Asymptotic dynamics of the alternate degrees of freedom for a two-mode system: An analytically solvable model*

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The composite systems can be non-uniquely decomposed into parts (subsystems). Not all decompositions (structures) of a composite system are equally physically relevant. In this paper we answer on theoretical ground why it may be so. We consider a pair of mutually un-coupled modes in the phase space representation that are subjected to the independent quantum amplitude damping channels. By investigating asymptotic dynamics of the degrees of freedom, we find that the environment is responsible for the structures non-equivalence. Only one structure is distinguished by both locality of the environmental influence on its subsystems and a classical-like description.

Keywords: amplitude dissipative channel, two-mode state, Kraus representation, alternate degrees of freedom

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1. Introduction

Realistic physical systems are composite, i.e. decomposable into smaller "parts" (subsystems). The set of the "subsystems" (i.e. of the degrees of freedom) of a composite system is not unique.

In classical physics, only one such set of subsystems (e.g. of the constituent particles) is usually considered physically relevant. The alternate decompositions (structures) of the composite system are typically considered non-realistic, a mathematical artifact. However, in the quantum mechanical context, the things may look different.

There is ongoing progress in distinguishing physical relevance of the alternate structures of a composite quantum system both on the foundational as well as on the level of application, cf. e.g. Refs. [1]–[12]. Regarding the foundational issues, the following question is of interest: which degrees of freedom of a composite system are accessible or can provide the above-mentioned classical description?[2,3,5,8,9,11,12] A closely related interpretational question reads: is there physically a fundamental set of the degrees of freedom of a composite system?[2,3,5,10] In the context of physical application, one can differently manipulate the different structures of a composite system, e.g., with the use of "entanglement swapping" for teleportation[1] or by targeting observables of a specific structure in order to avoid decoherence.[7] Quantum entanglement relativity[2–6] and relativity of the more general quantum correlations[9] open new possibilities in manipulating the quantum information hardware.[3,5,11] As a matter of fact, we just start to learn about the physical subtlety and possible usefulness of the concept of "quantum subsystem".

In this paper, we do not tackle the related deep questions. Rather, as a contribution to this new discourse in quantum theory, we stick to a concrete model that can be solved analytically and we provide some interesting observations.

We consider a pair of un-coupled modes in "phase space" representation (as a pair of non-interacting linear harmonic oscillators) independently subjected to the quantum amplitude damping channels.[13–16] We analytically (exactly) solve the Heisenberg equations of motion in the Kraus representation[13–19] and analyze the results obtained for the original as well as for some alternate degrees of freedom. We find that the environment non-equally "sees" the different structures. Particularly, only one structure is distinguished by the locality of the environmental influence on the structure's subsystems that provides a classical-like description of the subsystems.

This paper is arranged as follows. In Section 2 we re-derive the solutions to the Heisenberg equations for a pair of modes. Our derivation is specific as it is an exact calculation in the infinite-sum Kraus representation of the amplitude damping dynamics of the two-mode system. In Section 3 we introduce and analyze the alternate degrees of freedom (the alternate structures) for the pair of modes and we obtain the Heisenberg equations of motion for the new degrees of freedom. In Section 4 we emphasize the special characteristics of the original degrees of freedom that do not apply to the alternate degrees of freedom. Section 5 is conclusion.

2. The model

We consider the two uncoupled modes in the respective "phase space" representations,[16] i.e. as a pair of noninteracting linear oscillators, 1 and 2, with the respective frequen-

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cies and masses ω_1, ω_2, and m_1, m_2. The "phase space" position variables, x_1 and x_2, and the conjugate momentums, p_1 and p_2, respectively. The total Hilbert state space factorizes $\mathscr{H} = \mathscr{H}_1 \otimes \mathscr{H}_2$ and the total Hamiltonian $H = H_1 + H_2$, $H_i = p_i^2/2m_i + m_i\omega_i^2 x_i^2/2$, $i = 1,2$.

