Investigations into quantum correlation of coupled qubits in a squeezed vacuum reservoir*

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(Received 20 April 2012; revised manuscript received 10 July 2012)

In this paper, we investigate the quantum correlation of coupled qubits which are initially in maximally entangled mixed states in a squeezed vacuum reservoir. We compare and analyze the effects of squeezed parameters on quantum discord and quantum concurrence. The results show that in a squeezed vacuum reservoir, the quantum discord and quantum concurrence perform with completely opposite behaviors with the change of squeezed parameters. Quantum discord survives longer with the increase of squeezed amplitude parameter, but entanglement death is faster on the contrary. The results also indicate that the classical correlation of the system is smaller than quantum discord in a vacuum reservoir, while it is bigger than quantum discord in a squeezed vacuum reservoir. The quantum discord and classical correlation are more robust than quantum concurrence in the two reservoir environments, which indicates that the entanglement actually is easily affected by decoherence and quantum discord has a stronger ability to avoid decoherence in a squeezed vacuum reservoir.

Keywords: quantum correlation, coupled qubits, quantum discord, concurrence

PACS: 03.65.Ta, 03.65.Ud, 03.67.–a **DOI:** 10.1088/1674-1056/22/2/020305

1. Introduction

Quantum entanglement reflects a nonlocal quantum correlation between each subsystem of a quantum multi-system. It is the core resource to realize quantum information and quantum computing.[1–3] Based on the existing experiment conditions and technology, one of the problems people are interested in is that of how to prepare a kind of entangled source which is stable and has a strong environment disturbance-against-ability and a high entanglement degree.[4–8] However, entanglement is only a special form of quantum correlations. Recently, a lot of work has been devoted to the investigation of the dynamic behavior of quantum correlation in a quantum system. Quantum correlation, as another non-classical correlation which is different from entanglement, also plays an important role in quantum optics and quantum information. In order to characterize all the non-classical correlation, Ollivier and Zurek[9] defined the quantum correlation as 'Quantum Discord'. Datta *et al.*[10] and Xu *et al.*[11] proposed a model that can realize the quantum speedup algorithm without quantum entanglement. It indicates that this kind of no-entanglement-quantum correlation can also improve the efficiency of quantum communication.[12] In the dynamic behavior investigations of quantum and classical correlation in the Markovian and non-Markovian process, people found that it was a kind of asymptotic behavior that the quantum discord decayed with time in a Markovian environment, and this behavior formed a bright contrast with the possible entanglement sudden death phenomenon that appeared in entanglement dynamics. Obviously, if the quantum discord is treated as a quantum resource, the asymptotic decay action will be more advantageous to realize quantum information processing.

Obviously, quantum entanglement is only a special form of quantum correlation. Notice that the squeezing optical state or the Bose squeezing state has a significant effect. People measure the squeezing not only because it has specific physical meaning but also because it is relatively easy to measure. Therefore, the researchers have made in depth investigations into the relations between squeezing and quantum entanglement. They have found that the squeezing is closely related to entanglement and the entanglement can be judged to exist or not by measuring the spin squeezing of the atom ensemble. With the quantum discord facing more potential application values, people, of course, are seriously concerned with the relations between squeezing and quantum discord and its potential applications.

In the quantum information transporting process, a quantum system inevitably interacts with its environment because of the need of quantum control and other reasons. It will lead to the reduction of both quantum system entanglement and quantum discord. But the problem is, which one is more robust and stable? Based on this consideration, we measure the quantum correlations respectively by using the entanglement and quantum discord, and also investigate the quantum correlation behavior after the interactive coupled qubits have passed through the dissipative channel in a squeezed vacuum reservoir. We have found that when the system is initially in the

*Project supported by the National Natural Science Foundation of China (Grant No. 11164009).
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maximally entangled mixed state, the quantum discord and entanglement performed with completely opposite behavior with the change of squeezed parameters in the squeezed vacuum reservoir. This characteristic provides a feasible way for us to enhance the quantum discord of a system.

