Switching effects in superconductor/ferromagnet/superconductor graphene junctions*

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The Josephson effect in the superconductor/ferromagnet/superconductor (SFS) graphene Josephson junction is studied using the Dirac Bogoliubov-de Gennes (DBdG) formalism. It is shown that the SFS graphene junction drives 0–π transition with the increasing of $p = h_0 L/v_F\hbar$, which captures the effects of both the exchange field and the length of the junction; the spin-down current is dominant. The 0 state is stable for $p < p_c$ (critical value $p_c \approx 0.80$) and the π state is stable for $p > p_c$, where the free energy minima are at $\phi_g = 0$ and $\phi_g = \pi$, respectively. The coexistence of the 0 and π states appears in the vicinity of p_c.

Keywords: superconductor junction, grapheme, switching effect

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1. Introduction

The hybrid structures, consisting of a superconductor and a ferromagnet, have attracted considerable attention, as their devices have some novel functionalities.[1] One of most interesting effects is the formation of π states in superconductor/ferromagnet /superconductor (SFS) Josephson junctions.[2–4] At the interface between the superconductor and the ferromagnet, the Cooper pair penetrating into the ferromagnet has a finite momentum $Q \propto h_0/v_F$, where v_F is the Fermi velocity, because of the exchange splitting h_0 between the up-spin and the down-spin bands.[5] In SFS junctions, the order parameters in the two superconductors present different signs, when the thickness of the ferromagnet is about half of the period of the oscillation. This causes the current–phase relation to be shifted by π with respect to that of a normal Josephson junction. It is the so-called π junction.[5–12] Several experimental observations of the π state have been reported.[9–12]

Recently, there have been several experimental reports on proximity-induced superconductivity in graphene.[13,14] A measurable supercurrent was observed between regions of graphene under the influence of proximity-induced superconductivity. For graphene, the charge carriers are usually spin unpolarized. However, if graphene is growing on a ferromagnetic (FM) material[15–20] or is under an external magnetic field,[21] a spin split can be induced. Then the carriers are spin polarized and the Dirac points with different spins are split. The SFS graphene π junctions, in which a π-phase shift happens due to the ferromagnetic graphene induced by the proximity effect, were studied theoretically.[16,22–24] The oscillating critical current in the SFS graphene junction decays with a strong oscillatory dependence on both the exchange field and the junction width. However, the coexistence of the 0 and π states, and the free energies in the SFS graphene junction have not been studied so far.

In this paper, we study the Josephson effects in SFS graphene junctions at a finite temperature. Using the Dirac Bogoliubov-de Gennes (DBdG) formalism,[25] we calculate the Andreev levels, the Josephson currents, and the free energies. We also numerically analyze the quantities for the different values of p, which captures the effects of both the exchange field and the length of the junction, at different values of temperature T. The SFS graphene junctions drive 0–π transitions with the increasing of p. A coexistence of the 0 and π states may appear.

The rest of this paper is organized as follows. In the next section, we briefly introduce the theoretical approach that we use to obtain the Andreev levels dispersion, the Josephson currents, and the free energies. Then in Section 3, we report on our results of the numerical calculations. Finally in Section 4, we give a brief summary.

2. Theory and model

We consider an SFS graphene junction, as shown in Fig. 1, with region superconducting graphene (SG) in $x \leq -L$, region ferromagnet graphene (FG) in $-L \leq x \leq 0$, and re-

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gion superconducting graphene (SG) in $x \geq 0$. The propagation of quasiparticles is described by the Dirac Bogoliubov–de Gennes (DBdG) equation.[25] The SFS graphene junctions can be described by the DBdG equation in the presence of an exchange field h_0[16]

$$\begin{pmatrix} H_0 - \rho_\sigma h_0 & \rho_\sigma \Delta(T) \\ \rho_\sigma \Delta^*(T) & -H_0 - \rho_\sigma h_0 \end{pmatrix} \begin{pmatrix} u_\sigma \\ v_{\bar\sigma} \end{pmatrix} = \varepsilon_\sigma \begin{pmatrix} u_\sigma \\ v_{\bar\sigma} \end{pmatrix} \tag{1}$$

with

$$H_0 = -\mathrm{i}\hbar v_\mathrm{F}(\hat\sigma_x \partial_x + \hat\sigma_y \partial_y) - \mu_{\mathrm{S(F)}}. \tag{2}$$

