac{\beta}{\sqrt{\alpha^2+\beta^2}}|V\rangle,\quad |\varphi_1^{\perp}\rangle_a=\frac{\beta}{\sqrt{\alpha^2+\beta^2}}|H\rangle-\frac{\alpha}{\sqrt{\alpha^2+\beta^2}}|V\rangle,\tag{8}$$

and

$$|\varphi_2\rangle_c=\frac{\gamma}{\sqrt{\gamma^2+\beta^2}}|H\rangle+\frac{\beta}{\sqrt{\gamma^2+\beta^2}}|V\rangle,\quad |\varphi_2^{\perp}\rangle_c=\frac{\beta}{\sqrt{\gamma^2+\beta^2}}|H\rangle-\frac{\gamma}{\sqrt{\gamma^2+\beta^2}}|V\rangle,\tag{9}$$

i.e.,

$$\begin{aligned}|\Psi'\rangle &= \frac{\alpha\beta\gamma}{\sqrt{\alpha^2+\beta^2}\sqrt{\gamma^2+\beta^2}}(|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1}+|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1}+|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1})|\varphi_1\rangle_a|\varphi_2\rangle_c\\ &\quad+\Big(\frac{\alpha\beta^2}{\sqrt{\alpha^2+\beta^2}\sqrt{\gamma^2+\beta^2}}|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1}+\frac{\alpha\beta^2}{\sqrt{\alpha^2+\beta^2}\sqrt{\gamma^2+\beta^2}}|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1}\\ &\quad-\frac{\alpha\gamma^2}{\sqrt{\alpha^2+\beta^2}\sqrt{\gamma^2+\beta^2}}|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1}\Big)|\varphi_1\rangle_a|\varphi_2^{\perp}\rangle_c-\Big(\frac{\alpha^2\gamma}{\sqrt{\alpha^2+\beta^2}\sqrt{\gamma^2+\beta^2}}|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1}\\ &\quad+\frac{\beta^2\gamma}{\sqrt{\alpha^2+\beta^2}\sqrt{\gamma^2+\beta^2}}|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1}+\frac{\beta^2\gamma}{\sqrt{\alpha^2+\beta^2}\sqrt{\gamma^2+\beta^2}}|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1}\Big)|\varphi_1^{\perp}\rangle_a|\varphi_2\rangle_c\\ &\quad+\Big(\frac{\alpha^2\beta}{\sqrt{\alpha^2+\beta^2}\sqrt{\gamma^2+\beta^2}}|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1}+\frac{\beta^3}{\sqrt{\alpha^2+\beta^2}\sqrt{\gamma^2+\beta^2}}|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1}\\ &\quad+\frac{\beta\gamma^2}{\sqrt{\alpha^2+\beta^2}\sqrt{\gamma^2+\beta^2}}|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1}\Big)|\varphi_1^{\perp}\rangle_a|\varphi_2^{\perp}\rangle_c.\end{aligned}\tag{10}$$

As shown in Fig. 2, Alice uses the PBS_1 whose optical axis is placed at the angle φ_1, and two detectors D_1 and D_2 to complete the measurement on the photon in the spatial mode $d2$ with the basis $\{|\varphi_1\rangle_a,|\varphi_1^{\perp}\rangle_a\}$. With the same principle, Charlie uses the PBS_2 whose optical axis is placed at the angle φ_2, and two detectors D_3 and D_4 to complete the measurement on the photon in the spatial mode e_2 with the basis $\{|\varphi_2\rangle_c,|\varphi_2^{\perp}\rangle_c\}$. Here $\cos\varphi_1=\alpha/\sqrt{\alpha^2+\beta^2}$, and $\sin\varphi_1=-\beta/\sqrt{\alpha^2+\beta^2}$. $\cos\varphi_2=\gamma/\sqrt{\gamma^2+\beta^2}$, and $\sin\varphi_2=-\beta/\sqrt{\gamma^2+\beta^2}$. From Eq. (10), if Alice and Charlie obtain

the states $|\varphi_1\rangle_a$ and $|\varphi_2\rangle_c$ respectively when they measure their photons in the spatial modes d_2 and e_2, the remaining photon pair is essentially the maximally entangled W state $|\Phi\rangle_{a_1b_1c_1} = (|V\rangle|H\rangle|H\rangle + |H\rangle|V\rangle|H\rangle + |H\rangle|H\rangle|V\rangle)/\sqrt{3}$. The success probability is

$$P_0 = \frac{3|\alpha|^2|\beta|^2|\gamma|^2}{(|\alpha|^2+|\beta|^2)(|\beta|^2+|\gamma|^2)}. \quad (11)$$

