Effects of self-fields on dispersion relation and growth rate in two-stream electromagnetically pumped free electron laser with axial guide magnetic field

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A theory for a two-stream free-electron laser (FEL) with an electromagnetic wiggler (EMW) and axial guide magnetic field is developed. In the analysis, the effects of self-fields are taken into account. The growth rate is derived. The characteristics of the growth rate are studied numerically. The dependence of the normalized wave number, which corresponds to the maximum growth rate, on the cyclotron frequency is presented. The comparisons between the normalized maximum growth rate and its corresponding wave number normalized by employing the axial magnetic field, for the cases with and without self-fields in the two-stream FEL are studied numerically.

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1. Introduction

The free electron laser (FEL) uses a beam of relativistic electrons passing through a long transverse magnetic field to amplify electromagnetic radiation. Technical restrictions on conventional FELs have motivated researchers to study electromagnetically pumped FELs, which offer the possibility of achieving much smaller values of frequency. One of the major difficulties for an FEL operating with an electromagnetic wiggler (EMW) of sub-millimeter wavelength is its requirement of extremely high pump strength. One of the ways to overcome the difficulty with the pump strength is to use two beams.[1–3]

The two-stream FEL amplifiers have been studied extensively during the last few decades.[4–7] Botton and Ron[8] calculated the gain of an electromagnetic wave in a two-stream FEL. They demonstrated that the gain per pass is more than two orders of magnitudes stronger than that of the conventional FEL.

Although there is a considerable amount of literature on the theory of two-stream FEL, most treatments heretofore neglected the influence of the equilibrium self-fields of the electron beams. In the case of low energy beams, an axial magnetic field is usually used to guide the electron beam. In such a configuration, the equilibrium self-electric and self-magnetic fields generated by the charge and the current densities of the beams respectively can have considerable effects on the equilibrium orbits. The effects of self-fields on the electron trajectory and the electron gain in a two-stream FEL with an electromagnetic wave wiggler and an axial guide field have been investigated by Mehdian and Saviz.[9]

In this paper, a configuration of two-stream FEL in the presence of the axial guiding magnetic field is proposed and investigated. The rest of the paper is organized as follows. In Section 2, the dispersion relation is obtained employing the linear fluid theory. The numerical analysis of the dispersion relation and discussion are given in Section 3, and finally the conclusion is presented in Section 4.

2. Dispersion relation

An analysis of the wave propagation in the EMW with an axial guiding field B_0 and the self-electric and magnetic fields is performed based on the electron momentum transfer and energy equation,

$$\frac{\mathrm{d}\boldsymbol{p}_i}{\mathrm{d}t} = -e\Big(\boldsymbol{E} + \frac{\boldsymbol{\nu}_i \times \boldsymbol{B}}{c}\Big), \quad i = 1,2, \tag{1a}$$

$$\frac{\mathrm{d}\gamma_i}{\mathrm{d}t} = -\frac{e}{mc^2}(\boldsymbol{\nu}_i \cdot \boldsymbol{E}), \quad i = 1,2, \tag{1b}$$

along with the equation of continuity

$$\frac{\partial}{\partial t} n_i + \nabla \cdot (n_i \boldsymbol{\nu}_i) = 0, \quad i = 1,2 \tag{2}$$

and the Poisson equation

$$\frac{\partial^2 \phi}{\partial z^2} = -4\pi e \sum_{i=1}^{2} \delta n_i, \tag{3}$$

where ϕ is the space-charge potential associated with the perturbed density δn_i, and the wave equation for the transverse component of the vector potential,

$$\nabla^2 \boldsymbol{A}_{\mathrm{R}} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{A}_{\mathrm{R}}}{\partial t^2} = -\frac{4\pi}{c}\sum_{i=1}^{2} \boldsymbol{J}_{\perp i}. \tag{4}$$

The wiggler-induced transverse driving current is given by

$$\boldsymbol{j}_{\perp i} = -e(\delta n_i \boldsymbol{\nu}_{\perp 0i} + \delta \boldsymbol{\nu}_{\perp i} n_{0i}), \quad i = 1,2, \tag{5}$$