Here, H_0 is the single particle Hamiltonian, v_F represents the Fermi velocity with $v_\mathrm{F} = 10^6$ m/s, $\hat\sigma_x$ and $\hat\sigma_y$ are Pauli matrices in the pseudospin space of the sublattices, and $\mu_{\mathrm{S(F)}}$ is the Fermi energy; subscript $\sigma(\bar\sigma)$ refers to the real spin. If σ denotes up (down) spin and $\bar\sigma$ denotes down (up) spin $\rho_\uparrow$ is 1 and $\rho_\downarrow$ is -1. $(u_\sigma, v_{\bar\sigma}) \equiv \psi_\sigma$ is the energy eigenstate in the electron–hole space associated with the eigenvalue ε_σ. $\Delta(T)$ is set as the standard BCS temperature dependence.

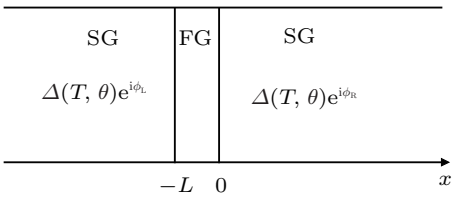


Fig. 1. Schematic illustration of an SFS graphene junction. The length of FG is L.

By solving the DBdG equation, we obtain the wave functions in each region of the model graphene SFS junction. For $x < -L$, we have

$$\begin{aligned} \Psi_{\mathrm{S,L}\sigma} &= r^e_{\mathrm{L}\sigma} \begin{pmatrix} u_\sigma \\ u_\sigma e^{\mathrm{i}(\pi-\theta_{\mathrm{e}\sigma})} \\ v_\sigma e^{-\mathrm{i}\varphi_\mathrm{L}} \\ v_\sigma e^{\mathrm{i}(\pi-\theta_{\mathrm{e}\sigma}-\varphi_\mathrm{L})} \end{pmatrix} e^{-\mathrm{i}q_\mathrm{e}\cos\theta_{\mathrm{e}\sigma}x} \\ &\quad + r^\mathrm{h}_{\mathrm{L}\bar\sigma} \begin{pmatrix} v_{\bar\sigma} \\ v_{\bar\sigma} e^{\mathrm{i}\theta_{\mathrm{h}\bar\sigma}} \\ u_{\bar\sigma} e^{-\mathrm{i}\varphi_\mathrm{L}} \\ u_{\bar\sigma} e^{\mathrm{i}(\theta_{\mathrm{h}\bar\sigma}-\varphi_\mathrm{L})} \end{pmatrix} e^{\mathrm{i}q_\mathrm{h}\cos\theta_{\mathrm{h}\bar\sigma}x}. \end{aligned} \tag{3}$$

For $-L < x < 0$

$$\begin{aligned} \Psi_{\mathrm{F}\sigma} &= a_\sigma \begin{pmatrix} 1 \\ e^{\mathrm{i}\theta_\sigma} \\ 0 \\ 0 \end{pmatrix} e^{\mathrm{i}p_{\mathrm{e}\sigma}\cos\theta_\sigma x} \\ &\quad + b_\sigma \begin{pmatrix} 1 \\ -e^{-\mathrm{i}\theta_\sigma} \\ 0 \\ 0 \end{pmatrix} e^{-\mathrm{i}p_{\mathrm{e}\sigma}\cos\theta_\sigma x} \\ &\quad + c_{\bar\sigma} \begin{pmatrix} 0 \\ 0 \\ 1 \\ e^{-\mathrm{i}\theta_A\bar\sigma} \end{pmatrix} e^{-\mathrm{i}p_{\mathrm{h}\bar\sigma}\cos\theta_{A\bar\sigma}x} \\ &\quad + d_{\bar\sigma} \begin{pmatrix} 0 \\ 0 \\ 1 \\ -e^{\mathrm{i}\theta_{A\bar\sigma}} \end{pmatrix} e^{\mathrm{i}p_{\mathrm{h}\bar\sigma}\cos\theta_{A\bar\sigma}x}. \end{aligned} \tag{4}$$

