Deterministic joint remote state preparation of arbitrary two- and three-qubit states*

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We propose a novel deterministic protocol that two senders are capable of remotely preparing arbitrary two- and three-qubit states for a remote receiver using EPR pairs and GHZ state as the quantum channel. Compared with the existing deterministic protocols [An *et al*. 2011 *Phys. Lett.* A **375** 3570 and Chen *et al*. 2012 *J. Phys.* A: *Math. Theor.* **45** 055303], the quantum resources and classical information in our scheme are decreased, and the whole operation process is simplified.

Keywords: joint remote state preparation, EPR pairs, GHZ state

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1. Introduction

With the development of quantum information, entangled states have been recognized as special physical resources to accomplish various intriguing tasks in quantum information processing such as quantum teleportation,[1] quantum dense coding,[2,3] quantum cryptography,[4] and so on. As one of these applications quantum teleportation is a process by which a quantum state, unknown to the participants, can be transmitted from one location to another by using classical communication and a previously shared entangled resource without the particle, the carrier of quantum information, being transmitted through the intervening space. It is an important task in quantum information processing. Remote state preparation (RSP) is the variant of quantum teleportation in which the sender knows the information of the quantum state to be teleported. Since the concept of RSP was first introduced by Lo,[5] there have been extensive investigations about remote preparation of a variety of quantum states.[6–15] However, in RSP, the sender knows all the information encoded in the quantum state, i.e., the coded information can be leaked out completely. In order to solve the problem of security, joint remote state preparation (JRSP) protocols[16–28] have come into being. In JRSP, more senders are involved in the operation, and all the information of the state is split among the senders so that all the senders must jointly perform the task but no one among them is able to identify the full content of the encoded information by himself.

So far, most of the protocols of JRSP are probabilistic, i.e., the success probability is less than one.[16–25] Chen *et al*.[23] proposed a protocol using six Einstein–Podolsky–Rosen (EPR) pairs as a shared quantum resource to jointly remotely prepare an arbitrary three-qubit state with the total success probability $1/8$ if the shared pairs are all maximally entangled. Zhan *et al*.[24] proposed several protocols for the joint remote preparation of four-qubit cluster-type states with real coefficients and complex coefficients by using six EPR pairs and two six-qubit entangled states as quantum channels respectively. In their protocols, the successful probabilities are $1/4$ and $1/16$ for the original state of real coefficients and complex coefficients, respectively. Very recently, An *et al*.[27] proposed a protocol using EPR pairs to deterministically, jointly, and remotely prepare an arbitrary single-qubit state and a two-qubit state. Chen *et al*.[28] proposed a protocol to deterministically, jointly, and remotely prepare an arbitrary three-qubit state via EPR pairs. Compared with the two deterministic protocols in Refs. [27] and [28], here we propose a deterministic protocol with less quantum resources and classical information, and the whole operation process is simplified.

In this paper, we use EPR pairs and Greenberger–Horne–Zeilinger (GHZ) states as the quantum channel to jointly and remotely prepare an arbitrary two-qubit state, and generalize it to the arbitrary three-qubit case. In Section 2, the JRSP protocol for an arbitrary two-qubit state is studied. The projective measurement basis for each sender has been displayed in detail, which is available for the desired state in both the real space and the complex space without any restriction. At the same time, classical communication and success probability of the JRSP have also been considered. Then, the generalization to the arbitrary three-qubit state is presented in Section 3 with a similar method. Section 4 is the discussion and summary.

2. Deterministic joint remote preparation of an arbitrary two-qubit state

We present a scheme for joint remote preparation of an arbitrary two-qubit entangled state among three participants.