Otherwise, if Alice and Charlie obtain other states, they cannot get the maximally entangled state. For example, if Alice obtains $|\varphi_1\rangle_a$ and Charlie obtains $|\varphi_2^{\perp}\rangle_c$, they will get another less-entangled state

$$\begin{aligned}|\Phi_1\rangle_{a_1b_1c_1} &= \frac{\alpha\beta^2}{\sqrt{\alpha^2+\beta^2}\sqrt{\gamma^2+\beta^2}}|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1}\\ &+ \frac{\alpha\beta^2}{\sqrt{\alpha^2+\beta^2}\sqrt{\gamma^2+\beta^2}}|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1}\\ &- \frac{\alpha\gamma^2}{\sqrt{\alpha^2+\beta^2}\sqrt{\gamma^2+\beta^2}}|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1}, \quad (12)\end{aligned}$$

with the probability of

$$P_1 = \frac{2|\alpha|^2|\beta|^4+|\alpha|^2|\gamma|^4}{(|\alpha|^2+|\beta|^2)(|\beta|^2+|\gamma|^2)}. \quad (13)$$

Equation (12) can be rewritten as

$$\begin{aligned}|\Phi_1\rangle_{a_1b_1c_1} &= \frac{\beta^2}{\sqrt{\gamma^4+2\beta^4}}|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1}\\ &+ \frac{\beta^2}{\sqrt{\gamma^4+2\beta^4}}|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1}\\ &- \frac{\gamma^2}{\sqrt{\gamma^4+2\beta^4}}|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1}. \quad (14)\end{aligned}$$

If Alice obtains the state $|\varphi_1^{\perp}\rangle_a$ and Charlie obtains the state $|\varphi_2\rangle_c$, they will get

$$\begin{aligned}|\Phi_2\rangle_{a_1b_1c_1} &= -\frac{\alpha^2}{\sqrt{\alpha^4+2\beta^4}}|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1}\\ &+ \frac{\beta^2}{\sqrt{\alpha^4+2\beta^4}}|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1}\\ &+ \frac{\beta^2}{\sqrt{\alpha^4+2\beta^4}}|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1}, \quad (15)\end{aligned}$$

with the probability of

$$P_2 = \frac{|\alpha|^4|\gamma|^2+2|\beta|^4|\gamma|^2}{(|\alpha|^2+|\beta|^2)(|\beta|^2+|\gamma|^2)}. \quad (16)$$

If Alice obtains the state $|\varphi_1^{\perp}\rangle_a$ and Charlie obtains the state $|\varphi_2^{\perp}\rangle_c$, they will get

$$\begin{aligned}|\Phi_3\rangle_{a_1b_1c_1} &= \frac{\alpha^2}{\sqrt{\alpha^4+\beta^4+\gamma^4}}|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1}\\ &+ \frac{\beta^2}{\sqrt{\alpha^4+\beta^4+\gamma^4}}|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1}\\ &+ \frac{\gamma^2}{\sqrt{\alpha^4+\beta^4+\gamma^4}}|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1}, \quad (17)\end{aligned}$$

with the probability of

$$P_3 = \frac{|\alpha|^4|\beta|^2+|\beta|^6+|\beta|^2|\gamma|^4}{(|\alpha|^2+|\beta|^2)(|\beta|^2+|\gamma|^2)}. \quad (18)$$

Interestingly, the states of Eqs. (14), (15), and (17) are all the less-entangled W states. We first compare the state of Eq. (17) with the original less-entangled W state of Eq. (3). They have essentially the same form. That is, we choose $\alpha' = \alpha^2/\sqrt{\alpha^4+\beta^4+\gamma^4}$, $\beta' = \beta^2/\sqrt{\alpha^4+\beta^4+\gamma^4}$, and $\gamma' = \gamma^2/\sqrt{\alpha^4+\beta^4+\gamma^4}$. In order to obtain the maximally entangled W state from Eq. (17), Alice and Charlie only need to perform the ECP in a second round, following the same principle described above.

