Electron capture rates for iron group nuclei at the surface of magnetar*

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Effects of an ultra-strong magnetic field on electron capture rates for ^{55}Co are analyzed in the nuclear shell model and under the Landau energy levels quantized approximation in the ultra-strong magnetic field, and the electron capture rates on 10 abundant iron group nuclei at the surface of a magnetar are given. The results show that electron capture rates on ^{55}Co are increased greatly in the ultra-strong magnetic field, by about 3 orders of magnitude generally. These conclusions play an important role in future study of the evolution of magnetars.

Keywords: electron capture, magnetar

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1. Introduction

Magnetars are neutron stars with ultra-strong magnetic fields, their general magnetic fields are in a range from 10^{13} Gs (1 Gs = 10^{-4} T) to 10^{15} Gs, which provides an ideal lab for all kinds of nuclear physics processes. Iron group nuclei abound in pre-supernova and neutron stars and on the surfaces of magnetars. Their electron captures play an important role in studying supernova mechanisms and the evolution of a magnetic field. In the core collapse process of Type II supernova and Type I b supernova, they absorb a large number of electrons and reduce the electron fraction of stellar interior, at the same time, they release a large number of neutrinos and take away the energy, and thus make the temperature of the stellar interior rapidly drop off. With the reduction of electronic number, radiation pressure of the stellar interior will be gradually weakened in the balance game with gravity, and thus quicken the core collapse of the stellar interior.[1]

For many years, electron capture on iron group nuclei has been extensively investigated. Aufderheide *et al.*[2] thought that ^{55}Co is one of the top ten nuclei, where the electron capture can affect supernova explosion. Later Hege *et al.*[3] pointed out that ^{55}Co can consume a large number of electrons and thus reduce internal electronic degeneracy pressure, which plays an important role in massive stars (25 $M_\odot$).[3] Electron captures on ^{55}Co in the supernova and presupernova have been calculated and discussed by Nab and Rahman[4] using proton–neutron quasi-particle random phase approximation (pn-QRPA) theory[4] and expanded pn-QRPA theory,[5] but they have not been related to the influence of an ultra-strong magnetic field on electron capture. Magnetic fields of most stars in the universe are all below 10^{12} Gs. A large number of studies show that the magnetic field below 10^{12} Gs has a tiny effect on electron capture, but for some magnetars ($> 10^{13}$ Gs), an ultra-strong magnetic field enhanced the homogeneity of electrons in momentum space distribution. Electrons are not uniformly distributed in the Fermi ball, but distributed in a group of cylinders in parallel with the magnetic field, therefore the orbital classical description of the free electron will no longer hold true; in this case the quantum effect must be considered. Peng and Tong,[6] Luo *et al.*,[7] and Dai *et al.*[8] have extensively investigated the electron capture in the strong magnetic field. The early research results indicated that an ultra-strong magnetic field has a great influence on electron capture: electron capture rate decreases significantly with the increasing of magnetic field. Recent studies[9,10] showed that in the ultra-strong magnetic field ($B \gg B_{\rm cr} = 4.414 \times 10^{13}$ Gs) the Fermi surface is elongated along the magnetic field direction from Fermi spherical into Landau cylindrical shape, and the level is perpendicular to the direction of magnetic field and quantized. At the time, the non-relativistic Landau energy level theory was revised.[11] Then, we found that the electron capture in the ultra-strong magnetic field did not decrease with the increase of magnetic field, but presented a trend of increase. The conclusion will affect the electron capture and β decay in the core of stars. In view of the importance of the iron group nuclide, the influence of an ultra-strong magnetic field on electron capture for iron group nuclei is analyzed in this paper.