We assume the oscillators are independently subjected to the quantum amplitude channels that can be described by mutually independent master equations (in the interaction picture for zero temperature):[14]

$$\begin{aligned}\dot{\rho}_1 &= \kappa_1(2a_1\rho_1 a_1^\dagger - a_1^\dagger a_1\rho_1 - \rho_1 a_1^\dagger a_1),\\ \dot{\rho}_2 &= \kappa_2(2a_2\rho_1 a_2^\dagger - a_2^\dagger a_2\rho_2 - \rho_2 a_2^\dagger a_2)\end{aligned} \tag{1}$$

with the respective "annihilation" ("creation") operators a_i ($a_i^\dagger$, $i = 1,2$) and, in general, with the different damping parameters $\kappa_i, i = 1,2$. The initial state is tensor product, $\rho_{12}(0) = \rho_1(0) \otimes \rho_2(0)$.

The solutions to Eq. (1) are well known both in the Schrödinger picture as well as in the Heisenberg picture, cf., e.g., Ref. [16]. As we need the observables (position and momentum operators) and their bi-linear forms, below we rederive the respective expressions that facilitates further analysis.

The master equations of Eq. (1) are known to be representable in the Kraus form:[13–16]

$$\rho_i(t) = \sum_{n=0}^{\infty} K_n^i(t)\rho_i(0)K_n^{i\dagger}(t), \quad i = 1,2 \tag{2}$$

with the completeness relation $\sum_{n=0}^{\infty} K_n^{i\dagger}(t)K_n^i(t) = I_i, i = 1,2, \forall t$. For the amplitude damping process, i.e. for the master equations Eq. (1), the Kraus operators read[13–16]

$$K_n^i(t) = \sqrt{\frac{(1-\mathrm{e}^{-2\kappa_i t})^n}{n!}}\,\mathrm{e}^{-\kappa_i t N_i} a_i^n;\ N_i = a_i^\dagger a_i,\ i = 1,2. \tag{3}$$

In the Heisenberg picture, the states ρ_i do not evolve in time. Then dynamics is presented for every oscillator's observable A in the Kraus representation:

$$A_i(t) = \sum_{n=0}^{\infty} K_n^{i\dagger}(t)A_i(0)K_n^i(t), \quad i = 1,2. \tag{4}$$

Sometimes, the infinite sum in Eq. (4) is approximated by a few first terms, e.g. in Ref. [20]. However, below we give the exact solutions to Eq. (4) without calling for or imposing any approximation. As the two oscillators dynamics are mutually independent, further on, we drop the index i thus providing the expressions relevant for both oscillators.

Substituting Eq. (3) into Eq. (4) one obtains

$$A(t) = \sum_{n=0}^{\infty} \frac{(1-\mathrm{e}^{-2\kappa t})^n}{n!} a^{\dagger n}\mathrm{e}^{-\kappa t N} A(0)\mathrm{e}^{-\kappa t N} a^n. \tag{5}$$

Bearing in mind:

$$x = \left(\frac{\hbar}{2m\omega}\right)^{1/2}(a + a^\dagger), \quad p = \mathrm{i}\left(\frac{m\hbar\omega}{2}\right)^{1/2}(a^\dagger - a), \tag{6}$$

we exchange $A(0)$ in Eq. (5) by a, $a^\dagger$, and $a^\dagger a$.

To simplify the calculation, we make use of the following generalization of the Baker–Hausdorff lemma[21]

$$\begin{aligned}\mathrm{e}^{-sA}B\mathrm{e}^{-sA} &= B - s\{A,B\} + \frac{s^2}{2!}\{A,\{A,B\}\}\\ &\quad -\frac{s^3}{3!}\{A,\{A,\{A,B\}\}\} + \cdots,\end{aligned} \tag{7}$$

where the curly brackets denote the anticommutator, $\{A,B\} = AB + BA$.

Then it is straightforward to obtain

$$\begin{aligned}\mathrm{e}^{-\kappa t N} a\,\mathrm{e}^{-\kappa t N} &= \mathrm{e}^{\kappa t} a\,\mathrm{e}^{-2\kappa t N},\\ \mathrm{e}^{-\kappa t N} a^\dagger \mathrm{e}^{-\kappa t N} &= \mathrm{e}^{-\kappa t} a^\dagger \mathrm{e}^{-2\kappa t N}.\end{aligned} \tag{8}$$

Returning Eq. (8) into the Heisenberg equations for a and $a^\dagger$, one obtains

$$A(t) = \mathrm{e}^{\pm\kappa t}\sum_{n=0}^{\infty}\frac{(1-\mathrm{e}^{-2\kappa t})^n}{n!} a^{\dagger n}A(0)\,\mathrm{e}^{-2\kappa t N} a^n \tag{9}$$

with the positive exponent for a and the negative exponent for $a^\dagger$.