Now let us discuss the states of Eqs. (14) and (15). Both $|\Phi_1\rangle_{a_1b_1c_1}$ and $|\Phi_2\rangle_{a_1b_1c_1}$ are the less-entangled states, but they are different from the $|\Phi_3\rangle_{a_1b_1c_1}$, for they essentially only have two different coefficients. Interestingly, these kinds of states are much easier to concentrate than the original state $|\Phi\rangle_{a_1b_1c_1}$, because one of the two parties only needs one single-photon to perform this ECP. We take the state of Eq. (14) as an example. If Alice and Charlie obtain Eq. (14), they first perform a phase-flip operation and make it become

$$\begin{aligned}|\Phi_1'\rangle_{a_1b_1c_1} &= \frac{\beta^2}{\sqrt{\gamma^4+2\beta^4}}|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1}\\ &+ \frac{\beta^2}{\sqrt{\gamma^4+2\beta^4}}|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1}\\ &+ \frac{\gamma^2}{\sqrt{\gamma^4+2\beta^4}}|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1}. \quad (19)\end{aligned}$$

Then the source of S_2 emits a single photon to Charlie, and the four-photon state can be written as

$$\begin{aligned}&|\Phi_1'\rangle_{a_1b_1c_1}\otimes\frac{1}{\sqrt{2}}(|H\rangle_{c_2}+|V\rangle_{c_2})\\ &= \Big(\frac{\beta^2}{\sqrt{\gamma^4+2\beta^4}}|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1}+\frac{\beta^2}{\sqrt{\gamma^4+2\beta^4}}|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1}+\frac{\gamma^2}{\sqrt{\gamma^4+2\beta^4}}|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1}\Big)\otimes\frac{1}{\sqrt{2}}(|H\rangle_{c_2}+|V\rangle_{c_2})\\ &= \frac{1}{\sqrt{2}}\Big(\frac{\beta^2}{\sqrt{\gamma^4+2\beta^4}}|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1}|H\rangle_{c_2}+\frac{\beta^2}{\sqrt{\gamma^4+2\beta^4}}|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1}|H\rangle_{c_2}+\frac{\gamma^2}{\sqrt{\gamma^4+2\beta^4}}|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1}|V\rangle_{c_2}\Big)\\ &+ \frac{1}{\sqrt{2}}\Big(\frac{\beta^2}{\sqrt{\gamma^4+2\beta^4}}|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1}|V\rangle_{c_2}+\frac{\beta^2}{\sqrt{\gamma^4+2\beta^4}}|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1}|V\rangle_{c_2}+\frac{\gamma^2}{\sqrt{\gamma^4+2\beta^4}}|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1}|H\rangle_{c_2}\Big). \quad (20)\end{aligned}$$

After passing through the PCM, if Charlie picks up the even parity state, equation (20) will collapse to

$$\frac{\beta^2}{\sqrt{\gamma^4+2\beta^4}}|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1}|H\rangle_{c_2} + \frac{\beta^2}{\sqrt{\gamma^4+2\beta^4}}|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1}|H\rangle_{c_2} + \frac{\gamma^2}{\sqrt{\gamma^4+2\beta^4}}|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1}|V\rangle_{c_2}. \quad (21)$$

We can rewrite the above state under the orthogonal basis as

$$|\varphi_2\rangle'_c = \frac{\gamma^2}{\sqrt{\gamma^4+\beta^4}}|H\rangle + \frac{\beta^2}{\sqrt{\gamma^4+\beta^4}}|V\rangle,$$
$$|\varphi_2^{\perp}\rangle'_c = \frac{\beta^2}{\sqrt{\gamma^4+\beta^4}}|H\rangle - \frac{\gamma^2}{\sqrt{\gamma^4+\beta^4}}|V\rangle. \quad (22)$$