2. Electron chemical potential and electron capture rate in an ultra-strong magnetic field

When the energy and momentum are $m_{\rm e}c^2$ and $m_{\rm e}c$ separately, we can obtain the Landau energy levels by solving the Dirac equation in an ultra-strong magnetic field,[11]

$$\varepsilon^2 = 1 + p_z^2 + (2n + 1 + \sigma)b, \tag{1}$$

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$$p_\perp = \sqrt{(2n+1+\sigma)b}, \tag{2}$$

where ε is the electron energy, p_z is the momentum along the direction of the applied magnetic field, $p_\perp$ is the momentum perpendicular to the direction of the applied magnetic field, n is the quantum number for Landau energy levels, $b = B/B_{\rm cr}$, and $\sigma = \pm 1$. The number of the electron gas is given by[12–15]

$$n_{\rm e} = \frac{\rho}{\mu_{\rm e}} = \frac{1}{4\pi^2 N_{\rm A}} \left(\frac{m_{\rm e}c}{\hbar}\right)^3 \sum_{n=0}^{\infty} p_\perp^2 \int_0^\infty (f_{-{\rm e}} - f_{+{\rm e}})] \, {\rm d}p_z, \tag{3}$$

where ρ is the mass density in units of g/cm^3, $\mu_{\rm e}$ is the average molecular weight of electron, $N_{\rm A}$ is the Avogadro constant, $f_{-{\rm e}}$ and $f_{+{\rm e}}$ are the electron and positron Fermi–Dirac distribution functions, respectively, and given by

$$f_{-{\rm e}} = \left[1 + \exp\left(\frac{\varepsilon_n - u_{\rm F} - 1}{KT}\right)\right]^{-1}$$

and

$$f_{+{\rm e}} = \left[1 + \exp\left(\frac{\varepsilon_n + u_{\rm F} + 1}{KT}\right)\right]^{-1},$$

where $u_{\rm F}$ is the electron chemical potential.

In the following, we introduce the calculation of electron capture rate briefly. Electron capture rate of a nucleus from initial state i to all final states f is given by[10]

$$\lambda_i = \sum_f \lambda_{if} = \sum_f \ln 2 B_{if} f(Q_{if}), \tag{4}$$

where $B_{if} = 10^{-3.596} M_{if}^2$ (M_{if} is the Gamow–Teller transition matrix element); $f(Q_{if})$ is the phase space factor; Q_{if} is the threshold energy of the electron capture reaction and can be given by $Q_{if} = Q_{00} + E_i - E_j$, with E_i and E_j being the excitation energies of the i-th state and f-th state, respectively, and $Q_{00} = M_{\rm p}C^2 - M_{\rm d}C^2$ with $M_{\rm d}$ and $M_{\rm p}$ being the mass of the daughter nucleus and the mass of the parent nucleus, respectively; $f(Q_{if})$ can be expressed as[12–14,16]

$$f(Q_{if}) = \frac{1}{4} \sum_{n=0}^{\infty} p_\perp^2 \int_0^\infty (Q_{if} + \varepsilon)^2 F(Z, \varepsilon) f_{-{\rm e}} \, {\rm d}p_z, \tag{5}$$

where $F(Z, \varepsilon)$ is the Coulomb wave correction; in this study we use the Schenter–Vogel approximation.[17] Taking into account all transitions from different initial states and the Brink hypothesis, the total electron capture rate of a nucleus is given by[10]

$$\lambda = \lambda_0 + \lambda_{\rm GT} = \ln 2 B_{\rm eff} f(Q_{00}) + \ln 2 B_{\rm GT} f(Q_{00} - E_{\rm GT}), \tag{6}$$

where $B_{\rm eff} = 1.65 \times 10^{-5}$, $B_{\rm GT} = 10^{-3.596} M_{\rm GT}^2$, $E_{\rm GT}$ is the resonance position and defined as[12–14]

$${\rm d}E_{\rm GT} = \Delta E_1 + \Delta E_2 + \Delta E_3, \tag{7}$$

where ΔE_1 is the energy difference between orbitals which the new neutron occupies in the GT resonance state and the ground state (estimated by using single particle energies), ΔE_2 is the particle–hole repulsion energy which must be supplied to pull out of the daughter ground state, ΔE_3 is the cost to break a neutron pair if there is an even number of neutrons in the daughter nucleus. $M_{\rm GT}$ is the GT transition matrix element at the resonance point. It can be expressed as[2]

$$M_{\rm GT}^2 = \frac{1}{2} \frac{n_i^{\rm p} n_f^{\rm h}}{2j_f + 1} \left| M_{\rm GT}^{\rm sp} \right|_{if}^2, \tag{8}$$

where $n_{\rm p}^i$ is the number of particles of the parent nucleus in the initial orbit, $n_{\rm h}^f$ is the single particle GT transition matrix element squared from the initial state to final state.

