Effects of transpiration on unsteady MHD flow of an upper convected Maxwell (UCM) fluid passing through a stretching surface in the presence of a first order chemical reaction*

Swati Mukhopadhyay[a)†], M. Golam Arif[b)], and M. Wazed Ali Pk[c)]

[a)] *Department of Mathematics, The University of Burdwan, Burdwan-713104, W.B., India*
[b)] *Institute of Business Administration, University of Rajshahi, Rajshahi-6205, Bangladesh*
[c)] *Department of Mathematics, University of Rajshahi, Rajshahi-6205, Bangladesh*

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The aim of this article is to present the effects of transpiration on the unsteady two-dimensional boundary layer flow of non-Newtonian fluid passing through a stretching sheet in the presence of a first order constructive/destructive chemical reaction. The upper-convected Maxwell (UCM) model is used here to characterize the non-Newtonian behavior of the fluid. Using similarity solutions, the governing nonlinear partial differential equations are transformed into ordinary ones and are then solved numerically by the shooting method. The flow fields and mass transfer are significantly influenced by the governing parameters. The fluid velocity initially decreases as the unsteadiness parameter increases and the concentration decreases significantly due to the increase in the unsteadiness. The effect of increasing values of transpiration (suction) and the Maxwell parameter is to suppress the velocity field; however, the concentration is enhanced as transpiration (suction) and the Maxwell parameter increase. Also, it is found that the fluid velocity decreases as the magnetic parameter increases; however, the concentration increases in this case.

Keywords: unsteady flow, MHD, upper convected Maxwell fluid, stretching surface transpiration, chemical reaction

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1. Introduction

The flow of a viscous fluid passing through a stretching sheet is a classical problem in fluid dynamics. In recent years, the laminar boundary layer flow induced by a stretching surface has attracted the attention of a large variety of researchers; as such, a flow analysis plays an important role in the boundary layer theory of Newtonian and non-Newtonian fluids and considerable interest has been evinced in the study of steady flows of a viscous incompressible fluid driven by a linearly stretching surface through a quiescent fluid. Many chemical engineering processes, like metallurgical process and the polymer extrusion process, involve the cooling of a molten liquid being stretched into a cooling system. In such processes, the fluid mechanical properties of the penultimate product would mainly depend on two things: the cooling liquid used and the rate of stretching. Some of the polymer fluids, such as polyethylene oxide and a polyisobutylene solution in ethane, having better electromagnetic properties, are recommended as their flow can be regulated by external magnetic fields.[1] Applications of stretching sheets can be found in areas such as paper production, hot rolling, glass blowing, continuous casting of metals, wire drawing, cooling of an infinite metallic plate in a cooling bath, the boundary layer along material handling conveyers, the aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film in the condensation processes, glass blowing, metal spinning, drawing plastic films and polymer extrusion.[2] Since the mechanical properties of the final product depend crucially on the rate of cooling or heating along the surface being stretched, Gupta and Gupta,[3] Vleggaar,[4] and Dutta *et al.*[5] examined the heat transfer characteristics of the boundary layer flow considered by Crane,[6] for cases with a constant wall temperature and constant wall heat flux (see Ref. [7]).

In recent years, the boundary layer flows of non-Newtonian fluids have been given considerable attention due to their ever increasing engineering applications. A vast majority of reactions involved specifically in polymer processing, food processing, biochemical industries, etc., are also typical examples of non-Newtonian behavior. The flow behaviors of non-Newtonian fluids are quite different from those of Newtonian fluids. Investigation on non-Newtonian fluid is necessary to obtain a thorough idea about non-Newtonian fluids and their various applications. The boundary layer flow of non-Newtonian fluid has wider applications, particularly in polymer depolarization, bubble columns, fermentation, composite processing, boiling, plastic foam processing, bubble absorption, and many others.[8] Non-Newtonian fluid flows generated by a stretching sheet have applications in extrusion of molten polymers through a slit die for the production of plastic sheets, the processing of food stuffs, paper production, and wire and fiber coating.[8] The fluids employed in material processing or protective castings are generally non-Newtonian. Due to the complexity of these fluids, no single constitutive