2. Model

Due to the fact that the quantum fluctuation of optical field amplitude in a squeezing state is less than that in a coherent state, the quantum state with squeezing effect has the prospect of wide application in optical communication fields. Since the 1870s, the squeezing state has been widely used in optical communication and precise measurement of the quantum optics. The investigation of the relation between the squeezing state and quantum correlation has become an important problem. The squeezed vacuum state is formed by the squeezing operator S acting on vacuum fields. It is a kind of non-classical optical field that has been prepared in experiment and is defined as

$$|0\rangle_g = S(\xi)|0\rangle, \tag{1}$$

$$S(\xi) = \exp\left[\frac{1}{2}\xi^* b^2 - \frac{1}{2}\xi b^{\dagger 2}\right], \tag{2}$$

where $\xi = r\mathrm{e}^{\mathrm{i}\theta}$, r is the squeezed amplitude parameter, θ is the squeezed angle (in the following discussion, $\theta = 0$), and $S^{\dagger}(\xi) = S^{-}(\xi) = S(-\xi)$.

In this paper, we model a system of coupled interactive qubits in the squeezed vacuum reservoir, and we suppose that the two qubits interact with the environment independently. In the interaction picture, the Hamiltonian of the model is ($k_{\mathrm{B}} = \hbar = 1$)[13]

$$\begin{aligned} H_{\mathrm{I}}(t) = {} & (\sigma_1^+ + \sigma_2^+)\sum_n g_n b_n \mathrm{e}^{\mathrm{i}(\varepsilon-\omega_n)t} \\ & + (\sigma_1^- + \sigma_2^-)\sum_n g_n^* b_n^{\dagger}\mathrm{e}^{-\mathrm{i}(\varepsilon-\omega_n)t}, \end{aligned} \tag{3}$$

where g_n is the interaction constant between the coupled qubits and the reservoir; b_n and $b_n^{\dagger}$ represent the annihilation and creation operator of the squeezed vacuum reservoir; $\sigma_j^{\pm}$ $(j = 1, 2)$ is the Pauli operator which is used to describe the qubits which obey the commutation and anticommutation algebra relation of Pauli operators.

Based on the quantum reservoir theory, the quantum master equation of the reduced density operator satisfies[14]

$$\begin{aligned} & \frac{\mathrm{d}}{\mathrm{d}t}\rho(t) \\ = {} & \sum_{k=A,B}\left[-\frac{\lambda}{2}\cosh^2(r)\left(\sigma_+^k\sigma_-^k\rho - 2\sigma_-^k\rho\sigma_+^k + \rho\sigma_+^k\sigma_-^k\right)\right. \\ & -\frac{\lambda}{2}\mathrm{e}^{-\mathrm{i}\theta}\sinh(2r)\sigma_-^k\rho\sigma_-^k \\ & -\frac{\lambda}{2}\sinh^2(r)\left(\sigma_-^k\sigma_+^k\rho - 2\sigma_+^k\rho\sigma_-^k + \rho\sigma_-^k\sigma_+^k\right) \\ & \left.-\frac{\lambda}{2}\mathrm{e}^{\mathrm{i}\theta}\sinh(2r)\sigma_+^k\rho\sigma_+^k\right], \end{aligned} \tag{4}$$

where λ is the decay ratio of the reservoir.

The solutions of the quantum master equation depend on the system initial state. We assume the system is initially in X-state as follows:[15]

$$\rho_{AB}(t) = \begin{pmatrix} \rho_{11}(t) & 0 & 0 & \rho_{14}(t) \\ 0 & \rho_{22}(t) & \rho_{23}(t) & 0 \\ 0 & \rho_{32}(t) & \rho_{33}(t) & 0 \\ \rho_{41}(t) & 0 & 0 & \rho_{44}(t) \end{pmatrix}. \tag{5}$$

We can see that the Bell state, the Werner state, and the maximally entangled mixed states, which are to be discussed in this paper, all belong to the X-state. For a given initial state, the solutions of the quantum master equation can be found in Appendix A.