For $x > 0$

$$\begin{aligned} \Psi_{\mathrm{S,R}\sigma} &= r^e_{\mathrm{R}\sigma} \begin{pmatrix} u_\sigma \\ u_\sigma e^{\mathrm{i}\theta_{\mathrm{e}\sigma}} \\ v_\sigma e^{-\mathrm{i}\varphi_\mathrm{R}} \\ v_\sigma e^{\mathrm{i}(\theta_{\mathrm{e}\sigma}-\varphi_\mathrm{R})} \end{pmatrix} e^{\mathrm{i}q_\mathrm{e}\cos\theta_{\mathrm{e}\sigma}x} \\ &\quad + r^\mathrm{h}_{\mathrm{R}\bar\sigma} \begin{pmatrix} v_{\bar\sigma} \\ v_{\bar\sigma} e^{\mathrm{i}(\pi-\theta_{\mathrm{h}\bar\sigma})} \\ u_{\bar\sigma} e^{-\mathrm{i}\varphi_\mathrm{R}} \\ u_{\bar\sigma} e^{\mathrm{i}(\pi-\theta_{\mathrm{h}\bar\sigma}-\varphi_\mathrm{R})} \end{pmatrix} e^{-\mathrm{i}q_\mathrm{h}\cos\theta_{\mathrm{h}\bar\sigma}x}, \end{aligned} \tag{5}$$

where

$$u_\sigma = \sqrt{\frac{1}{2}\left(1 + \frac{\sqrt{\varepsilon_\sigma^2 - |\Delta(T)|^2}}{\varepsilon_\sigma}\right)},$$

$$v_\sigma = \sqrt{\frac{1}{2}\left(1 - \frac{\sqrt{\varepsilon_\sigma^2 - |\Delta(T)|^2}}{\varepsilon_\sigma}\right)},$$

$$p_{\mathrm{e}\sigma} = (\varepsilon + \mu_\mathrm{F} + \rho_\sigma h_0)/\hbar v_\mathrm{F},$$

$$p_{\mathrm{h}\sigma} = (\varepsilon - \mu_\mathrm{F} + \rho_\sigma h_0)/\hbar v_\mathrm{F},$$

$$p_{\mathrm{h}\bar\sigma}\sin\theta_{A\bar\sigma} = p_{\mathrm{e}\sigma}\sin\theta_\sigma, \quad q_{\mathrm{e,h}} = \mu_\mathrm{s}/\hbar v_\mathrm{F},$$

$$q_i \sin\theta_{i\sigma} = p_{\mathrm{e}\sigma}\sin\theta_\sigma, \quad i = \mathrm{e, h}, \quad \phi = \phi_\mathrm{L} - \phi_\mathrm{R}.$$

The boundary conditions at the interfaces are given by[25]

$$\Psi_{\mathrm{F}\sigma}(x = -L) = \Psi_{\mathrm{S,L}\sigma}(x = -L), \tag{6}$$

$$\Psi_{\mathrm{S,R}\sigma}(x = 0) = \Psi_{\mathrm{F}\sigma}(x = 0). \tag{7}$$

The strategy for calculating the Josephson currents and free energies in the SFS graphene junction is first to obtain the energy spectrum for the Andreev-bound state in the FG region, which is done by matching the wave functions given by Eqs. (3)–(5) with the boundary conditions described by Eqs. (6) and (7) at the two S/F interfaces, and then to solve it for obtaining the allowed eigenstates of energy ε_σ with nonzero solution. The general expression for the Josephson current can then be written as[16]

$$I(\phi) = \frac{2e}{\hbar}\sum_\sigma \int_{-\pi/2}^{\pi/2} \mathrm{d}\theta \cos\theta \frac{\mathrm{d}\varepsilon_\sigma(\phi)}{\mathrm{d}\phi} f(\varepsilon_\sigma/k_\mathrm{B}T), \tag{8}$$

where $f(x) = 1/(e^{x/k_\mathrm{B}T} + 1)$ is the Fermi–Dirac distribution function and k_B is the Boltzmann constant. The free energy is given as[26]

$$F(\phi) = -kT \sum_{\sigma=\uparrow\downarrow} \int_{-\pi/2}^{\pi/2} \mathrm{d}\theta \cos\theta \ln\{2\cosh[\varepsilon_\sigma(\phi)/2k_\mathrm{B}T]\}. \tag{9}$$

The minimum of the SFS graphene junction free energy may determine the ground-state phase difference ϕ_g. The Andreev level in an SFS graphene junction is in the form of

$$\varepsilon_\sigma = \Delta_0 \sqrt{\frac{X + \rho_\sigma \sqrt{X^2 - 4Y[A\cos^2(\phi/2) - B]^2}}{2Y}}. \quad (10)$$