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†Corresponding author. E-mail: smmath08@gmail.com
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equation exhibiting all of the properties of such fluids is available. Several models can be found in this regard. The vast majority of non-Newtonian fluid models are concerned with simple models, like the power law and grade two or three (Refs. [9]–[15]). Though these models are very simple, they deviate greatly from reality in result. In modelling fluids with shear-dependent viscosity, the power-law model has a vast usage; however, it fails to predict the effects of elasticity. The fluids of grade two or three can calculate the effects of elasticity; however, in these models, the viscosity is not shear-dependent and also they are unable to predict the effects of stress relaxation. The Maxwell model, a subclass of rate type fluids, can predict the stress relaxation; therefore, this model has become more popular. In this model, also, the complicating effects of shear-dependent viscosity have been excluded, which enables one to focus solely on the effect of the fluid elasticity on the characteristics of its boundary layer.[16] Thermoplastic polymers in the vicinity of their melting temperature, fresh concrete (neglecting its aging), numerous metals at a temperature close to their melting point, geomaterials, etc. behave typically as Maxwell fluids. The mathematical model of Maxwell fluid has served as a simplified description of dilute polymeric solutions/fluids and has also been used to describe the rheology of flour dough as well as other fluids, including glycerin, toluene, crude oil, etc. The Maxwell model uses convected derivatives instead of time derivatives.[17]

Recently, the subject of hydromagnetics has attracted the attention of many authors due to its ever increasing applications in various fields. Magneto-hydrodynamics (MHD) was originally applied to astrophysical and geophysical problems, where it is still very important, but more recently to the problem of fusion power, where the application is the creation and containment of hot plasmas by electromagnetic forces, since material walls would be destroyed. Astrophysical problems include solar structure, especially in the outer layers, the solar wind bathing the earth and other planets and interstellar magnetic fields.[18] In modern metallurgical and metalworking processes, the study of the MHD flow of an electrically conducting fluid is of considerable interest. To purify the molten metals from non-metallic inclusions, hydromagnetic techniques are used. This type of problem is very useful for polymer technology and metallurgy. Hayat *et al.*[19,20] constructed analytic solutions for the MHD flow of a UCM fluid over a porous stretching sheet and also for the unsteady MHD flow in a rotating Maxwell fluid through a porous medium.

The separation of the boundary layer is associated with large energy losses and in most applications, in the form of lift loss and drag increase, it adversely affects the aerodynamic loads. Therefore, the control of the flow separation is very important in most of the systems involving fluid flow, underwater vehicles, and turbo-machinery. Both passive and active devices are used to control the flow separation.[21] The process of suction or injection (blowing) also has its importance in many engineering applications, such as in the design of thrust bearing and radial diffusers and thermal oil recovery. To remove reactants in chemical processes, suction is used whereas blowing is used to add reactants, cool the surfaces, prevent corrosion or scaling, and reduce the drag.[22] Injection or withdrawal of fluid through a porous bounding heated or cooled wall can lead to the enhancement of the heating or cooling of the system and can help to delay the transition from a laminar flow.[23]

The phenomenon of mass transfer is the movement of mass from one locality to another in the system. The mass transfer phenomenon is used in various scientific disciplines for different systems and mechanisms that involve the molecular and convective transport of atoms and molecules. The evaporation of water, the diffusion of chemical contamination in rivers and oceans from natural or artificial sources, and the separation of chemicals in the distillation procedure are some common examples of mass transfer.[24] Actually, mass transfer determines the rate at which the separation is possible. In view of these facts, various investigators, such as Andersson *et al.*,[25] Kendoush,[26] Takhar *et al.*,[27] and Liu[28] were interested in mass transfers under different aspects. The transport of the mass and momentum with chemical reactive species in the second grade fluid flow caused by a linear stretching sheet was discussed by Akyildiz *et al.*[29] Cortell[30] discussed mass transfers with chemically reactive species for two classes of viscoelastic fluids over a porous stretching sheet. Mass transfer is widely used in chemical engineering problems. Generally, the chemical species transfer between two phases through an interface or diffusion through a phase. The driving force for mass transfer is the difference between the concentrations.[24] Recently, Mukhopadhyay and Bhattacharyya[31] discussed the mass transfer effect on the Maxwell fluid flow passing through an unsteady stretching sheet. Bhattacharyya *et al.*[32] discussed the MHD boundary layer flow for the diffusion of a chemically reactive species. In this paper, they also analyzed the combined effects of unsteadiness and suction/blowing in the presence of a first order constructive/destructive chemical reaction.

In most of these above-mentioned studies, the flow, temperature and concentration fields are considered to be in a steady state. However, due to a sudden stretching of the flat sheet or by a step change of the temperature of the sheet, etc, heat and mass transfer can be unsteady. The inviscid flow is developed instantaneously when the surface is impulsively stretched with a certain velocity. However, near the sheet, the flow in the viscous layer is developed slowly and it becomes a fully developed steady flow after a certain instant of time. Many authors, viz. Andersson *et al.*,[33] Dandapat *et al.*,[34,35] Elbashbeshy and Bazid,[36] Sharidan *et al.*,[37] Ali and Magyari,[38] Tsai *et al.*,[39] Chamkha *et al.*,[40] El-Aziz[41] studied the problems for unsteady stretching surface under different conditions by using a similarity method to transform governing time-dependent boundary layer equations into a set of ordinary differential equations. Recently, Mukhopadhyay[42,43] obtained similarity solutions

for unsteady mixed convection flow passing through a stretching sheet and also discussed the effects of partial slip on unsteady mixed convection flow passing through a stretching surface. In another paper, Mukhopadhyay[44] analyzed the effects of suction on an unsteady mixed convection slip flow passing through a porous stretching sheet. Recently, Bhattacharyya *et al.*[45] analyzed the effects of slip on an unsteady boundary layer stagnation point flow passing through a stretching sheet.