For $A \equiv a^\dagger$ one directly obtains

$$a^\dagger(t) = \mathrm{e}^{-\kappa t} a^\dagger \sum_{n=0}^{\infty}\frac{(1-\mathrm{e}^{-2\kappa t})^n}{n!} a^{\dagger n}\mathrm{e}^{-2\kappa t N} a^n = \mathrm{e}^{-\kappa t} a^\dagger; \tag{10}$$

the last equality in Eq. (10) follows from Eq. (8) and from the completeness relation for the Kraus operators.

For $a(t)$ we obtain

$$\begin{aligned}a(t) &= \mathrm{e}^{\kappa t}\sum_{n=0}^{\infty}\frac{(1-\mathrm{e}^{-2\kappa t})^n}{n!}[a^{\dagger n}, a]\,\mathrm{e}^{-2\kappa t N} a^n\\ &\quad + \mathrm{e}^{\kappa t} a\sum_{n=0}^{\infty}\frac{(1-\mathrm{e}^{-2\kappa t})^n}{n!} a^{\dagger n}\mathrm{e}^{-2\kappa t N} a^n,\end{aligned} \tag{11}$$

where $[A,B] = AB - BA$ is the commutator.

With the aid of $[a^{\dagger n}, a] = -n a^{\dagger n-1}$, equation (11) reads

$$a(t) = -\mathrm{e}^{\kappa t}\sum_{n=0}^{\infty} n\frac{(1-\mathrm{e}^{-2\kappa t})^n}{n!} a^{\dagger n-1}\mathrm{e}^{-2\kappa t N} a^n + \mathrm{e}^{\kappa t} a. \tag{12}$$

As it is easy to prove for the sum in Eq. (12):

$$\sum_{n=0}^{\infty} n\frac{(1-\mathrm{e}^{-2\kappa t})^n}{n!} a^{\dagger n-1}\mathrm{e}^{-2\kappa t N} a^n = (1-\mathrm{e}^{-2\kappa t})a,$$

one finally obtains

$$a(t) = \mathrm{e}^{-\kappa t} a. \tag{13}$$

In completely the same way the terms $a^2(t)$, $a^{\dagger 2}(t)$ as well as $(a^\dagger a)(t)$ can be calculated to obtain

$$\begin{aligned}a^2(t) &= \sum_{n=0}^{\infty} K_n^\dagger(t)a^2 K_n(t) = \mathrm{e}^{-2\kappa t} a^2,\\ a^{\dagger 2}(t) &= \sum_{n=0}^{\infty} K_n^\dagger(t)a^{\dagger 2} K_n(t) = \mathrm{e}^{-2\kappa t} a^{\dagger 2},\\ (a^\dagger a)(t) &= \sum_{n=0}^{\infty} K_n^\dagger(t)a^\dagger a K_n(t) = \mathrm{e}^{-2\kappa t} a^\dagger a.\end{aligned} \tag{14}$$

Now we can write the desired solutions to the Heisenberg equations for the oscillator phase-space observables as

follows:

$$x(t) = \mathrm{e}^{-\kappa t}x, \quad p(t) = \mathrm{e}^{-\kappa t}p,$$
$$x^2(t) = \mathrm{e}^{-2\kappa t}x^2 + \frac{\hbar}{2m\omega}(1-\mathrm{e}^{-2\kappa t}),$$
$$p^2(t) = \mathrm{e}^{-2\kappa t}p^2 + \frac{m\hbar\omega}{2}(1-\mathrm{e}^{-2\kappa t}). \tag{15}$$

From Eq. (15) we directly obtain the asymptotic solutions:

$$\lim_{t\to\infty} x(t) = 0 = \lim_{t\to\infty} p(t), \ \lim_{t\to\infty} x^2(t) = \frac{\hbar}{2m\omega},$$
$$\lim_{t\to\infty} p^2(t) = \frac{m\hbar\omega}{2}, \tag{16}$$

while equation (16) gives rise directly to

$$\lim_{t\to\infty} \Delta x(t)\Delta p(t) = \frac{\hbar}{2}; \tag{17}$$

in Eq. (17) it is clearly seen that the standard deviations of the respective observables appear.