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Assume that the two senders (Alice and Bob) need to jointly and remotely prepare a state for a remote receiver (Charlie). The state we intend to prepare is in a general form of a two-qubit state given by[19]

$$|\Phi\rangle = \sum_{j=0}^{3} a_j \mathrm{e}^{\mathrm{i}\lambda_j}|j\rangle, \tag{1}$$

with any parameters $\{a_j, \lambda_j\} \in R$ and satisfying the normalization condition $\sum_{j=0}^{3} a_j^2 = 1$, while $|0\rangle \equiv |00\rangle$, $|1\rangle \equiv |01\rangle$, $|2\rangle \equiv |10\rangle$, $|3\rangle \equiv |11\rangle$ are logograms of the four two-qubit orthonormal states in their computational basis. Herein, the state $|\Phi\rangle$ is known partially to Alice and Bob, and we also assume that Alice has complete information of the coefficients a_j, while Bob knows the coefficients $\mathrm{e}^{\mathrm{i}\lambda_j}$. The receiver Charlie does not have any information about the desired state $|\Phi\rangle$ at all. In this situation, neither Alice nor Bob alone can help Charlie to reconstruct the desired state. Yet, to faithfully transmit the full secret to the remote receiver the two senders must cooperate correctly.

The two senders and the receiver initially share two EPR pairs and one GHZ state as the quantum channel, which can be written as

$$|\Theta\rangle = \frac{1}{\sqrt{2}} \bigotimes_{i=1}^{2} |q\rangle_{A_iB_i} (|000\rangle + |111\rangle)_{123}, \tag{2}$$

where $|q\rangle_{A_iB_i} = (|00\rangle + |11\rangle)_{A_iB_i}/\sqrt{2}$. The qubits A_1 and 1 belong to Alice, the qubits B_1, B_2, and 2 belong to Bob, while the remaining qubits A_2 and 3 belong to Charlie as shown in Fig. 1.

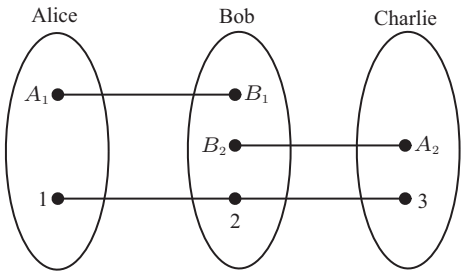


Fig. 1. The quantum channel via two EPR pairs and one GHZ state for two senders Alice and Bob to jointly and remotely prepare a general two-qubit state Eq. (1) for a remote receiver Charlie. In this figure as well as in all other figures, a qubit is represented by a black dot and entangled qubits are connected by solid lines.

At first, Alice measures her qubits A_1 and 1 in the basis $\{|\psi_n\rangle; n = 0,1,2,3\}$. Because Alice only knows $\{a_j\}$, so we define $\{|\psi_n\rangle\}$ as

$$\begin{pmatrix} |\psi_0\rangle_{A_11} \\ |\psi_1\rangle_{A_11} \\ |\psi_2\rangle_{A_11} \\ |\psi_3\rangle_{A_11} \end{pmatrix} = \begin{pmatrix} a_0 & a_1 & a_2 & a_3 \\ a_1 & -a_0 & a_3 & -a_2 \\ -a_2 & a_3 & a_0 & -a_1 \\ a_3 & a_2 & -a_1 & -a_0 \end{pmatrix} \begin{pmatrix} |0\rangle_{A_11} \\ |1\rangle_{A_11} \\ |2\rangle_{A_11} \\ |3\rangle_{A_11} \end{pmatrix}. \tag{3}$$

After making the measurement in the basis $\{|\psi_n\rangle\}$, Alice transmits her measurement results exactly not only to Charlie but also to Bob by broadcasting 2 bits of classical information. According to the broadcasted outcomes, Bob and Charlie perform the corresponding operation k shown in Table 1, so Bob and Charlie are always able to obtain the same collapsed state as the measurement result $|\psi_0\rangle$ no matter what the measurement result of Alice is.

Table 1. Corresponding operation k performed by Bob and Charlie depending on Alice's measurement result. I is the identity operator, σ_x is the bit-flip operator and σ_z is the phase-flip one.