Our purpose is to compare and investigate the dynamic behavior differences in an open system when the entanglement and quantum discord are respectively used to measure the quantum correlation. We adopt the concurrence entanglement defined by Wootters[16] to measure the system entanglement. Under X-state,

$$\begin{aligned} C(\rho_{AB}) = {} & 2\max\{0, |\rho_{14}(t)| - \sqrt{\rho_{22}(t)\rho_{33}(t)}, |\rho_{23}(t)| \\ & - \sqrt{\rho_{11}(t)\rho_{44}(t)}\}. \end{aligned} \tag{6}$$

On the other hand, there are more general quantum correlations that may be present even for separable states. Recently, researchers introduced the notion, called quantum discord (QD), as a capture of all nonclassical correlations between bipartite quantum systems.[17] QD is defined as the difference in quantum version between two classically equivalent definitions of mutual information and can be given as

$$QD(\rho_{AB}) = I(\rho_{AB}) - CC(\rho_{AB}), \tag{7}$$

where $I(\rho_{AB}) = S(\rho_A) + S(\rho_{\mathrm{B}}) - S(\rho_{AB})$ is the quantum mutual information and $CC(\rho_{AB})$ is the classical correlation between the two subsystems. As discussed in Refs. [18] and [19], the classical correlation $CC(\rho_{AB}) = \max_{\{B_k\}}[S(\rho_A) - S(\rho_{AB}|\rho_{\mathrm{B}})]$, where $\{B_k\}$ is a set of von Neumann measurements performed on subsystem B locally, $S(\rho_{AB}|\{B_k\}) = \sum_k p_k S(\rho_k)$ is the quantum conditional entropy, $\rho_k = (I_A \otimes B_k)\rho(I_A \otimes B_k)/\mathrm{Tr}(I_A \otimes B_k)\rho(I_A \otimes B_k)$ is the conditional density operator corresponding to the outcome denoted by k, and $p_k = \mathrm{Tr}(I_A \otimes B_k)\rho(I_A \otimes B_k)$. Here, I_A is the identity operator performed on subsystem A. Its calculation is based on a complex maximization procedure and obtaining an analytic expression of $CC(\rho_{AB})$ for general states is not an easy task. However, for the X state described by the density matrix (5), $QD(\rho_{AB})$ and $CC(\rho_{AB})$ are given as[20]

$$CC(\rho_{AB}) = \max(CC_1, CC_2), \tag{8}$$

$$QD(\rho_{AB}) = \min(QD_1, QD_2), \tag{9}$$

where

$$CC_j = H(\rho_{11}+\rho_{22}) - D_j,$$
$$QD_j = H(\rho_{11}+\rho_{33}) + \sum_{k=1}^{4} \lambda_k Log_2 \lambda_k + D_j,$$

and

$$D_1 = H(\tau), \quad D_2 = -\sum_{j=1}^{4} \rho_{jj} \log_2 \rho_{jj} - H(\rho_{11}+\rho_{33}),$$
$$H(\rho_{11}+\rho_{22}) = -(\rho_{11}+\rho_{22}) \log_2 (\rho_{11}+\rho_{22}) - [1-(\rho_{11}+\rho_{22})] \log_2 [1-(\rho_{11}+\rho_{22})],$$
$$\tau = \frac{1+\sqrt{[1-2(\rho_{33}+\rho_{44})]^2 + 4(|\rho_{14}|+|\rho_{23}|)^2}}{2},$$

with λ_k being the four eigenvalues of $\rho_{AB}(t)$.

3. Discussion

Our purpose is to compare and investigate the dynamic behavior differences in an open system between the quantum correlations which are respectively measured by the entanglement and quantum discord. The system dynamic evolution behavior always begins with a certain initial quantum state. Therefore, in this section, we are going to discuss the most representative quantum initial states: one is the maximally entangled mixed states and the other is the pure Bell state with a maximal entanglement.

In practice, one will more often need to deal with mixed states than with pure ones. From the point of view of entanglement exploitation, one should be interested in maximally entangled mixed states (MEMS) ρ_M, which are the basic constituents of all quantum communication protocols. The maximally entangled mixed state (MEMS) has been recently experimentally encountered. We will focus our attention on these kinds of states here. Thus, we consider that the initial design of two-qubits is in the MEMS, i.e.,[21]

$$\rho_M(0) = \begin{pmatrix} g(x) & 0 & 0 & x/2 \\ 0 & 1-2g(x) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ x/2 & 0 & 0 & g(x) \end{pmatrix}, \qquad (10)$$

with $g(x) = 1/3$ for $0 \le x \le 2/3$, and $g(x) = x/2$ for $2/3 \le x \le 1$. The quantum concurrence of the $\rho_M(0)$ is given by $C(0) = x$.