A reaction is said to be of the first order if the rate of reaction is directly proportional to the concentration itself. It is known that several chemical reactions can be approximated by the first-order or pseudo first-order reaction.[46] Even zeroth and second-order reactions can be approximated satisfactorily by the first order reaction up to 60% conversion if we are concerned mainly with the changes of compositions with the reaction time.[46] Therefore, the first-order chemical reaction plays a significant role in chemical engineering. Despite the overwhelming importance and frequent occurrence of non-Newtonian behavior of fluid in industry and technology, no attempt has been made so far to analyze the MHD upper convected Maxwell fluid flow and mass transfer passing through an unsteady stretching porous surface in the presence of a first order constructive/destructive chemical reaction. This work aims to fill the gap in the existing literature. An attempt is made to study the effects of suction/blowing on a non-Newtonian Maxwell fluid flow and mass transfer in the presence of a first order constructive/destructive chemical reaction. The fluid electrically conducts in the presence of a magnetic field. Similarity solutions are obtained and the reduced ordinary differential equations are solved numerically using the shooting method. The effects of the governing parameters on the velocity and concentration fields are investigated and analyzed with the help of their graphical representations.

2. Problem formulation

We consider a laminar boundary-layer two-dimensional flow and the mass transfer of an incompressible electrically conducting non-Newtonian Maxwell fluid (in the presence of a transverse magnetic field) over an unsteady stretching porous sheet. The flow is permeated by a non-uniform magnetic field of strength $B_1 = B_0/\sqrt{1-\alpha t}$ applied in the direction normal to the surface, where B_0 is the initial strength of the magnetic field. Let $C_{\rm w}$ be the concentration at the sheet and C_∞ the concentration far away from the sheet. Also, the reaction of species is assumed to be the first order homogeneous chemical reaction of rate k_1.[24]

The unsteady fluid and mass flows start at $t = 0$. The sheet emerges out of a slit at origin ($x = 0$, $y = 0$) (see Fig. 1) and moves with non-uniform velocity $U(x,t) = bx/(1-\alpha t)$ where b and α are positive constants with dimensions (time)$^{-1}$, b is the initial stretching rate and $b/(1-\alpha t)$ is the effective stretching rate, which is increasing with time. In the case of polymer extrusion, the material properties of the extruded sheet may vary with time.

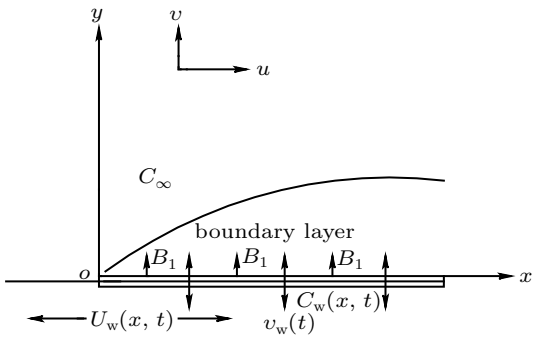


Fig. 1. Sketch of the physical problem.

The governing equations of such a type of flow are, in the usual notations, expressed as[47]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right) = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_1^2}{\rho}\left(u + \lambda v\frac{\partial u}{\partial y}\right), \tag{2}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - k_1(C - C_\infty). \tag{3}$$

For the upper convected Maxwell (UCM) fluid model used in this paper, no unsteady terms for the shear stress term need to be included, as mentioned by Alizadeh-Pahlavan and Sadeghy.[47] Therefore, no unsteady terms appear for the coefficient of λ.

Here, u and v are the components of velocity respectively in the x and y directions, ρ is the fluid density, where ν is the kinematic viscosity of the fluid, σ is the conductivity of the fluid, C is the concentration of the species of the fluid, D is the diffusion coefficient of the diffusing species in the fluid and $k_1(t) = k_0/(1-\alpha t)$ is the time-dependent reaction rate; $k_1 > 0$ refers to the destructive reaction, whereas $k_1 < 0$ denotes the constructive reaction, k_0 is a constant, $\lambda = \lambda_0(1-\alpha t)$ is the relaxation time , and λ_0 is a constant.