Then Charlie lets his photon in mode e_2 pass through the PBS_2, whose optical axis is placed at the angle φ'_2. The detectors D_3 and D_4 are used to complete the measurement on the photon in the mode e_2. Here $\cos\varphi'_2 = \gamma^2/\sqrt{\gamma^4+\beta^4}$ and $\sin\varphi'_2 = -\beta^2/\sqrt{\gamma^4+\beta^4}$. If Charlie obtains the state $|\varphi_2\rangle'_c$, they will obtain the maximally entangled state. If Charlie obtains $|\varphi_2^{\perp}\rangle'_c$, they will obtain another less-entangled state of the form

$$\frac{\beta^4}{\sqrt{\gamma^4+2\beta^4}\sqrt{\gamma^4+\beta^4}}|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1} + \frac{\beta^4}{\sqrt{\gamma^4+2\beta^4}\sqrt{\gamma^4+\beta^4}}|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1} - \frac{\gamma^4}{\sqrt{\gamma^4+2\beta^4}\sqrt{\gamma^4+\beta^4}}|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1}, \quad (23)$$

which can be reconcentrated into a maximally entangled W state in a third round. Certainly, from Eq. (20), if Charlie obtains an odd parity state, he needs to perform a bit-flip operation. Then the following steps are the same as described above. In this way, they can ultimately obtain a maximally entangled W state by repeating this ECP. On the other hand, if equation (15) is obtained, it can also be reconcentrated in a second round and only Alice needs to perform the ECP similar to Charlie's operation described above. Therefore, it can be found that all the remaining less-entangled states can be reconcentrated. If the remaining state is Eq. (14), only Charlie needs to repeat this ECP. If the remaining state is Eq. (15), only Alice needs to repeat this ECP, and if the remaining state is Eq. (17), both Alice and Charlie should repeat the whole ECP.

4. Entanglement transformation efficiency

Thus far, we have fully described this ECP, which is based on the LOCC. It is well known that LOCC cannot increase entanglement. Therefore, the ECP must be the entanglement transformation. In Ref. [35], we once calculated the entanglement transformation efficiency for GHZ state concentration. However, the W state has a different entanglement structure from the GHZ state. Thus, here we calculate the entanglement transformation efficiency after performing the ECP. The entanglement of the GHZ state is a global entanglement. That is, if we trace over any one particle, the remaining particles do not entangle anymore. But the W state is the entanglement between each particle. That is, if we trace over any one particle, the remaining two particles are still entangled. Therefore, for a W state described in Eq. (3), the entanglement of three-tangle is equal to 0.[55]

In this paper, we use the concurrence described in Refs. [55] and [56] to calculate the entanglement between each of the particles.

Before performing the ECP, the concurrence between particles A and B in Eq. (3) is $C_{AB} = 2|\alpha\beta|$. The concurrence between particles A and C in Eq. (3) is $C_{AC} = 2|\alpha\gamma|$, and the concurrence of the two subsystems A and BC is $C_{A(BC)} = 2|\alpha|\sqrt{|\beta|^2+|\gamma|^2}$. We denote the entanglement transformation efficiency as

$$\eta_{AB} = \frac{C'_{AB}}{C_{AB}},\ \eta_{AC} = \frac{C'_{AC}}{C_{AC}},\ \eta_{A(BC)} = \frac{C'_{A(BC)}}{C_{A(BC)}}. \quad (24)$$

Here

$$C'_{AB} = \frac{2}{3}P_0 + \frac{2|\beta|^4}{|\gamma|^4+2|\beta|^4}P_1 + \frac{2|\alpha|^2|\beta|^2}{|\alpha|^4+2|\beta|^4}P_2 + \frac{2|\alpha|^2|\beta|^2}{|\alpha|^4+|\beta|^4+|\gamma|^4}P_3. \quad (25)$$