Here, $p = h_0 L/v_F\hbar$ captures the effects of both the exchange field h_0 and the length L of the junction,

$$Y = 4\sin^2\theta \sin^2(p\cos\theta) + \cos^4\theta,$$
$$X = Y - [2\sin^2(p\cos\theta) - \cos^2\theta]\cos^2\theta\cos\phi,$$
$$A = \cos^2\theta, \text{ and } B = \sin^2(p\cos\theta).$$

3. Numerical results

We present the values of the energy of Andreev state ε_σ and the Josephson currents, each as a function of phase difference ϕ for different values of $p = h_0 L/v_F\hbar$ with $T/T_c = 0.2$. As seen in Figs. 2(a), 2(c), and 2(e), with the increasing of p, the order relations between the spin-up energy $\varepsilon_\uparrow$ and the spin-down energy $\varepsilon_\downarrow$ do not vary, where $\varepsilon_\uparrow > \varepsilon_\downarrow$. The oscillatory period of the Andreev state energy is 2π in the cases of $p = 0.2$ and $p = 1.8$, but the period is π in the case of $p = 0.8$. For the case of $p = 0.2$, the spin-down current exhibits a maximum at $0 < \varphi < \pi$ and for the spin-up case, the maximum appears at $\pi < \varphi < 2\pi$ [see Fig. 2(b)]. The spin-down current is in a 0 phase while the spin-up current is in a π phase. The Andreev energy levels for spin-up and spin-down quasiparticles in an SFS graphene junction depend on the direction of spin; the spin-down current is dominant, as seen in Figs. 2(b), 2(d), and 2(f). The Josephson current exhibits a maximum at $0 < \varphi < \pi$, which is in a 0 phase. As $p = 1.8$, the Josephson current flows in the opposite direction and exhibits a maximum at $\pi < \varphi < 2\pi$, which is in a π phase. The SFS graphene junction drives the 0–π transition with the increasing of p. This means that an efficient supercurrent switch may be realized by tuning the exchange field.

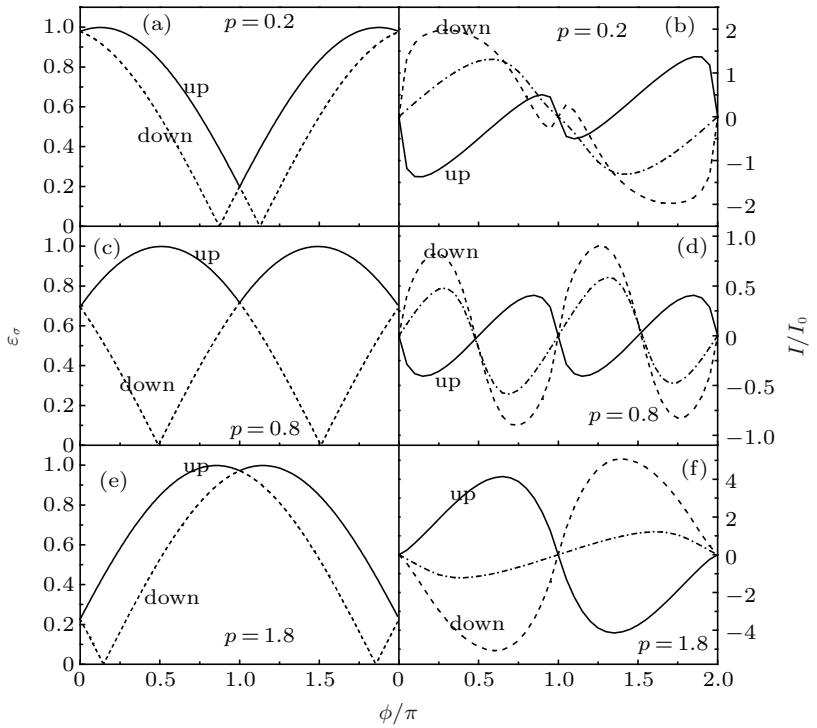


Fig. 2. Energies of the Andreev states and the Josephson currents each as a function of the phase difference for different spin directions. A spin-up case is denoted by the solid line, a spin-down case by the dash line, and a spin-up +spin-down case by the dash–dot line. Here, $T/T_c = 0.2$.