Nonzero value of the covariance function, $C = \langle A_1A_2\rangle - \langle A_1\rangle\langle A_2\rangle$, reveals correlations in the total $1+2$ system's state, ρ_{12}; the observables $A_i, i = 1,2$, refer to the two oscillators, while the symbol $\langle *\rangle$ denotes the state averaging. In the Heisenberg picture, $\langle A_1(t)A_2(t)\rangle = \mathrm{tr}_{12}A_1(t)A_2(t)\rho_{12}(0)$. In general, $C = 0$ does not guarantee the absence of correlations. However, as we are interested in the Gaussian states and in the (gaussianity-preserving), the amplitude damping channel,[13–19] the zero value of the covariance function implies the absence of any correlations between the two modes.

Picking A_1 from the set $\{x_1, p_1\}$ and A_2 from the set $\{x_2, p_2\}$, one can form the covariance functions C_{ij} for the different combinations. For the independent channels considered here, one Kraus operator for the total system, $K^1_m(t)\otimes K^2_n(t)$, with the completeness relation $\sum_{m,n=0}^{\infty} K^{1\dagger}_m(t)\otimes K^{2\dagger}_n(t)K^1_m(t)\otimes K^2_n(t) = I_{12}, \forall t$.

Now, with the definition

$$A_1(t)A_2(t) = \sum_{m,n=0}^{\infty} K^{1\dagger}_m(t)A_1(0)K^1_m(t)\otimes K^{2\dagger}_n(t)A_2(0)K^2_n(t) \tag{18}$$

and with the aid of Eq. (15), one easily obtains

$$\lim_{t\to\infty} C_{ij}(t) = \lim_{t\to\infty} \mathrm{e}^{-(\kappa_1+\kappa_2)t}C_{ij}(0) = 0, \quad \forall i,j. \tag{19}$$

The expressions (16) and (19) directly provide the following conclusion: asymptotic state for the two-mode system is tensor-product (cf. Eq. (19)) of the minimal-uncertainty (cf. Eq. (17)) Gaussian states.

3. The alternate degrees of freedom

We introduce the alternate degrees of freedom, X_A and ξ_B, with the respective conjugate momentums, P_A and π_B; $[X_A, P_A] = \mathrm{i}\hbar$, $[\xi_B, \pi_B] = \mathrm{i}\hbar$. The formal subsystems A and B define an alternate structure, $A+B$, for the composite system $C \equiv 1+2$. The total Hilbert state space, $\mathscr{H} = \mathscr{H}_A\otimes\mathscr{H}_B$, while the total Hamiltonian, $H = H_A + H_B + H_{AB}$.

Let us for simplicity consider the linear canonical transformations (LCTs):

$$X_A = \sum_i \alpha_i x_i, \ P_A = \sum_j \gamma_j p_j,$$
$$\xi_B = \sum_m \beta_m x_m, \ \pi_B = \sum_n \delta_n p_n, \tag{20}$$

which give rise to the constraints

$$\sum_i \alpha_i\gamma_i = 1 = \sum_i \beta_i\delta_i, \quad \sum_i \alpha_i\delta_i = 0 = \sum_i \beta_i\gamma_i. \tag{21}$$

In general, the LCTs change the tensor-product factorization of the total-system's Hilbert state space, the form of the Hamiltonian (typically, the subsystems A and B interact to each other) as well as the form of the total system's state.[2,7,9] As to the later, the absence of correlations (quantum or classical) for e.g. the $1+2$ structure, can give rise to some kind of correlations for the $A+B$ structure–the recently observed correlations relativity.[9]

Despite the fact that the Kraus operator $K^1_m\otimes K^2_n$ is composed of the local Kraus operators for the $1+2$ structure, due to Eq. (20), this becomes non-local operation for the $A+B$ structure, $K^1_m\otimes K^2_n \neq K^A_p\otimes K^B_q$. In other words: while there are two independent environments for the 1 and 2 oscillators, the systems A and B share the same environment.