The C'_{AB} is the concurrence between particles A and B after performing this ECP. From Eq. (25), it comprises four terms. The first term $2P_0/3$ means that after concentration, they obtain the maximally entangled W state with the probability of P_0 and it describes the concurrence between A and B for the maximally entangled W state. The second term and other terms are analogous with the first one. For instance, the second term means that with the probability of P_1, they obtain the state of Eq. (14), and item $2|\beta|^4/(|\gamma|^4+2|\beta|^4)$ describes the concurrence between A and B if they obtain such a state. C'_{AC} and $C_{A(BC)}$ are analogous with C'_{AB}. C'_{AC} means the concurrence between particles A and C after the ECP and $C'_{A(BC)}$ means the concurrence between subsystems A and BC after performing the ECP.

$$C'_{AC} = \frac{2}{3}P_0 + \frac{2|\beta|^2|\gamma|^2}{|\gamma|^4+2|\beta|^4}P_1 + \frac{2|\alpha|^2|\beta|^2}{|\alpha|^4+2|\beta|^4}P_2 + \frac{2|\alpha|^2|\gamma|^2}{|\alpha|^4+|\beta|^4+|\gamma|^4}P_3, \quad (26)$$

$$C'_{\mathrm{A(BC)}} = \frac{2\sqrt{2}}{3}P_0 + \frac{2|\beta|^2}{\sqrt{|\gamma|^4+2|\beta|^4}}\sqrt{\frac{|\beta|^4}{|\gamma|^4+2|\beta|^4}+\frac{|\beta|^4}{|\gamma|^4+2|\beta|^4}}P_1 + \frac{2|\alpha|^2}{\sqrt{|\alpha|^4+2|\beta|^4}}\sqrt{\frac{|\beta|^4}{|\alpha|^4+2|\beta|^4}+\frac{|\beta|^4}{|\alpha|^4+2|\beta|^4}}P_2$$
$$+\frac{2|\alpha|^2}{\sqrt{|\alpha|^4+|\beta|^4+|\gamma|^4}}\sqrt{\frac{|\beta|^4}{|\alpha|^4+|\beta|^4+|\gamma|^4}+\frac{|\gamma|^4}{|\alpha|^4+|\beta|^4+|\gamma|^4}}P_3. \quad (27)$$

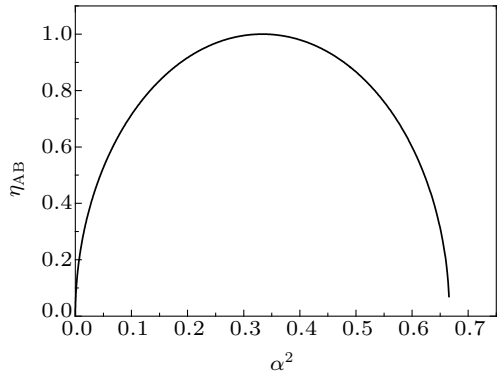


Fig. 3. The relationship between the entanglement transformation efficiency η_{AB} of the present ECP and the initial coefficient of the less-entangled state α^2, when $\gamma = 1/\sqrt{3}$. We make $\alpha \in (0, \sqrt{2/3})$. η_{AB} can reach 1 when $\alpha = \beta = \gamma = 1/\sqrt{3}$.

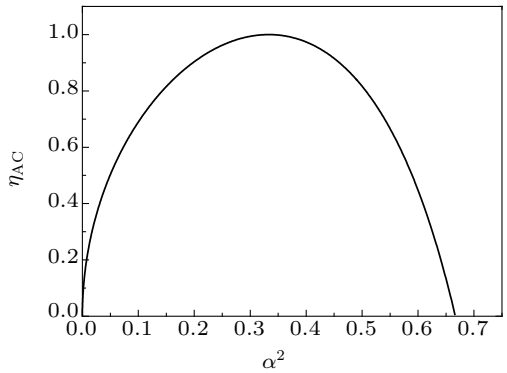


Fig. 4. The relationship between the entanglement transformation efficiency η_{AC} of the present ECP and the initial coefficient of the less-entangled state α^2, when $\gamma = 1/\sqrt{3}$. We make $\alpha \in (0, \sqrt{2/3})$.