Figure 3 shows the plots of the Josephson current I and the free energy F versus the phase difference φ for different values of T and p. At low temperature $T/T_c = 0.1$, the Andreev reflection is dominant and deviations from the sinusoidal $I(\phi)$ law are pronounced. At high temperature $T/T_c = 0.8$, the current–phase relation is quite close to $I(\phi) = \pm I_c \sin\phi$, where $\pm$ signs correspond to 0 and π junctions, respectively. The oscillation amplitude I_c in $I(\phi)$ decreases as p increases from 0.2 to the critical value $p_c \approx 0.80$. For $p > p_c$, the supercurrent flows in the opposite direction in Figs. 3(a) and 3(c), and I_c starts to increase. The free energy minimum of the ground-state is at $\varphi_g = 0$ for small value of p, where 0 state is stable, however the free energy minimum is at $\varphi_g = \pi$ for large value of p, where π state is stable. This is the signature of the π junction with a ground-state phase difference $\varphi_g = \pi$, as seen in Figs. 3(b) and 3(d). The multinode oscillations in $I(\phi)$ appear in the vicinity of p_c, where 0 and π states are equally stable, so the 0 and π states may coexist.

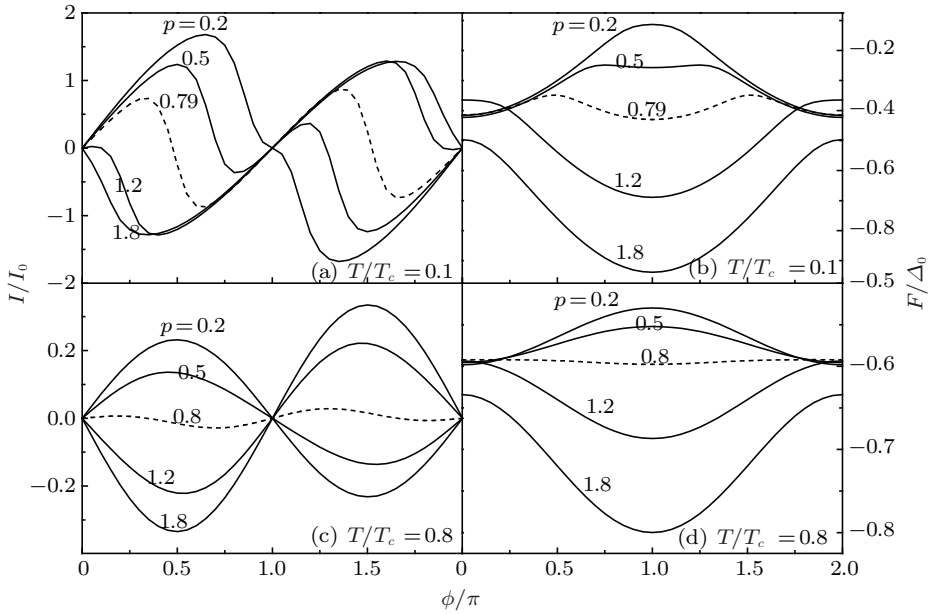


Fig. 3. Plots of Josephson current and free energy versus phase difference for different values of p at $T/T_c = 0.1$ (panels (a) and (b)) and $p_c = 0.79$, and $T/T_c = 0.80$ (panels (c) and (d)) and $p_c = 0.8$.

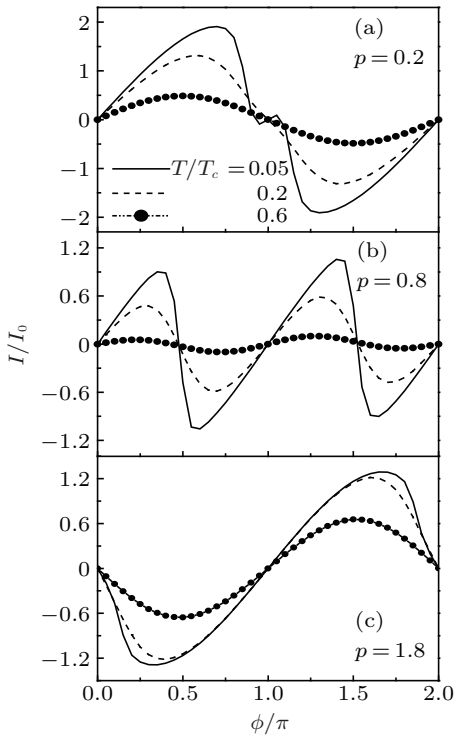


Fig. 4. Plots of Josephson current versus phase difference for different temperatures: $T/T_c = 0.05$ (solid line), 0.2 (dash line), and 0.6 (dash–dot line).