In the following, we are going to discuss how the squeezed amplitude parameter affects the quantum entanglement (line-c), the quantum discord (line-qd), and the classical correlation (line-cc) while the quantum system is initially in an MEMS. Meanwhile we are going to compare the different influences from a vacuum reservoir and a squeezed vacuum reservoir. Figures 1–5 are the most representative evolution curves we choose.

Figures 1 and 2 reflect the influences on entanglement, quantum discord, and classical correlation from the vacuum reservoir and squeezed vacuum reservoir. The figures show that whether in the vacuum reservoir or in the squeezed vacuum reservoir, the evolutions of entanglement and quantum discord are similar. They monotonically evolve into zero because of the dissipation effect, but the decay of quantum discord with time is obviously slower than the quantum concurrence. So we can know that the quantum discord is more robust than the quantum entanglement. Then, quantum discord survives longer than the quantum entanglement especially in the squeezed vacuum reservoir. This indicates that quantum discord adapts to survive in the squeezed vacuum reservoir. In addition, the decrease of initial concurrence of quantum state will make the survival times of both quantum concurrence and quantum discord shorter and this can also be seen from Figs. 3 and 4 clearly. Finally, by comparing Figs. 1 and 2, we see that in the vacuum reservoir, the system classical correlation is always smaller than the quantum discord, but in the squeezed vacuum reservoir, the system classical correlation is always larger than the quantum discord. In the process of decay, the classical correlation is always more robust than the quantum concurrence in both reservoir environments.

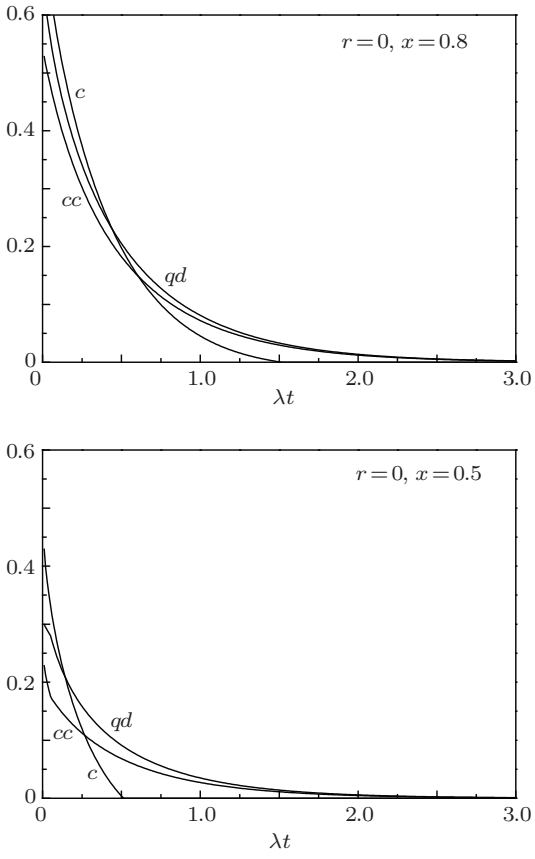


Fig. 1. Time evolutions of concurrence, quantum discord, and classical correlation in the vacuum reservoir when the qubits are initially in the MEMS.