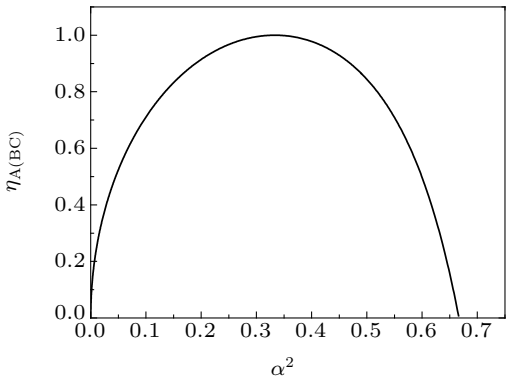


Fig. 5. The relationship between the entanglement transformation efficiency $\eta_{A(BC)}$ of the present ECP and the initial coefficient of the less-entangled state α^2, when $\gamma = 1/\sqrt{3}$. We make $\alpha \in (0, \sqrt{2/3})$.

We calculated the entanglement transformation efficiency η_{AB}, η_{AC}, and $\eta_{A(BC)}$ altered by the coefficient α. Here, we choose $\gamma = 1/\sqrt{3}$ and make $\alpha \in (0, \sqrt{2/3})$. The relationships between the coefficient α^2 and the entanglement transformation efficiency η_{AB}, η_{AC}, and $\eta_{A(BC)}$ are shown in Figs. 3, 4, and 5, respectively. It is interesting to see that the η_{AB}, η_{AC}, and $\eta_{A(BC)}$ are not fixed values but all increase with the initial entanglement. They can reach the maximal value 1 when $\alpha = \beta = \gamma = 1/\sqrt{3}$. These results are consistent with the results shown in Ref. [35]. Interestingly, we also calculated the entanglement transformation efficiency η_{AB}, η_{AC}, and $\eta_{A(BC)}$ altered by the coefficient α^2 when $\gamma = 1/4$, as shown in Fig. 6, Fig. 7, and Fig. 8, respectively. In this case, the original state is not the maximally entangled state. From Fig. 6, η_{AB} can also reach the max value 1 when $\alpha^2 = \beta^2 = 1 - (1/4)^2/2 = 15/32$. But as shown in Fig. 7, η_{AC} cannot reach 1 when we change α^2. Our numerical simulation shows that the max value of $\eta_{AC} \approx 0.7078$ when $\alpha^2 \approx 0.6757$. In Fig. 8, it is shown that $\eta_{A(BC)} = 1$ when $\alpha^2 = 15/32$, which is similar to Fig. 6.

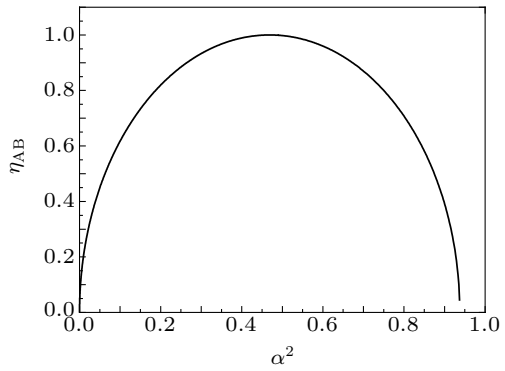


Fig. 6. The relationship between the entanglement transformation efficiency η_{AB} of the present ECP and the initial coefficient of the less-entangled state α^2, when $\gamma = 1/4$. We make $\alpha \in (0, \sqrt{15/32})$.

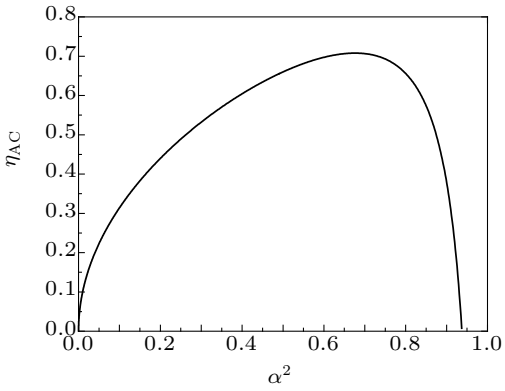


Fig. 7. The relationship between the entanglement transformation efficiency η_{AC} of the present ECP and the initial coefficient of the less-entangled state α^2, when $\gamma = 1/4$. We make $\alpha \in (0, \sqrt{15/32})$.