Alice's result	$k_{B_1A_2B_223}$
$\|\psi_0\rangle_{A_11}$	$I\otimes I\otimes I\otimes I\otimes I$
$\|\psi_1\rangle_{A_11}$	$I\otimes I\otimes I\otimes \sigma_x\otimes \sigma_x\sigma_z$
$\|\psi_2\rangle_{A_11}$	$\sigma_z\sigma_x\otimes I\otimes I\otimes I\otimes \sigma_z$
$\|\psi_3\rangle_{A_11}$	$\sigma_x\sigma_z\otimes I\otimes I\otimes \sigma_x\otimes \sigma_x$

Afterwards, Bob first applies $\mathrm{CNOT}_{B_1B_2}$ on his two qubits B_1 and B_2 and then measures qubit B_2 in its computational basis $\{|0\rangle_{B_2}, |1\rangle_{B_2}\}$. Then, the system composed of Bob and Charlie collapses into

$$|\Theta'\rangle = \frac{1}{2\sqrt{2}} |m\rangle_{B_2} |\phi_m\rangle_{B_1A_223}, \tag{4}$$

where

$$|\phi_0\rangle_{B_1A_223} = a_0|00\rangle + a_1|03\rangle + a_2|30\rangle + a_3|33\rangle, \tag{5}$$

$$|\phi_1\rangle_{B_1A_223} = a_0|10\rangle + a_1|13\rangle + a_2|20\rangle + a_3|23\rangle, \tag{6}$$

with a probability $p_m = 1/2$ for any value of possible m ($m = 0$ or 1). Bob transmits his measurement result exactly to Charlie by broadcasting 1 bit of classical information. Next, Bob starts measuring his qubits B_1 and 2 in the basis $\{|\varphi_s\rangle; s = 0,1,2,3\}$, which is determined by the parameters $\mathrm{e}^{\mathrm{i}\lambda_j}$. Without loss of generality, we set $\lambda_0 = 0$, so the basis $\{|\varphi_s\rangle\}$ can be constructed as[27]

$$\begin{pmatrix} |\varphi_0\rangle_{B_12} \\ |\varphi_1\rangle_{B_12} \\ |\varphi_2\rangle_{B_12} \\ |\varphi_3\rangle_{B_12} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & \mathrm{e}^{-\mathrm{i}\lambda 1} & \mathrm{e}^{-\mathrm{i}\lambda_2} & \mathrm{e}^{-\mathrm{i}\lambda_3} \\ 1 & -\mathrm{e}^{-\mathrm{i}\lambda_1} & \mathrm{e}^{-\mathrm{i}\lambda_2} & -\mathrm{e}^{-\mathrm{i}\lambda_3} \\ 1 & -\mathrm{e}^{-\mathrm{i}\lambda_1} & -\mathrm{e}^{-\mathrm{i}\lambda_2} & \mathrm{e}^{-\mathrm{i}\lambda_3} \\ 1 & \mathrm{e}^{-\mathrm{i}\lambda_1} & -\mathrm{e}^{-\mathrm{i}\lambda_2} & -\mathrm{e}^{-\mathrm{i}\lambda_3} \end{pmatrix} \times \begin{pmatrix} |0\rangle_{B_12} \\ |1\rangle_{B_12} \\ |2\rangle_{B_12} \\ |3\rangle_{B_12} \end{pmatrix}. \tag{7}$$

Then $|\phi_m\rangle_{B_1A_223}$ is expressed by $\{|\varphi_s\rangle; s = 0,1,2,3\}$ in the form

$$|\phi_m\rangle_{B_1A_223} = \frac{1}{2} \sum_{s=0}^{3} |\varphi_s\rangle_{B_12} r_{ms}^{\dagger} |\Phi\rangle_{A_23}, \tag{8}$$

with r_{ms} an operator as listed in Table 2.

Obviously, with an equal probability p_s= 1/4, Bob will obtain a state $|\varphi_s\rangle_{B_12}$ with $s = (0,1,2,3)$, which he also needs to announce publicly. By now, the total classical communication cost of Bob's work process is 3 bits.