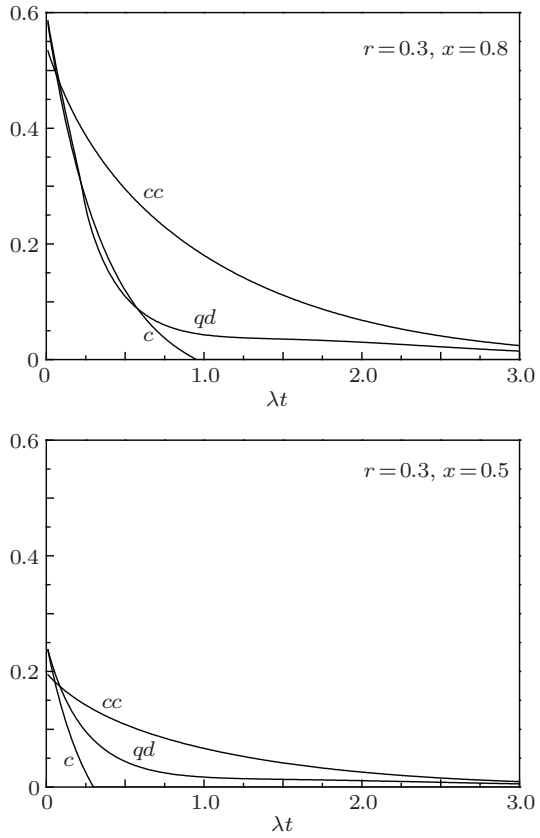


Fig. 2. Time evolutions of concurrence, quantum discord, and classical correlation in the squeezed vacuum reservoir when the qubits are initially in the MEMS.

Figure 3 shows the effect of the squeezed amplitude parameter on the entanglement when the coupled qubits are in the squeezed vacuum reservoir. From Fig. 3, we find that in the squeezed vacuum reservoir, the increase of the squeezed amplitude parameter will make the survival time of entanglement shorter.

Figure 4 shows that the squeezed amplitude parameter affects the quantum discord when the coupled qubits are in the squeezed vacuum reservoir. We can obviously see from Fig. 4 that in the squeezed vacuum reservoir, the quantum discord and entanglement have a similar trend of evolution with time, but they have big differences with the increase of the squeezed amplitude parameter since the quantum discord is a measure of quantum correlation. The increase of the squeezed amplitude parameter can prolong the survival time of quantum discord. Comparing Figs. 3 and 4, we discover that when the quantum system is initially in an MEMS, the quantum discord is more robust than the quantum entanglement in the squeezed vacuum reservoir.

Figure 5 reflects the influences on entanglement, quantum discord, and classical correlation from the vacuum reservoir and squeezed vacuum reservoir, while the system is initially in the pure Bell state with a maximal entanglement. The figure shows that in the squeezed vacuum reservoir quantum, the quantum discord is more robust than the quantum entanglement. This result is just the same as the result of the initial MEMS. In the vacuum reservoir, the time evolutions of system classical correlation and quantum discord are almost the same.

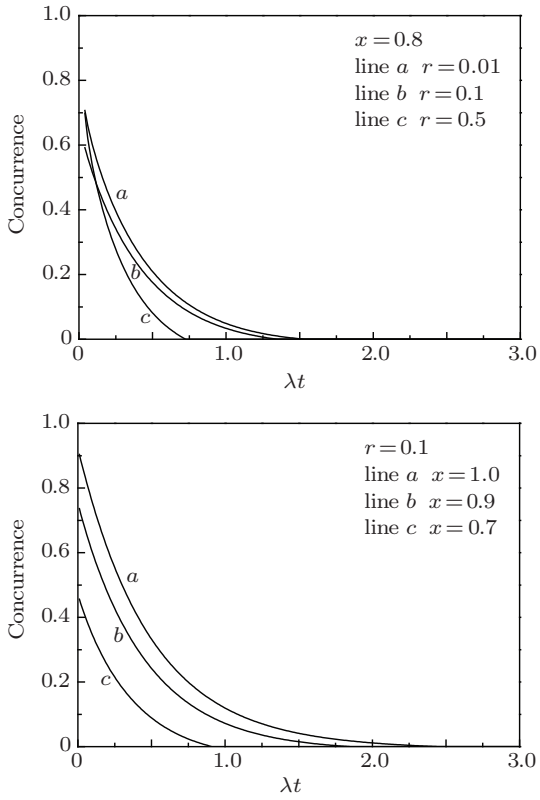


Fig. 3. Curves of quantum concurrence versus λt in the squeezed vacuum state.