2.1. Boundary conditions

As is well known, in case of polymer extrusion, the material properties of the extruded sheet may vary with time; therefore, the appropriate boundary conditions for the problem are given by

$$u = U(x,t),\ \ v = v_{\rm w}(t),\ \ C = C_{\rm w}(x,t) \quad \text{at} \quad y = 0, \tag{4}$$

$$u \to 0,\ \ C \to C_\infty \quad \text{as} \quad y \to \infty, \tag{5}$$

where the concentration of the surface of the sheet is similarly assumed to vary both along the sheet and with time, in accordance with $C_{\rm w}(x,t) = C_\infty + bx(1-\alpha t)^{-2}$. If b is positive (negative) and is in proportion to x, the wall concentration $C_{\rm w}(x,t)$ increases (reduces). Moreover, the amount of increase (reduction) in the concentration along the sheet increases with time. Here, $v_{\rm w}(t) = -v_0/(\sqrt{1-\alpha t})$ is the velocity of suction ($v_0 > 0$) or blowing ($v_0 < 0$). The expressions for $U(x,t)$, $C_{\rm w}(x,t)$, $v_{\rm w}(t)$, $\lambda(t)$, $k_1(t)$ are valid for time $t < \alpha^{-1}$.

2.2. Method of solution

We now introduce the following relations for u, v, and ϕ as

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad \text{and} \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}, \tag{6}$$

where ψ is the stream function.

We introduce

$$\eta = \sqrt{\frac{c}{\nu(1-\alpha t)}}y, \quad \psi = \sqrt{\frac{\nu c}{(1-\alpha t)}}xf(\eta)$$

and $C = C_\infty + bx(1-\alpha t)^{-2}\phi(\eta)$.

With the help of the above relations, the governing equations finally reduce to

$$M\left(\frac{\eta}{2}f'' + f'\right) + f^{/2} - ff'' + \beta\left(f^2 f''' - 2ff'f''\right) = f''' - Ha(f' - \beta ff''), \tag{7}$$

$$M\left(\frac{1}{2}\eta\phi' + 2\phi\right) + f'\phi - f\phi' = \frac{1}{Sc}\phi'' - \gamma\phi, \tag{8}$$

where $M = \alpha/b$ is the unsteadiness parameter, $\beta = b\lambda_0$ is the Maxwell parameter, $Ha = \sigma B_0^2/(\rho b)$ is the magnetic parameter, $Sc = \nu/D$ is the Schmidt number, and $\gamma = k_0/b$ is the reaction rate parameter. Here, $\gamma > 0$ represents the destructive reaction, $\gamma = 0$ corresponds to no reaction, and $\gamma < 0$ refers to the generative reaction.

The boundary conditions (5) and (6) then become

$$f' = 1, \quad f = S, \quad \phi = 1 \quad \text{at} \quad \eta = 0, \tag{9}$$

$$f' \to 0, \quad \phi \to 0 \quad \text{as} \quad \eta \to \infty. \tag{10}$$

Here, $S = v_0/\sqrt{\nu b}$ is the suction/blowing parameter. S corresponds to suction or blowing accordingly, as $S > 0$ or, $S < 0$.

Further, with $M = 0$ and $\beta = 0$, Eq. (7) reduces to the corresponding momentum equation of Andersson *et al.*[48] for a special case with $n = 1$ and also to that of Prasad *et al.*[49] with constant physical properties, i.e., with $\beta = 0$, $\beta_1 = 0$, $\varepsilon = 0$, $\theta_r \to \infty$.

3. Numerical method for solution

The above equations (7) and (8) along with boundary conditions (9) and (10) are solved by converting them into an initial value problem.[50,51] We set

$$f' = z, \quad z' = p, \quad \phi' = q,$$

$$p' = \left(M\frac{\eta}{2}p + Mz + z^2 - 2\beta fzp - fp + Haz - Ha\beta fp\right)/(1 - \beta f^2), \tag{11}$$

$$q' = Sc\left(\frac{M}{2}\eta q + 2M\phi + z\phi - fq - \gamma\phi\right) \tag{12}$$

with the boundary conditions

$$f(0) = S, \quad f'(0) = 1, \quad \phi(0) = 1. \tag{13}$$

In order to integrate Eqs. (11) and (12) as an initial value problem, a value for $p(0)$, i.e., $f''(0)$ and $q(0)$, i.e. $\phi'(0)$ are required but no such values are given at the boundary. The suitable guess values for $f''(0)$ and $\phi'(0)$ are chosen and then integration is carried out. The calculated values for f' and ϕ at $\eta = 10$ (say) are compared with the given boundary condition $f'(10) = 0$ and $\phi(10) = 0$ and the estimated values, $f''(0)$ and $\phi'(0)$, are adjusted to give a better approximation for the solution.

Taking the series of values for $f''(0)$ and $\phi'(0)$, the fourth order classical Runge–Kutta method with step-size $h = 0.01$ is applied. The above procedure is repeated until the results up to the desired degree of accuracy, 10^{-5}, are obtained.