Table 2. The reconstruction operator r_{ms} in Eq. (8) versus m and s. I is the identity operator, σ_x is the bit-flip operator, and σ_z is the phase-flip one.

s	r_{ms} $(m=0)$	r_{ms} $(m=1)$
0	$I\otimes I$	$\sigma_x\otimes I$
1	$I\otimes\sigma_z$	$\sigma_x\otimes\sigma_z$
2	$\sigma_z\otimes\sigma_z$	$\sigma_z\sigma_x\otimes\sigma_z$
3	$\sigma_z\otimes I$	$\sigma_z\sigma_x\otimes I$

In the end, Charlie's job is simply to perform r_{ms} on his qubits A_2 and 3 to obtain the desired state $|\Phi\rangle_{A_23}$ according to the outcomes m and s. Since Charlie is always able to reconstruct $|\Phi\rangle_{A_23}$ by performing corresponding operations r_{ms}, our JRSP protocol is deterministic and its total success probability p_t is

$$p_t=\sum_{m=0}^{1}\sum_{s=0}^{3}p_m p_s=8\times\frac{1}{2}\times\frac{1}{4}=1. \tag{9}$$

In our protocol of deterministic joint remote preparation of an arbitrary two-qubit state, the senders send the measurement results to the receiver with a cost of 5 classical bits of communication, including 2 classical bits from Alice for distinguishing the outcome $|\psi_n\rangle$ and 3 classical bits from Bob for showing the exact outcome of his measurement.

3. Deterministic joint remote preparation of an arbitrary three-qubit state

We now turn to discussing the JRSP process for an arbitrary three-qubit state. Suppose Alice and Bob wish to help Charlie remotely prepare a three-qubit state in the following form[28]

$$|\Psi\rangle=\sum_{j'=0}^{7}a_{j'}\,\mathrm{e}^{\mathrm{i}\lambda_{j'}}|j'\rangle, \tag{10}$$

where any parameters $\{a_{j'},\lambda_{j'}\}\in R$ with the normalization condition $\sum_{j'=0}^{7}a_{j'}^2=1$, while $|0\rangle\equiv|000\rangle$, $|1\rangle\equiv|001\rangle$, $|2\rangle\equiv|010\rangle$, $|3\rangle\equiv|011\rangle$, $|4\rangle\equiv|100\rangle$, $|5\rangle\equiv|101\rangle$, $|6\rangle\equiv|110\rangle$, $|7\rangle\equiv|111\rangle$ are logograms of the eight three-qubit orthonormal states in their computational basis. We assume that the coefficients $a_{j'}$ are known to Alice, while Bob has complete knowledge of the coefficients $\mathrm{e}^{\mathrm{i}\lambda_{j'}}$. Thus, neither one of the two senders (Alice or Bob) alone can help the receiver Charlie, who has no idea of the desired state $|\Psi\rangle$ at all, to reconstruct the desired state, unless they could cooperate together. In order to remotely prepare the state of Eq. (10), the two senders and the receiver initially need to share four EPR pairs and one GHZ state as the quantum channel, which can be written as

$$|\Upsilon\rangle=\frac{1}{\sqrt{2}}\bigotimes_{i=1}^{4}|q\rangle_{A_iB_i}(|000\rangle+|111\rangle)_{123}. \tag{11}$$

The qubits A_1, A_3, and 1 belong to Alice, the qubits B_1, B_2, B_3, B_4, and 2 belong to Bob, while the remaining qubits A_2, A_4, and 3 belong to Charlie as shown in Fig. 2. Alice measures qubits A_1, A_3, and 1 in the basis $\{|\psi'_{n'}\rangle;n'=0,1,...,7\}$, which is defined by $\{a_{j'}\}$ with the formula [23]