3. Analysis and discussion

Figures 1 and 2 show respectively changes of electron capture rates for ^{55}Co with magnetic field at different temperature-density points. It can be seen from the two figures that electron capture rates increase obviously with the increasing of magnetic field. When magnetic field strength is less than 10^{13} Gs, the changes of electron capture rates are all small, and almost negligible; when magnetic field strength is greater than 10^{13} Gs, the rates increase obviously, about 4 orders of magnitude.

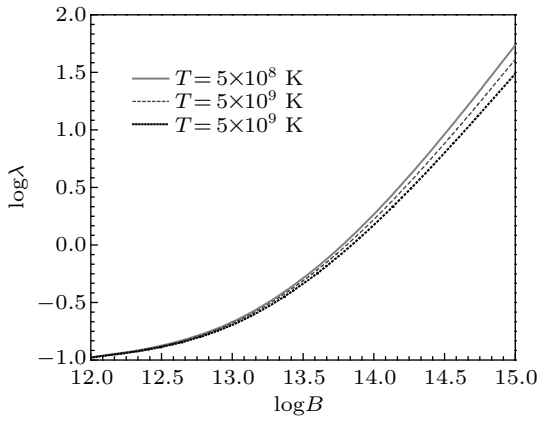


Fig. 1. Electron capture rates for ^{55}Co in an ultra-strong magnetic field ($\rho/\mu_{\rm e} = 5 \times 10^7$ mol/cm^3 and $T = 5 \times 10^8$, 1×10^9, 5×10^9 K.

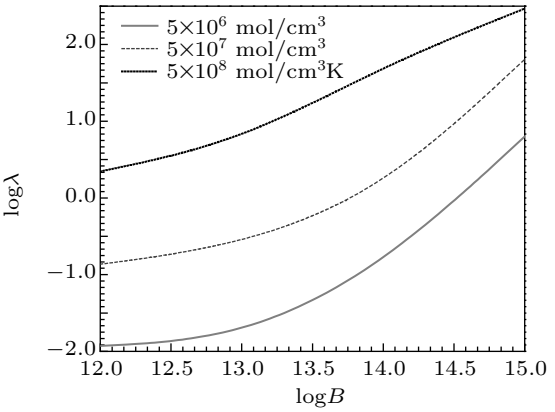


Fig. 2. Electron capture rates for ^{55}Co in an ultra-strong magnetic field ($T = 5 \times 10^8$ K and $\rho/\mu_{\rm e} = 5 \times 10^6$, 5×10^7, 5×10^8 mol/cm^3).

Figures 3 and 4 show respectively changes of electron capture rates for ^{55}Co with temperature in the cases of different densities under the condition of no magnetic field or $B = 4.414 \times 10^{15}$ Gs. It can be seen from the two figures that at the same temperature, the higher the density, the higher its electron capture rate is. This is because the density makes the electronic number increase, and probability of electron capture is increased, then electron capture rate is increased correspondingly. Under the condition of no magnetic field, electron capture rate for ^{55}Co increases with the increasing of temperature, and decreases with the increasing of temperature under the condition of $B = 4.414 \times 10^{15}$ Gs. An ultra-strong magnetic field has an obvious effect on electron capture rate, and makes it increase 1–4 orders of magnitude. This is because electronic Fermi energy increases with the increasing of magnetic field, the number of electrons whose energy is greater than the capture threshold Q value will increase, and electron capture rate increases correspondingly.

Table 1 shows the changes of electron capture rates for ten abundant nuclei with ultra-strong magnetic field at typical temperature-density point. It can be seen from the table that an ultra-strong magnetic field has a great effect on electron capture rates for iron group nuclei, and makes them increase 3 orders of magnitude and more.