4. Verification of the results for simple cases

In order to validate the method used in this study and to judge the accuracy of our analysis, comparisons are made with the available results corresponding to the skin-friction coefficient $f''(0)$ for the steady motion of Newtonian fluid in the absence of suction/blowing, with those of Andersson *et al.*[48] (with $n = 1$) and Prasad *et al.*[49] of constant physical properties and also with the exact results. The comparisons are presented in Table 1 and it is found that the results are in excellent agreement. Moreover, the skin-friction coefficient $f''(0)$ for an unsteady flow of viscous incompressible Newtonian fluid ($\beta = 0$) is compared with the available results of Sharidan *et al.*[37] and Chamkha *et al.*[40] in Table 2 and the results are also in good agreement with their results. Furthermore, the values of $f''(0)$ for a steady flow of viscous incompressible non-Newtonian Maxwell fluid for variable values of Maxwell parameter β in the absence/presence of a magnetic field are compared with the available results of Sadeghy *et al.*[52] and Abel *et al.*[53] in Table 3 and the results are found to be in good agreement with each other.

Table 1. Comparison among the values of skin friction $[-f''(0)]$ for different values of the magnetic parameter Ha for a steady case ($M = 0$) for Newtonian fluid ($\beta = 0$) in the absence of suction/blowing ($S = 0$).

	$Ha = 0$	$Ha = 0.5$	$Ha = 1$	$Ha = 1.5$	$Ha = 2$
Exact solution	1.000	1.2247	1.414235	1.58811388	1.73205
Andersson *et al.*[48] for $n = 1$	1.000000	1.2249	1.4140	1.58100	1.73200
Prasad *et al.*[49] with constant physical properties	1.000174	1.224753	1.4144499	1.581139	1.732203
Present study	1.000173	1.224753	1.414450	1.581140	1.732203

Table 2. Values of $[-f''(0)]$ for various values of unsteadiness parameter M with $\beta = 0$.

M	Sharidan *et al.*[37]	Chamakha *et al.*[40]	Present study
0.8	1.261042	1.261512	1.261479
1.2	1.377722	1.378052	1.377850

Table 3. Values of $[-f''(0)]$ for different values of Maxwell parameter β.

β	$Ha = 0$			$Ha = 0.2$	
	Sadeghy *et al.*[52]	Abel *et al.*[53]	Present study	Abel *et al.*[53]	Present study
0	1.0000	0.999962	0.999963	1.095445	1.095444
0.2	1.0549	1.051948	1.051949	1.188270	1.188270
0.4	1.10084	1.101850	1.101851	1.275878	1.275878
0.6	1.15016	1.150163	1.150162	1.358733	1.358732
0.8	1.19872	1.196692	1.196693	1.437369	1.437369

5. Results and discussion

In order to study the behavior of velocity and concentration fields for Maxwell fluid, a comprehensive numerical computation is carried out for various values of the parameters that describe the flow characteristics; the results are reported in Figs. 2(a)–7(c).

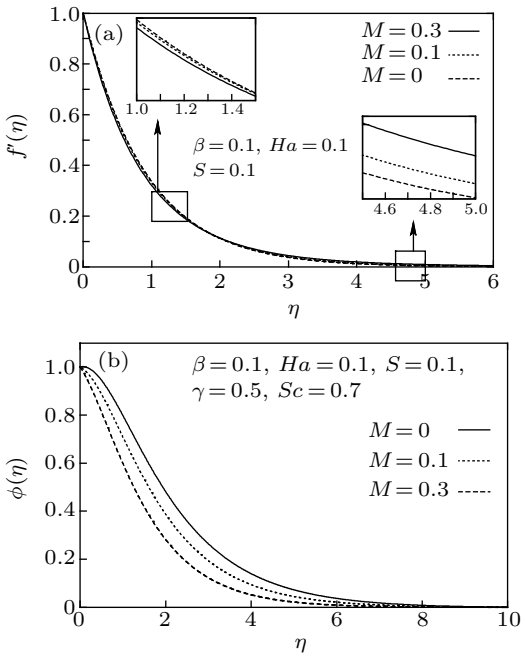


Fig. 2. (a) Velocity profiles and (b) concentration profiles for different values of unsteadiness parameter M.

Figure 2(a) exhibits the velocity profiles for several values of unsteadiness parameter M. It is seen that the velocity along the sheet decreases initially with the increase of unsteadiness parameter M, which implies that the thickness of the momentum boundary layer accordingly decreases near the wall; however, away from the wall the fluid velocity increases with the increase of unsteadiness. $M = 0$ indicates the steady case.