$$\begin{pmatrix}|\psi'_0\rangle_{A_1A_31}\\|\psi'_1\rangle_{A_1A_31}\\|\psi'_2\rangle_{A_1A_31}\\|\psi'_3\rangle_{A_1A_31}\\|\psi'_4\rangle_{A_1A_31}\\|\psi'_5\rangle_{A_1A_31}\\|\psi'_6\rangle_{A_1A_31}\\|\psi'_7\rangle_{A_1A_31}\end{pmatrix}=\begin{pmatrix}a_0&a_1&a_2&a_3&a_4&a_5&a_6&a_7\\a_1&-a_0&a_3&-a_2&a_5&-a_4&a_7&-a_6\\a_2&-a_3&-a_0&a_1&-a_6&a_7&a_4&-a_5\\a_3&a_2&-a_1&-a_0&a_7&a_6&-a_5&-a_4\\a_4&-a_5&a_6&-a_7&-a_0&a_1&-a_2&a_3\\a_5&a_4&-a_7&-a_6&-a_1&-a_0&a_3&a_2\\a_6&-a_7&-a_4&a_5&a_2&-a_3&-a_0&a_1\\a_7&a_6&a_5&a_4&-a_3&-a_2&-a_1&-a_0\end{pmatrix}\times\begin{pmatrix}|0\rangle_{A_1A_31}\\|1\rangle_{A_1A_31}\\|2\rangle_{A_1A_31}\\|3\rangle_{A_1A_31}\\|4\rangle_{A_1A_31}\\|5\rangle_{A_1A_31}\\|6\rangle_{A_1A_31}\\|7\rangle_{A_1A_31}\end{pmatrix}. \tag{12}$$

After making the measurement in the basis $\{|\psi'_{n'}\rangle\}$, Alice transmits her measurement result exactly not only to Charlie but also to Bob by broadcasting 3 bits of classical information. Bob and Charlie perform corresponding operation k shown in Table 3 depending on the broadcasted outcomes, so Bob and Charlie are always able to obtain the same collapsed state as the result $|\psi'_0\rangle$ is.

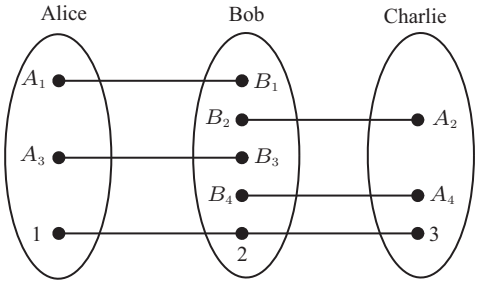


Fig. 2. The quantum channel via four EPR pairs and one GHZ state for two senders Alice and Bob to jointly and remotely prepare a general three-qubit state Eq. (10) for a remote receiver Charlie.

Table 3. Corresponding operation k performed by Bob and Charlie depending on Alice's measurement result. I is the identity operator, σ_x is the bit-flip operator and σ_z is the phase-flip one.

Alice's result	$k_{B_1A_2B_2B_3A_4B_423}$
$\|\psi'_0\rangle_{A_1A_31}$	$I\otimes I\otimes I\otimes I\otimes I\otimes I\otimes I$
$\|\psi'_1\rangle_{A_1A_31}$	$I\otimes I\otimes I\otimes I\otimes I\otimes\sigma_x\otimes\sigma_x\sigma_z$
$\|\psi'_2\rangle_{A_1A_31}$	$\sigma_z\otimes I\otimes I\otimes\sigma_x\sigma_z\otimes I\otimes I\otimes\sigma_z\otimes I$
$\|\psi'_3\rangle_{A_1A_31}$	$I\otimes I\otimes\sigma_x\otimes\sigma_z\otimes I\otimes I\otimes\sigma_x\otimes\sigma_x$
$\|\psi'_4\rangle_{A_1A_31}$	$\sigma_x\sigma_z\otimes I\otimes I\otimes I\otimes I\otimes I\otimes\sigma_z\otimes I$
$\|\psi'_5\rangle_{A_1A_31}$	$\sigma_x\sigma_z\otimes I\otimes I\otimes\sigma_z\otimes I\otimes I\otimes\sigma_x\otimes\sigma_x$
$\|\psi'_6\rangle_{A_1A_31}$	$\sigma_x\otimes I\otimes I\otimes\sigma_x\sigma_z\otimes I\otimes I\otimes\sigma_z\otimes I$
$\|\psi'_7\rangle_{A_1A_31}$	$\sigma_x\sigma_z\otimes I\otimes I\otimes\sigma_x\otimes I\otimes I\otimes\sigma_x\otimes\sigma_x$