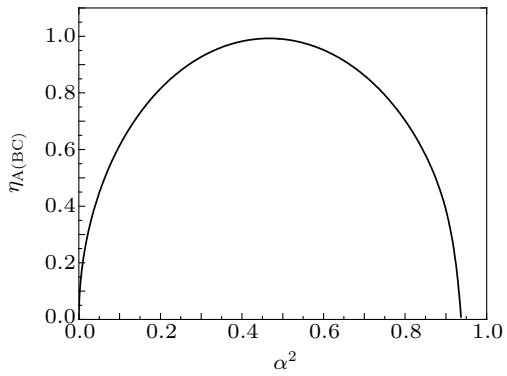


Fig. 8. The relationship between the entanglement transformation efficiency $\eta_{A(BC)}$ of the present ECP and the initial coefficient of the less-entangled state α^2, when $\gamma = 1/4$. We make $\alpha \in (0, \sqrt{15/32})$.

5. Discussion and summary

Thus far, we have fully explained this ECP. In this paper, the most important element of the ECP is the PCM as shown in Fig. 1, which is constructed by weak cross-Kerr nonlinearity. We resort to the general homodyne–heterodyne measurement of the phase shift of the coherent state to obtain the information of the parity. If the cross-Kerr nonlinearity is weak, we should let α be large to make $\alpha\theta > 1$, where α is the amplitude of the coherent state. Certainly, this requirement can be satisfied with a large amplitude of the coherent state, even with the weak nonlinearity.[44,45,52,53] At present, in the optical single photon regime, it is still quite a controversial assumption to have a clean cross-Kerr nonlinearity. Natural cross-Kerr nonlinearity is only $\tau \approx 10^{-18}$ in the optical single-photon regime.[57] Gea-Banacloche argued that it is impossible to obtain a giant Kerr effect with a single-photon wave packet,[58] which is consistent with Shapiro and Razavi.[59,60] Fortunately, the PCM in our ECP does not require a strong nonlinearity and it can work for weak values of the cross-Kerr coupling. On the other hand, Hofmann *et al.* pointed out that with a single two-level atom in a one-sided cavity, a π phase shift can be reached.[61] He *et al.* showed that the high fidelities, nonzero conditional phases and high photon numbers are compatible if the transverse-mode effects can be suppressed.[50] Feizpour *et al.* also showed that it is possible to amplify a cross-Kerr phase shift to an observable value.[62] Recently, Zhu and Zhang discussed that coupled with a four-level, double Λ-type configuration, giant cross-Kerr nonlinearities were also obtained with nearly vanishing optical absorption.[63]

In summary, we presented an optimal ECP for an arbitrary less-entangled three-photon W state. Alice and Charlie exploited the same PCM based on the cross-Kerr nonlinearity to extend the three-photon W state to a five-photon system. They projected the additional two photons with a suitable orthogonal basis. By detecting the state of the two additional photons, the three-photon system could be divided into two groups. One was in the maximally entangled W state and the others were in another partially entangled W state which could be reconcentrated in the next round. By iterating this protocol several times, it could reach a higher success probability. This ECP is quite different from the conventional ECPs for the W state. First, it is focused on an arbitrary less-entangled W state and does not resort to the unitary evolution. Second, it only requires single photons of the same form $(|H\rangle + |V\rangle)/2$. Third, this PCM can work on the weak regime of cross-Kerr nonlinearity, which is suitable for current technology. Fourth, this ECP can be repeated to obtain a higher success probability, which makes it optimal. We hope this ECP may be useful in practical quantum information processing.

Acknowledgement

We thank Dr. Zhao Bao-Kui of Jilin University for helpful discussion.

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