Of course, the composite system's state and the Hamiltonian are unique in every instant in time for every possible structure. Consequently, dynamics of the total system is described by unique master equation for the total state ρ_{12}. However, for the above distinguished reasons, the separation of the dynamics is not in general possible for the alternate subsystems A and B. Deriving master equations for the subsystems A and B from the master equations (1) is as yet largely intact. Rigorously, the Kraus operators for the $A+B$ structure can follow only from the master equation for the $A+B$ structure. Fortunately enough, these Kraus operators are not necessary for our consideration. As emphasized above: dynamics of the total system is uniquely defined by the infinite sum of the form of Eq. (18), not by the particular Kraus operators. Therefore, it suffices to know the dynamics in the terms of one structure, i.e., the $1+2$ structure, Eq. (18). So, with the use of Eq. (15), we can still draw some conclusions regarding the total system's state relative to the $A+B$ structure.

With the use of Eqs. (18) and (20)

$$X_A(t) = \alpha_i x_i(t), \ \xi_B(t) = \beta_i x_i(t),$$
$$P_A(t) = \gamma_i p_i(t), \ \pi_B(t) = \delta_i p_i(t), \tag{22}$$

in Eq. (22), we assume summation over the repeated indices. On the other hand:

$$X^2_A(t) = \alpha_i\alpha_j(x_ix_j)(t), \quad P^2_A(t) = \gamma_i\gamma_j(p_ip_j)(t), \tag{23}$$

where equation (18) gives $(x_ix_j)(t) \equiv \sum_{m,n} K^{1\dagger}_m(t)\otimes K^{2\dagger}_n(t)x_ix_jK^1_m(t)\otimes K^2_n(t)$; and analogously for the subsystem B.

Equations (22) and (23) give rise to the standard deviation

$$\begin{aligned}(\Delta X_A(t))^2 &= \mathrm{tr}_{12}\,\alpha_i\alpha_j(x_ix_j)(t)\rho_{12}(0) - (\mathrm{tr}_{12}\,\alpha_i x_i(t)\rho_{12}(0))^2\\ &= \alpha_i\alpha_j\langle(x_ix_j)(t)\rangle - \alpha_i\alpha_j\langle x_i(t)\rangle\langle x_j(t)\rangle\\ &= \sum_i \alpha_i^2(\Delta x_i(t))^2\\ &\quad + \sum_{i\neq j}\left(\alpha_i\alpha_j\langle(x_ix_j)(t)\rangle - \alpha_i\alpha_j\langle x_i(t)\rangle\langle x_j(t)\rangle\right)\\ &= \sum_i \alpha_i^2(\Delta x_i(t))^2. \end{aligned} \tag{24}$$

The last equality follows from the observation that, for $i\neq j$, the locality of the Kraus operators for the $1+2$ structure, Eq. (18), gives rise to the equality $\langle(x_ix_j)(t)\rangle = \langle x_i(t)\rangle\langle x_j(t)\rangle$; remind: the initial state $\rho_{12}(0) = \rho_1(0)\otimes\rho_2(0)$. In complete analogy, one obtains the standard deviations for P_A, ξ_B, and π_B. Then easily follow the products of the asymptotic standard deviations

$$\begin{aligned}\Delta X_A(\infty)\Delta P_A(\infty) &= \sqrt{\left(\frac{\alpha_1^2\hbar}{2m_1\omega_1} + \frac{\alpha_2^2\hbar}{2m_2\omega_2}\right)\left(\frac{\gamma_1^2 m_1\hbar\omega_1}{2} + \frac{\gamma_2^2 m_2\hbar\omega_2}{2}\right)} \geq \frac{\hbar}{2},\\ \Delta\xi_B(\infty)\Delta\pi_B(\infty) &= \sqrt{\left(\frac{\beta_1^2\hbar}{2m_1\omega_1} + \frac{\beta_2^2\hbar}{2m_2\omega_2}\right)\left(\frac{\delta_1^2 m_1\hbar\omega_1}{2} + \frac{\delta_2^2 m_2\hbar\omega_2}{2}\right)} \geq \frac{\hbar}{2}. \end{aligned} \tag{25}$$

On the other hand, given the more general considerations,[9] one can expect that an alternate structure $A+B$ is described by correlation of their subsystems, A and B. This expectation can here be tested by considering the covariance functions, e.g.

$$C = \langle X_A(t)\xi_B(t)\rangle - \langle X_A(t)\rangle\langle\xi_B(t)\rangle. \tag{26}$$

Substituting Eqs. (15) and (22) into Eq. (26), one finds in the asymptotic limit

$$C = \alpha_i\beta_i(\Delta x_i(\infty))^2 = \alpha_1\beta_1\frac{\hbar}{2m_1\omega_1} + \alpha_2\beta_2\frac{\hbar}{2m_2\omega_2}. \tag{27}$$