Afterwards Bob first applies $\mathrm{CNOT}_{B_1B_2}$ and $\mathrm{CNOT}_{B_3B_4}$ on his qubits (B_1,B_2), (B_3,B_4) and then measures qubits B_2 and B_4 in its computational basis $\{|00\rangle_{B_2B_4},|01\rangle_{B_2B_4},|10\rangle_{B_2B_4},|11\rangle_{B_2B_4}\}$. Then, the system composed of Bob and Charlie collapses into

$$|\Upsilon'\rangle=\frac{1}{4\sqrt{2}}|m'l\rangle_{B_2B_4}|\phi'_{m'l}\rangle_{B_1A_2B_3A_423}, \tag{13}$$

where

$$
\begin{aligned}
&|\phi'_{00}\rangle_{B_1A_2B_3A_4}23 \\
&= a_0|00\rangle + a_1|03\rangle + a_2|14\rangle + a_3|17\rangle \\
&\quad + a_4|60\rangle + a_5|63\rangle + a_6|74\rangle + a_7|77\rangle, \qquad (14)\\
&|\phi'_{01}\rangle_{B_1A_2B_3A_4}23 \\
&= a_0|04\rangle + a_1|07\rangle + a_2|10\rangle + a_3|13\rangle \\
&\quad + a_4|64\rangle + a_5|67\rangle + a_6|70\rangle + a_7|73\rangle, \qquad (15)\\
&|\phi'_{10}\rangle_{B_1A_2B_3A_4}23 \\
&= a_0|20\rangle + a_1|23\rangle + a_2|34\rangle + a_3|37\rangle \\
&\quad + a_4|40\rangle + a_5|43\rangle + a_6|54\rangle + a_7|57\rangle, \qquad (16)\\
&|\phi'_{11}\rangle_{B_1A_2B_3A_4}23 \\
&= a_0|24\rangle + a_1|27\rangle + a_2|30\rangle + a_3|33\rangle \\
&\quad + a_4|44\rangle + a_5|47\rangle + a_6|50\rangle + a_7|53\rangle, \qquad (17)
\end{aligned}
$$

with a probability $p_{m'l} = 1/4$ for any pair of possible m' ($m' = 0$ or 1) and l ($l = 0$ or 1). Bob transmits his measurement result exactly to Charlie by broadcasting 2 bits of classical information.

In next preparation step, Bob measures his qubits B_1, B_3, and 2 in the basis $\{|\varphi'_{s'}\rangle; s' = 0,1,\ldots,7\}$. Bob must make use of the parameter $e^{i\lambda_{j'}}$, which he is supposed to know. Without loss of generality, we set $\lambda_0 = 0$, so we define $\{|\varphi'_{s'}\rangle\}$ as