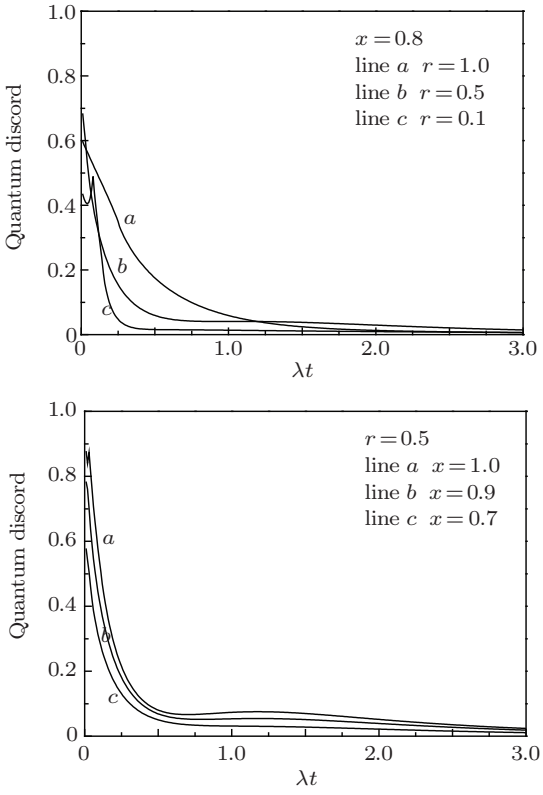


Fig. 4. Curves of quantum discord versus λt in the squeezed vacuum state.

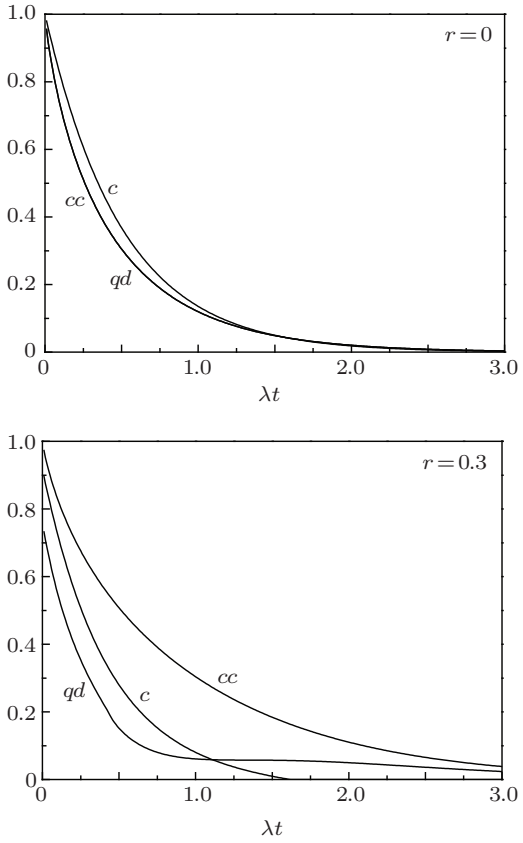


Fig. 5. Time evolutions of concurrence, quantum discord, and classical correlation in the vacuum reservoir or the squeezed vacuum reservoir, when two qubits are initially in the state $(|00\rangle \pm |11\rangle)/\sqrt{2}$.

In the squeezed vacuum reservoir, the quantum entanglement and quantum discord have opposite behavior trends with the change of squeezed parameter, which is a very strange phenomenon. From the surface, though both quantum discord and entanglement can be used as a measure of quantum correlation, they have many internal differences. In other words, the quantum discord and quantum entanglement are both the measure of quantum correlation, however, the quantum entanglement reflects only the nonlocal correlation of the system, while the quantum discord, on the other hand, represents the whole quantum correlation of the two-qubit system. The results show that there is exactly another kind of correlation which, is not quantum-entanglement. This kind of quantum correlation can be represented only by quantum discord instead of the measure of entanglement such as the quantum concurrence.

4. Conclusions

In this paper, we investigated and compared how the squeezed vacuum reservoir influences the quantum discord and concurrence when the system is initially in an MEMS or a Bell state. We find that it is difficult for the quantum concurrence to survive in a squeezed vacuum reservoir with high squeezed amplitude. On the contrary, the quantum discord is suited to survive in a squeezed vacuum reservoir with high squeezed amplitude. At the same time, we find that for the two qubits initially entangled, the initial entanglement can suppress the decay of quantum correlation, which means it is beneficial to prolonging the survival time of quantum discord and improving its robustness.