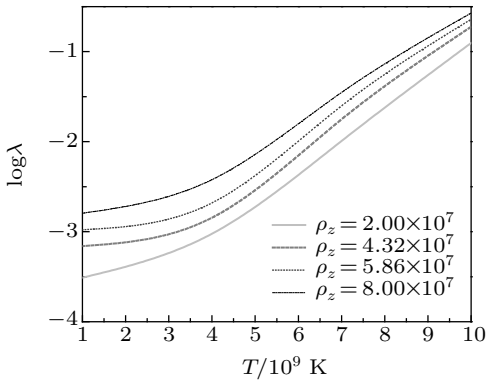


Fig. 3. Electron capture rates for ^{55}Co under no magnetic field ($T = (1 \sim 10) \times 10^9$ K, $\rho/\mu_e = (2 \sim 8) \times 10^7$ mol/cm^3).

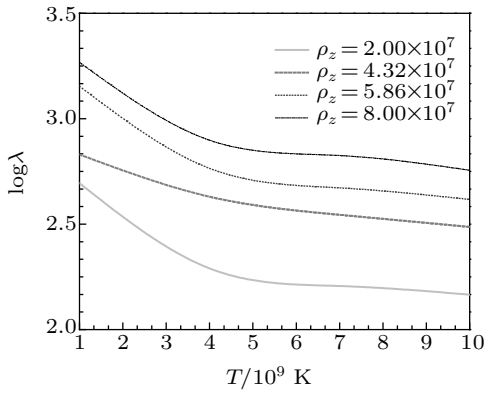


Fig. 4. Electron capture rates for ^{55}Co ($T = (1 \sim 10) \times 10^9$ K, $\rho/\mu_e = (2 \sim 8) \times 10^7$ mol/cm^3, $B = 4.414 \times 10^{15}$ Gs).

Table 1. Changes of electron capture rates for ten nuclei with magnetic field ($\rho = 5.86 \times 10^7$ g/cm^3, Y_e=0.47, $T = 3.4 \times 10^9$ K).

B	^{51}Cr	^{52}Mn	^{53}Mn	^{54}Mn	^{53}Fe	^{54}Fe	^{57}Fe	^{55}Co	^{57}Co	^{56}Ni
4.414×10^{13}	0.64019	1.07276	0.66993	0.31008	1.69545	0.23246	0.18014	0.13072	0.53191	0.13891
8.828×10^{13}	0.48975	0.88048	0.51986	0.22383	1.42841	0.16512	0.26559	0.19539	0.39785	0.10623
2.207×10^{14}	0.28747	0.60520	0.31467	0.12671	1.03407	0.10459	0.46983	0.36427	0.22645	0.10573
4.414×10^{14}	0.17043	0.45024	0.19640	0.18783	0.81438	0.06252	0.57214	0.44692	0.12420	0.09584
8.828×10^{14}	0.08449	0.19643	0.06571	0.25135	0.35639	0.41081	2.09119	1.79024	0.12849	0.66726
2.207×10^{15}	2.88244	3.33659	2.37395	3.74360	1.15259	4.61886	9.61357	8.85732	3.18686	5.45344
4.414×10^{15}	13.1739	9.67329	11.2341	14.2747	7.99446	16.3334	25.5981	24.1368	13.2687	17.4315
8.828×10^{15}	42.7233	33.4671	36.6415	42.2649	30.0301	46.1626	61.6946	58.4642	40.1233	45.8740

4. Conclusions

Electron capture rates of stellar interior in an ultra-strong magnetic field are restudied. It can be seen that in an ultra-strong magnetic field on the surface of a magnetar, electron capture rate increases more obviously than in no magnetic field, by $3 \sim 5$ orders of magnitude. In view of importance of electron capture rates in the evolution process of compact stars including neutron stars, an ultra-strong magnetic field is the most important factor in the study of internal physical processes and evolution processes of magnetars, it has important significance in studying the internal physics process of neutron stars, supernova explosion mechanism, cooling process of neutron stars, evolution process of magnetars, etc. Further research will be carried out in the future.

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