4. Analysis and discussion of the results

The model of Section 2 bears some important features. First, we do not assume interaction between the two modes (oscillators). For the coupled modes, even Markovianity of the evolution requires justification.[19,22] Second, we assume the two independent (mutually noninteracting) environments for the two modes. While this simplification is mathematically welcome,[18,19,22] it is physically remarkable: the total environment locally (i.e. mutually independently) influences the two modes (oscillators). This possibility to separate the total environment into two parts locally "monitoring" the two modes (thus giving rise to Eq. (1)) does not apply to the alternate subsystems of the open system C.

The asymptotic quantum state for the $1+2$ structure is "classical": it is tensor-product of a pair of the "coherent" (the minimal-uncertainty Gaussian) states. The amplitude damping is a CP map[17–19] and therefore cannot induce any (quantum or classical) correlations for the input tensor-product states.[23] On the other hand, the "coherent" states are arguably the most classical of all quantum states[16,18,19] (and the references therein). These states are also known to represent the most robust (the "preferred") states of a system described by the master equation Eq. (1).[24] Thereby, (in the asymptotic limit), one can imagine the pair $1+2$ as a pair of "individual", mutually distinguishable and non-correlated systems bearing the (robust) quantum states of their own. In a sense, this is a definition of "classical systems". So, one can say the environment composed of the two noninteracting parts that provide mutually independent (local) amplitude damping processes makes the $1+2$ structure special.

It is worth repeating: the Kraus operators Eq. (3) are nonlocal for any (non-trivial) alternate structure $A+B$. Interestingly enough, a special choice of the parameters can provide the classicality also for an alternate structure. For the resonant oscillators ($\omega_1=\omega_2=\omega$) of equal masses ($m_1=m_2=m$), one obtains equalities in Eq. (25) and $C=0$ in Eq. (27) for the A appearing as the center of mass of the two oscillators ($\alpha_1 = 1/2 = \alpha_2$) and for the B appearing as the "internal" degree of freedom ($\beta_1 = 1 = -\beta_2$). This model, as the simplest one possible for a pair of oscillators, is often analyzed, cf. e.g. Refs. [8], [25], and [26]. However, already for the oscillators of non-equal masses,[22] or of the equal masses but non-resonant,[27] this cannot be obtained. Thereby and therefore, due to the local character of the environmental influence, the classical-like structure $1+2$ is special, i.e., distinguished in the set of the possible structures of the composite system C.

In the more general context, our considerations exhibit usefulness of the simple (or simplified) models: Analytic solutions for the simple models can still may serve for obtaining some information about the more elaborate models, if the two are mutually linked via the proper LCTs. The proper LCTs performed on the model (on the structure) $1+2$ can introduce another pair of harmonic oscillators, A and B. The new oscillators can differ from the original ones in a number of instances. That is not only their respective masses and frequencies may differ, but (as distinct from the original $1+2$) the new oscillators share the same environment and can still be in mutual interaction.[2,10] If the analytic solutions to the Heisenberg equations for the original pair are known, one can easily obtain the analytic solutions to the Heisenberg equations for the "new" oscillators. Therefore the structure- (i.e. the LCTs-) based considerations provide a new method for investigating

the open bipartite systems and their dynamics–just start from a simple model. To this end, the details will be presented elsewhere.

Markovian dynamics is expected to provide the "classical" description of an open system in the asymptotic limit.[28] In this regard, our considerations provide a new lesson: for the model considered, the "classicality" is a matter of a special structure of the composite system C. The special structure is chosen by the composite system's environment, and particularly by the condition of local influence of the total environment on the constituent subsystems.

This classical-like picture changes for the isolated composite systems.[10] On the other hand, the alternate degrees of freedom of the open quantum systems may bear nontrivial physical interest and use. To this end, we refer the reader to the relevant literature[2–12,25,27] and the references therein.

5. Conclusion

It is a phenomenological fact: not all the observables (e.g. the degrees of freedom) of a composite system are equally accessible in a laboratory. With the use of a simple model of an open two-mode system we show why it may be so. Our considerations exhibit that, in the asymptotic limit, there is only one set of the degrees of freedom that exhibits a classical-like description as a consequence of a local environmental influence.

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