Appendix A

When the system is initially in X-state, the solution of Eq. (4) in initial state (5) is direct, and the density matrix elements of the time evolution are given by

$$\rho_{11}(t) = A_1 \tanh^4 r - A_2 \mathrm{e}^{-f(t)} \tanh^2 r + A_4 \mathrm{e}^{-2f(t)}, \quad \text{(A1)}$$

$$\rho_{22}(t) = A_1 \tanh^2 r - A_2 \mathrm{e}^{-f(t)} \operatorname{sech}^2 r - A_3 \mathrm{e}^{-f(t)} - A_4 \mathrm{e}^{-2f(t)}, \quad \text{(A2)}$$

$$\rho_{33}(t) = A_1 \tanh^2 r + A_3 \mathrm{e}^{-f(t)} - A_4 \mathrm{e}^{-2f(t)}, \quad \text{(A3)}$$

$$\rho_{44}(t) = 1 - \rho_{11}(t) - \rho_{22}(t) - \rho_{33}(t), \quad \text{(A4)}$$

where

$$A_1 = \frac{\cosh^4 r}{\cosh^2(2r)}, \quad \text{(A5)}$$

$$A_2 = \frac{1}{\cosh^2(2r)} \left[\frac{\cosh^2 r}{\operatorname{sech}(2r)} (\rho_{44}(0) - \rho_{11}(0)) - \cosh^2 r \right], \quad \text{(A6)}$$

$$A_3 = \frac{1}{\cosh^2(2r)} \left[\frac{1}{2\operatorname{sech}^2(2r)} (\rho_{33}(0) - \rho_{22}(0)) - \frac{1}{2\operatorname{sech}(2r)} (\rho_{44}(0) - \rho_{11}(0)) + \frac{1}{2} \right], \quad \text{(A7)}$$

$$A_4 = \frac{1}{\cosh^2(2r)} \left[\frac{\sinh^2 r}{\operatorname{sech}(2r)} \rho_{44}(0) + \frac{\cosh^2 r}{\operatorname{sech}(2r)} \rho_{11}(0) - \frac{1}{4} \sinh(2r) \right], \quad \text{(A8)}$$

$$f(t) = \lambda t \cosh(2r). \quad \text{(A9)}$$

Appendix B

When the system is initially in X-state, the solution of Eq. (4) in initial state (5) is direct, and the density matrix elements of the time evolution are given by

$$\rho_{23}(t) = \mathrm{e}^{-f(t)}[(B_1 \mathrm{e}^{-y(t)} - B_2 \mathrm{e}^{+y(t)}) + (B_3 - B_4)], \quad \text{(B1)}$$

$$\rho_{32}(t) = \mathrm{e}^{-f(t)}[(B_1 \mathrm{e}^{-y(t)} - B_2 \mathrm{e}^{+y(t)}) - (B_3 - B_4)], \quad \text{(B2)}$$

$$\rho_{14}(t) = \mathrm{e}^{-f(t)}[(B_1 \mathrm{e}^{-y(t)} + B_2 \mathrm{e}^{+y(t)}) - (B_3 + B_4)], \quad \text{(B3)}$$

$$\rho_{14}(t) = \mathrm{e}^{-f(t)}[(B_1 \mathrm{e}^{-y(t)} + B_2 \mathrm{e}^{+y(t)}) + (B_3 + B_4)], \quad \text{(B4)}$$

where

$$4B_1 = \rho_{23}(0) + \rho_{32}(0) + \rho_{14}(0) + \rho_{41}(0), \quad \text{(B5)}$$

$$2B_2 = 2B_1 - \rho_{23}(0) - \rho_{32}(0), \quad \text{(B6)}$$

$$2B_3 = 2B_1 - \rho_{32}(0) - \rho_{14}(0), \quad \text{(B7)}$$

$$2B_4 = 2B_1 - \rho_{23}(0) - \rho_{14}(0), \tag{B8}$$

$$y(t) = \lambda t\, \mathrm{e}^{\mathrm{i}\theta} \sinh(2r). \tag{B9}$$

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