$$
\begin{pmatrix}
|\varphi'_0\rangle_{B_1B_3}2\\ |\varphi'_1\rangle_{B_1B_3}2\\ |\varphi'_2\rangle_{B_1B_3}2\\ |\varphi'_3\rangle_{B_1B_3}2\\ |\varphi'_4\rangle_{B_1B_3}2\\ |\varphi'_5\rangle_{B_1B_3}2\\ |\varphi'_6\rangle_{B_1B_3}2\\ |\varphi'_7\rangle_{B_1B_3}2
\end{pmatrix}
= \frac{1}{2}
\begin{pmatrix}
1 & e^{-i\lambda_1} & e^{-i\lambda_2} & e^{-i\lambda_3} & e^{-i\lambda_4} & e^{-i\lambda_5} & e^{-i\lambda_6} & e^{-i\lambda_7}\\
1 & -e^{-i\lambda_1} & e^{-i\lambda_2} & -e^{-i\lambda_3} & e^{-i\lambda_4} & -e^{-i\lambda_5} & e^{-i\lambda_6} & -e^{-i\lambda_7}\\
1 & -e^{-i\lambda_1} & -e^{-i\lambda_2} & e^{-i\lambda_3} & -e^{-i\lambda_4} & e^{-i\lambda_5} & e^{-i\lambda_6} & -e^{-i\lambda_7}\\
1 & e^{-i\lambda_1} & -e^{-i\lambda_2} & -e^{-i\lambda_3} & e^{-i\lambda_4} & e^{-i\lambda_5} & -e^{-i\lambda_6} & -e^{-i\lambda_7}\\
1 & -e^{-i\lambda_1} & e^{-i\lambda_2} & -e^{-i\lambda_3} & -e^{-i\lambda_4} & e^{-i\lambda_5} & -e^{-i\lambda_6} & e^{-i\lambda_7}\\
1 & e^{-i\lambda_1} & -e^{-i\lambda_2} & -e^{-i\lambda_3} & -e^{-i\lambda_4} & -e^{-i\lambda_5} & e^{-i\lambda_6} & e^{-i\lambda_7}\\
1 & -e^{-i\lambda_1} & -e^{-i\lambda_2} & e^{-i\lambda_3} & e^{-i\lambda_4} & -e^{-i\lambda_5} & -e^{-i\lambda_6} & e^{-i\lambda_7}\\
1 & e^{-i\lambda_1} & e^{-i\lambda_2} & e^{-i\lambda_3} & -e^{-i\lambda_4} & -e^{-i\lambda_5} & -e^{-i\lambda_6} & -e^{-i\lambda_7}
\end{pmatrix}
\begin{pmatrix}
|0\rangle_{B_1B_3}2\\ |1\rangle_{B_1B_3}2\\ |2\rangle_{B_1B_3}2\\ |3\rangle_{B_1B_3}2\\ |4\rangle_{B_1B_3}2\\ |5\rangle_{B_1B_3}2\\ |6\rangle_{B_1B_3}2\\ |7\rangle_{B_1B_3}2
\end{pmatrix}. \qquad (18)
$$

Then $|\phi'_{m'l}\rangle_{B_1A_2B_3A_4}23$ is expressed by $\{|\varphi'_{s'}\rangle; s' = 0,1,\ldots,7\}$ in the form

$$
|\phi'_{m'l}\rangle_{B_1A_2B_3A_4}23 = \frac{1}{2\sqrt{2}}\sum_{s'=0}^{7}|\varphi'_{s'}\rangle_{B_1B_3}2\, r^{\dagger}_{m'ls'}|\Psi\rangle_{A_2A_4}3, \quad (19)
$$

with $r_{m'ls'}$ an operator which is listed in Table 4. Bob also needs to announce the outcome $s' = (0,1,...,7)$ publicly, then he will obtain a state $|\varphi'_{s'}\rangle_{B_1B_3}2$ with an equal probability $p_{s'}$= 1/8 independent of s'. By now, 5 bits of classical information are required in Bob's work process.

Table 4. The reconstruction operator $r_{m'ls'}$ in Eq. (19) versus m', l, and s'. I is the identity operator, σ_x is the bit-flip operator, and σ_z is the phase-flip one.