Figure 2(b) shows the effects of the unsteadiness parameter on the solute distribution. From this figure, it is noticed that the concentration at a particular point is found to decrease significantly as the unsteadiness parameter increases. The rate of mass transfer (from the fluid to the sheet) decreases as M increases. As the unsteadiness parameter M increases, less mass is transferred from the fluid to the sheet; hence, the concentration $\phi(\eta)$ decreases [Fig. 2(b)]. Since the fluid flow is caused solely by the stretching sheet and the sheet surface concentration is higher than the free stream concentration, the velocity and concentration decrease with the increase in η. Concentration overshoot is noted for the steady case. As the concentration gradient at the wall is positive, so there is a mass transfer from the fluid to the sheet, i.e., mass absorption occurs. That is why the overshoot in the concentration profiles is noted near the surface.

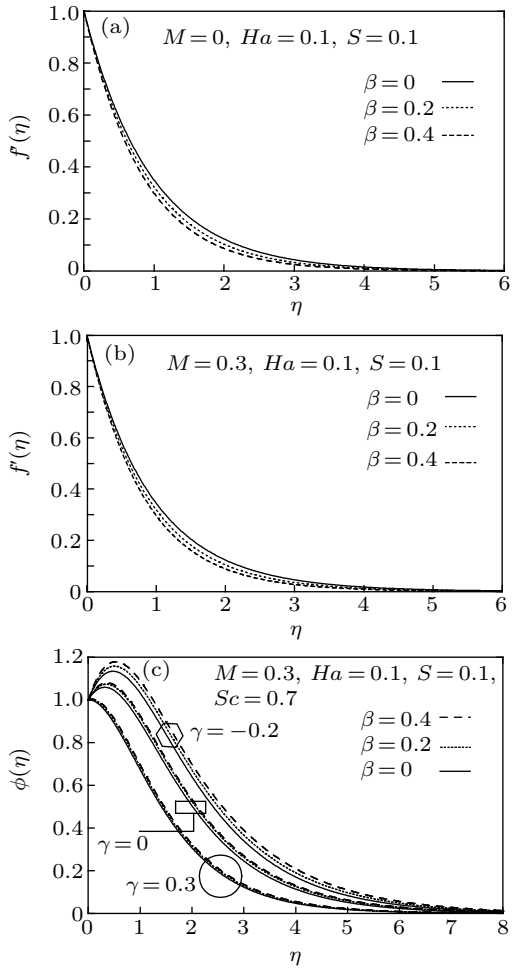


Fig. 3. Velocity profiles for different values of Maxwell parameter β for the cases of steady motion (a) and unsteady motion (b); concentration profiles for variable Maxwell parameter β for unsteady motion (c).

The effects of Maxwell parameter β on the velocity and concentration profiles for steady motion as well as unsteady motion are clearly exhibited in Figs. 3(a)–3(c) respectively. Here $\beta = 0$ gives the result for a viscous incompressible New-

tonian fluid. The effect of increasing values of β is to reduce the velocity and hence the boundary layer thickness decreases [Figs. 3(a) and 3(b)]. The velocity curves in Figs. 3(a) and 3(b) show that the rate of transport is considerably reduced with the increase of β. The effect of increasing β leads to the enhancement of the concentration field in the steady motion as well as unsteady motion [Fig. 3(c)]. The thickening of the solute boundary layer occurs due to the increase in the elasticity stress parameter. It can also be seen from Figs. 3(a) and 3(b) that the momentum boundary layer thickness decreases as β increases; hence this induces an increase in the absolute value of the velocity gradient at the surface. The value of the concentration gradient at the surface increases with the increase of β, as shown in Fig. 3(c). Thus, the mass transfer rate at the surface increases with the increase of β. The concentration field ϕ also decreases for large values of β in case of a generative chemical reaction ($\gamma < 0$). However, the magnitude of ϕ is larger in the case of ($\gamma < 0$) than in the case of a destructive chemical reaction ($\gamma > 0$). It is also found that the concentration field decreases for several values of β in all cases ($\gamma = 0$, $\gamma > 0$ and $\gamma < 0$). A chemical reaction causes the rate of interfacial mass transfer to increase. The reaction reduces the local concentration, thus increasing its concentration gradient and its flux.

The effects of magnetic parameter Ha on the velocity and concentration fields are exhibited in Figs. 4(a) and 4(b) respectively. With the increase of the magnetic field, the fluid velocity decreases [Fig. 4(a)] but the concentration increases in this case. The transverse magnetic field, applied in the direction normal to the flow direction, has a tendency to create a dragging force known as the Lorentz force, which opposes the motion of the fluid.