The current–phase relationships for different temperatures are shown in Fig. 4. The Josephson current is a decaying function of the temperature, but finally the sign of the current is not changed. The SFS graphene junction does not undergo 0–π transition with the increasing of the temperature.

4. Summary

In this paper, we investigated the Josephson currents and free energies in SFS graphene junctions. The SFS graphene junction drives 0–π transition with the increasing of p; the spin-down current is dominant. The free energy minimum of the ground-state is at $\varphi_g = 0$ for the small value of p, where the 0 state is stable; however, the free energy minimum is at $\varphi_g = \pi$ for the large value of p, where the π state is stable. The 0 and π states may coexist in the vicinity of p_c. The exchange splitting could be adjusted by means of some external source, such as the gate voltage or external electrical fields. These results imply that the supercurrent across the junction may be tuned in a controllable manner. These properties can provide new clues for the supercurrent switching.

References

[1] Buzdin A I 2005 *Rev. Mod. Phys.* **77** 935
[2] Bulaevskii L N, Kuzii V V and Sobyanin A A 1977 *JETP Lett.* **25** 290
[3] Buzdin A I, Bulaevskii L N and Panyukov V 1982 *JETP Lett.* **35** 178
[4] Radovic Z, Lazarides N and Flytzanis N 2003 *Phys. Rev. B* **68** 014501
[5] Demler E A, Arnold G B and Beasley M R 1997 *Phys. Rev. B* **55** 15174
[6] Xu M J and Dou X M 2010 *Chin. Phys. B* **19** 067301
[7] Yu H L, Wang Z G and Pong J 2008 *Chin. Phys. B* **17** 4627
[8] Li X W 2007 *Chin. Phys.* **16** 3514
[9] Ryazanov V V, Oboznov V A, Rusanov A Yu, Veretennikov A V, Golubov A A and Aarts J 2001 *Phys. Rev. Lett.* **86** 2427
[10] Kontos T, Aprili M, Lesueur J, Genêt F, Stephanidis B and Boursier R 2002 *Phys. Rev. Lett.* **89** 137007
[11] Frolov S M, Van H D J, Oboznov V A, Bolginov V V and Ryazanov V V 2004 *Phys. Rev. B* **70** 144505
[12] Bauer A, Bentner J, Aprili M, Della R M L, Reinwald M, Wegscheider W and Strunk C 2004 *Phys. Rev. Lett.* **92** 217001

[13] Heersche H B, Jarillo-Herrero P, Oostinga J B, Vandersypen L M K and Morpurgo A F 2007 *Nature* **446** 56
[14] Du X, Skachko I and Andrei E Y 2008 *Phys. Rev. B* **77** 184507
[15] Haugen H, Huertas-Hernando D and Brataas A 2008 *Phys. Rev. B* **77** 115406
[16] Linder J, Yokoyama T, Huertas-Hernando D and Sudbo A 2008 *Phys. Rev. Lett.* **100** 187004
[17] Zhang Q, Fu D, Wang B, Zhang R and Xing D Y 2008 *Phys. Rev. Lett.* **101** 047005
[18] Son Y W, Cohen M L and Louie S G 2006 *Nature* **444** 347
[19] Kan E J, Li Z, Yang J and Hou J G 2007 *Appl. Phys. Lett.* **91** 243116
[20] Tian H Y and Wang J 2012 *Chin. Phys. B* **21** 017203
[21] Abanin D A, Morozov S V, Ponomarenko L A, Gorbachev R V, Mayorov A S, Katsnelson M I, Watanabe K, Taniguchi T, Novoselov K S, Levitov L S and Geim A K 2011 *Science* **332** 328
[22] Moghaddam A G and Zareyan M 2008 *Phys. Rev. B* **78** 115413
[23] Soodchomshom B, Tang I M and Hoonsawat R 2008 *Physica C* **468** 2361
[24] Liang Q F, Yu Y, Wang Q H and Dong J M 2008 *Phys. Rev. Lett.* **101** 187002
[25] Beenakker C W J 2006 *Phys. Rev. Lett.* **97** 067007
[26] Annunziata G, Enoksen H, Linder J, Cuoco M, Noce C and Sudbø A 2011 *Phys. Rev. B* **83** 144520