s'	$r_{m'ls'}(m'=0,l=0)$	$r_{m'ls'}(m'=0,l=1)$	$r_{m'ls'}(m'=1,l=0)$	$r_{m'ls'}(m'=1,l=1)$
0	$I\otimes I\otimes I$	$I\otimes\sigma_x\otimes I$	$\sigma_x\otimes I\otimes I$	$\sigma_x\otimes\sigma_x\otimes I$
1	$I\otimes I\otimes\sigma_z$	$I\otimes\sigma_x\otimes\sigma_z$	$\sigma_x\otimes I\otimes\sigma_z$	$\sigma_x\otimes\sigma_x\otimes\sigma_z$
2	$\sigma_z\otimes\sigma_z\otimes\sigma_z$	$\sigma_z\otimes\sigma_z\sigma_x\otimes\sigma_z$	$\sigma_z\sigma_x\otimes\sigma_z\otimes\sigma_z$	$\sigma_z\sigma_x\otimes\sigma_z\sigma_x\otimes\sigma_z$
3	$I\otimes\sigma_z\otimes I$	$I\otimes\sigma_z\sigma_x\otimes I$	$\sigma_x\otimes\sigma_z\otimes I$	$\sigma_x\otimes\sigma_z\sigma_x\otimes I$
4	$\sigma_z\otimes I\otimes\sigma_z$	$\sigma_z\otimes\sigma_x\otimes\sigma_z$	$\sigma_z\sigma_x\otimes I\otimes\sigma_z$	$\sigma_z\sigma_x\otimes\sigma_x\otimes\sigma_z$
5	$\sigma_z\otimes\sigma_z\otimes I$	$\sigma_z\otimes\sigma_z\sigma_x\otimes I$	$\sigma_z\sigma_x\otimes\sigma_z\otimes I$	$\sigma_z\sigma_x\otimes\sigma_z\sigma_x\otimes I$
6	$I\otimes\sigma_z\otimes\sigma_z$	$I\otimes\sigma_z\sigma_x\otimes\sigma_z$	$\sigma_x\otimes\sigma_z\otimes\sigma_z$	$\sigma_x\otimes\sigma_z\sigma_x\otimes\sigma_z$
7	$\sigma_z\otimes I\otimes I$	$\sigma_z\otimes\sigma_x\otimes I$	$\sigma_z\sigma_x\otimes I\otimes I$	$\sigma_z\sigma_x\otimes\sigma_x\otimes I$

Finally, as seen from Eq. (19), the desired state Eq. (10) is always able to be reconstructed by Charlie's application of $r_{m'ls'}$ on his qubits A_2, A_4, and 3. Our JRSP protocol is deterministic and its total success probability p_t is

$$
p_t = \sum_{m'=0}^{1}\sum_{l=0}^{1}\sum_{s'=0}^{7} p_{m'l}p_{s'} = 32\times\frac{1}{4}\times\frac{1}{8} = 1. \qquad (20)
$$

In our protocol of deterministic joint remote preparation of an arbitrary three-qubit state, the senders send the measurement results to the receiver with 8 classical bits of communication cost, including 3 classical bits from Alice for distinguishing the outcome $|\psi'_{n'}\rangle$ and 5 classical bits from Bob for showing the exact outcome of his measurement.

4. Discussion and summary

Considering the protocol for deterministic joint remote preparation of an arbitrary two-qubit state, reference [27] used four EPR pairs as the shared quantum resource with the receiver performing a set of two-qubit computational basis measurements, and the total classical communication cost was 6 bits. However, our protocol requires two EPR pairs and one GHZ state as the quantum channel. The second sender (Bob) only needs to perform single-qubit computational basis mea-

surement and 5 bits of classical information are required in the whole work process. While comparing Ref. [28] with our present protocol for deterministic joint remote preparation of an arbitrary three-qubit state, we find that the former takes 12 qubits as the quantum resources and delivers 9 bits of classical information among the three participants, and the receiver needs to perform a set of three-qubit computational basis measurements, whereas the latter only needs 11 qubits quantum resources and 8 bits of classical information delivered by participants, and the second sender (Bob) only needs to perform two-qubit computational basis measurement.

In conclusion, we have presented a JRSP protocol of arbitrary two- and three-qubit states by using EPR pairs and GHZ state as the quantum channel. Compared with the existing protocols, our protocol is deemed to be obtained deterministically. The quantum resources and classical information in our protocol are decreased, respectively, and the whole operation process is simplified.

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