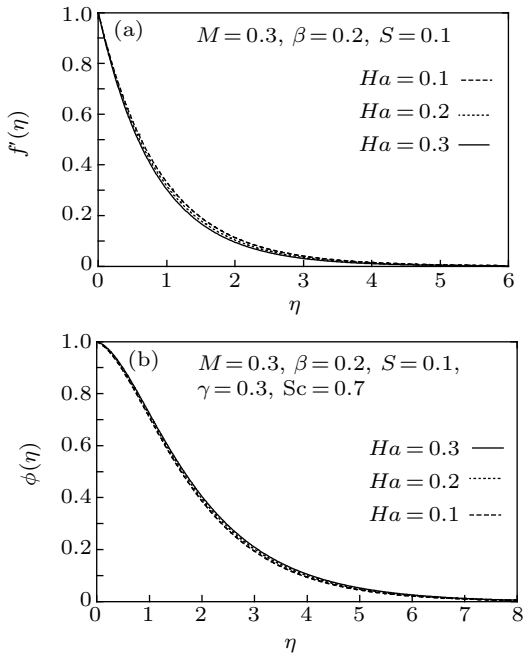


Fig. 4. Velocity (a) and concentration (b) profiles for different values of magnetic parameter Ha.

Effects of transpiration (suction/blowing) parameter S on the velocity and concentration fields are exhibited in Figs. 5(a) and 5(b) respectively. Fluid velocity and concentration are both found to decrease as the value of transpiration (suction/blowing) parameter S increases. With the increase of suction ($S > 0$), fluid velocity is found to decrease. That is, the effect of suction is to reduce the fluid velocity in the boundary layer and in turn, the wall-shear stress decreases. An increase in suction causes progressive thinning of the boundary layer.[54] However, the injection ($S < 0$) has an opposite effect. Figure 5(b) exhibits that the concentration at a particular point in the boundary layer also decreases as S increases ($S > 0$). This causes a decrease in the rate of mass transfer. This is quite the opposite in the case for blowing ($S < 0$). The solute boundary layer is thinner in the case of suction ($S > 0$) than in the case of impermeability ($S = 0$) whereas it is thicker than in the case of blowing ($S < 0$).

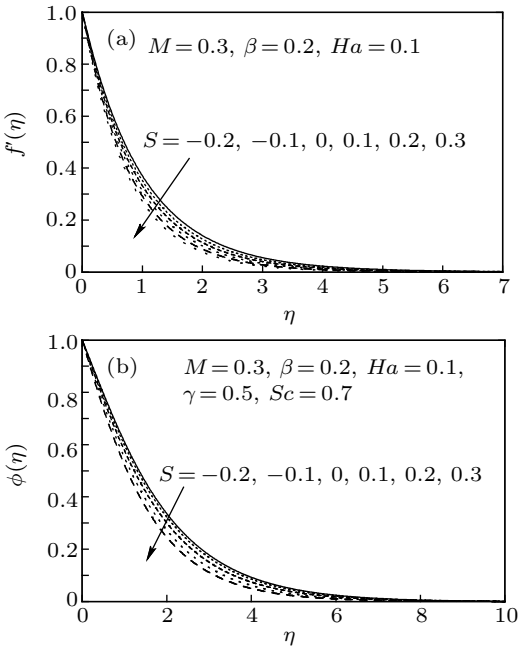


Fig. 5. Velocity (a) and concentration (b) profiles for different values of suction parameter S.

Figure 6(a) exhibits the nature of concentration profiles for variable values of Schmidt number Sc. It is noted that the concentration decreases as Sc increases. Moreover, the solute boundary layer thickness decreases as the Schmidt number increases. The Schmidt number is inversely proportional to the diffusion coefficient (D) and thus an increase in the values of Sc results in a decrease of the diffusion coefficient. Consequently, with an increase of the diffusion coefficient, the concentration and concentration boundary layer thickness increase. This effect is more prominent in case of suction ($S > 0$) than in the cases of impermeability ($S = 0$) and blowing ($S < 0$) [Fig. 6(a)]. Figure 6(b) illustrates the effects of reaction rate parameter γ on concentration profiles. Concentration increases as reaction rate parameter γ increases. Concentration overshoots are noted for all values of γ considered in this study

[Fig. 6(b)]. It is seen that concentration at a particular point increases for the generative chemical reaction ($\gamma < 0$), whereas it decreases for the destructive chemical reaction ($\gamma > 0$). Note that the change in the case of a generative chemical reaction

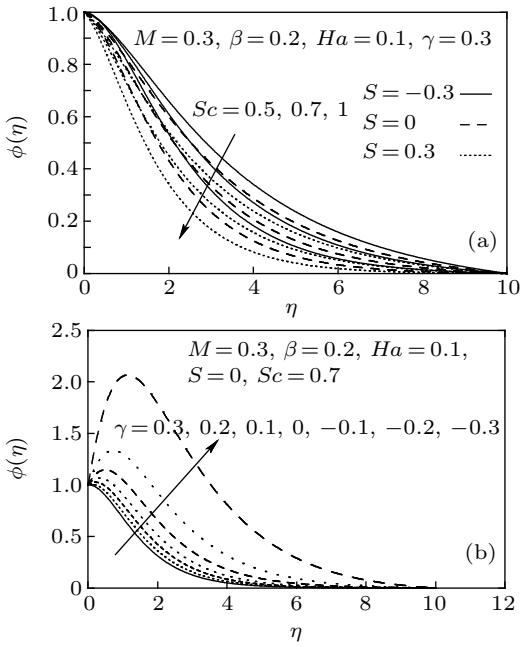


Fig. 6. Concentration profiles for different values of (a) Schmidt number Sc and (b) reaction rate parameter γ.

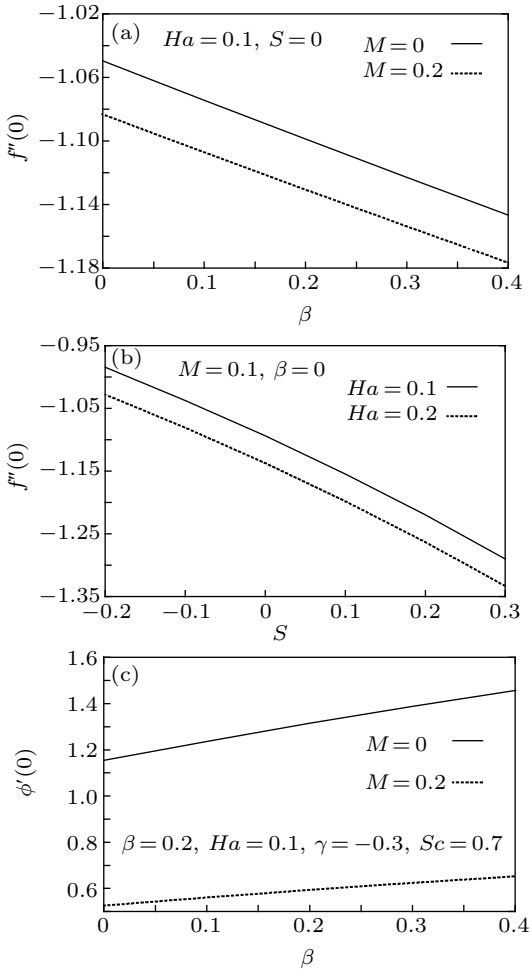


Fig. 7. Variations of $[f''(0)]$ (a) with Maxwell parameter β for two values of unsteadiness parameter M and (b) with suction/blowing parameter S for two values of magnetic parameter Ha; (c) variations of the mass transfer coefficient with Maxwell parameter β for two values of unsteadiness parameter M.

($\gamma < 0$) is larger than in the case of the destructive chemical reaction ($\gamma > 0$). In other words, the reaction-rate parameter is a decelerating agent in this case. The concentration boundary layer decreases in the case of a destructive chemical reaction. This is due to the fact that the conversion of the species takes place as a result of the chemical reaction and thereby reduces the concentration in the boundary-layer. Thus, for a reactive concentration distribution, the reaction rate is vital in controlling the solute boundary layer formed near the sheet.

Furthermore, the effects of unsteadiness parameter M and Maxwell parameter β on the velocity gradient at the wall and mass transfer coefficient are presented in Figs. 7(a) and 7(c). The velocity gradient at the wall $[f''(0)]$ decreases as unsteadiness parameter M and Maxwell parameter β increase, but the mass transfer rate at the surface $[\phi'(0)]$ increases with the increase of β, but decreases as M increases. The effects of suction/blowing and the magnetic parameter on $[f''(0)]$ are presented in Fig. 7(b). From this figure, it is clear that $[f''(0)]$ decreases as magnetic parameter Ha and suction/blowing parameter S increase. A drop in the value of $[f''(0)]$, as investigated in this paper, has an important implication that in free coating operations, the elastic properties of the coating formulations may be beneficial to the whole process.[54]

6. Conclusions

Similarity solutions for the unsteady MHD boundary layer flow of a Maxwell fluid and the mass transfer over a porous stretching surface in the presence of a first order constructive/destructive chemical reaction are obtained. The main findings of this investigation are listed as follows:

(i) the fluid velocity decreases initially due to the increase in the unsteadiness parameter but increases finally. The concentration also decreases significantly in this case.

(ii) With the increase in suction, magnetic parameter and Maxwell parameter, the fluid velocity is found to decrease; however, the opposite nature of concentration is noted in such cases.

(iii) The concentration field ϕ decreases when Sc increases. The concentration field ϕ has opposite results for destructive ($\gamma > 0$) and generative ($\gamma < 0$) chemical reactions.

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