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where δn_i and n_{0i} are the perturbed and unperturbed beam densities respectively, $\boldsymbol{\nu}_{\perp 0i}$ is the transverse velocity induced by the wiggler field and $\delta\boldsymbol{\nu}_{\perp i}$ is the transverse velocity induced by the radiation field. The stimulated radiation can be given by

$$\boldsymbol{A}_{\rm r} = A_{\rm r}(\mathrm{e}^{\mathrm{i}(kz-\omega t)}\hat{e}_+ + \mathrm{e}^{-\mathrm{i}(kz-\omega t)}\hat{e}_-). \tag{6a}$$

The EM wave as a pump wiggler field is described by

$$\boldsymbol{A}_{\rm w} = A_{\rm w}(\mathrm{e}^{\mathrm{i}(k_{\rm w}z+\omega t)}\hat{e}_- + \mathrm{e}^{-\mathrm{i}(k_{\rm w}z+\omega t)}\hat{e}_+), \tag{6b}$$

where $\hat{e}_\pm = (\hat{e}_+ \pm \mathrm{i}\hat{e}_-)/2$.

The fields $\boldsymbol{E}$ and $\boldsymbol{B}$ are expressed as

$$\boldsymbol{E}(z,t) = -\frac{\partial\phi}{\partial z}\hat{e}_z - \frac{1}{c}\frac{\partial(\boldsymbol{A}_{\rm w}+\boldsymbol{A}_{\rm r})}{\partial t} + \boldsymbol{E}_{\rm s}(x), \tag{7a}$$

$$\boldsymbol{B}(z,t) = B_0\hat{e}_z + \frac{\partial}{\partial z}\left[\hat{e}_z \times (\boldsymbol{A}_{\rm w}+\boldsymbol{A}_{\rm r})\right] + \boldsymbol{B}_{\rm s}(x), \tag{7b}$$

where $\boldsymbol{A}_{\rm r}$ is the vector potential of the radiation field and $\boldsymbol{A}_{\rm w}$ is the vector potential of the wiggler field.

The beam equilibrium self-electric and self-magnetic fields are

$$\boldsymbol{E}_{\rm s}(x) = -2\pi e\sum_{k=1}^{2} n_k(x\hat{e}_x + y\hat{e}_y), \tag{8a}$$

$$\boldsymbol{B}_{\rm s}(x) = -2\pi e\sum_{k=1}^{2} n_k\beta_{zk}(-y\hat{e}_x + x\hat{e}_y). \tag{8b}$$

For the stable motion of electron, we have

$$\boldsymbol{\nu}_{\perp 0i} = \nu_{\perp 0i}\frac{e\boldsymbol{A}_{\rm w}}{mc}, \quad i = 1,2, \tag{9}$$

where

$$\nu_{\perp 0i} = \frac{(\omega_{\rm w}+k_{\rm w}\nu_{\|i})^2}{\omega_{b1}^2+\omega_{b2}^2-(\omega_{b1}^2\beta_{\|1}+\omega_{b2}^2\beta_{\|2})\beta_{\|i}-\Omega_0(\omega_{\rm w}+k_{\rm w}\nu_{\|i})+\gamma_0(\omega_{\rm w}+k_{\rm w}\nu_{\|i})^2}, \quad i = 1,2. \tag{10}$$

Substituting Eqs. (6a), (7b), and (8b) into Eq. (1a), we obtain the transverse perturbed velocities of the two streams as follows:

$$\delta\boldsymbol{\nu}_{\perp i} = \delta\nu_{\perp i}\frac{e\boldsymbol{A}_{\rm R}}{mc}, \quad i = 1,2, \tag{11}$$

where

$$\delta\nu_{\perp i} = \frac{(\omega-k\nu_{\|i})^2}{\omega_{b1}^2+\omega_{b2}^2-(\omega_{b1}^2\beta_{\|1}+\omega_{b2}^2\beta_{\|2})\beta_{\|i}-\Omega_0(\omega-k\nu_{\|i})+\gamma_0(\omega-k\nu_{\|i})^2}, \quad i = 1,2. \tag{12}$$

Taking the axial component of Eq. (1a) and using the energy equation (Eq. (1b)), we find that

$$\delta\nu_{zi} = \frac{-\mathrm{i}}{[(k+k_{\rm w})\nu_{\|i}-(\omega-\omega_{\rm w})]} \times\left\{\frac{e}{m\gamma_{0i}\gamma_{zi}^2}\frac{\partial\phi}{\partial z}+\frac{\mathrm{i}e}{m\gamma_{0i}^2}D_i\phi_p(z,t)\right\}, \quad i = 1,2. \tag{13a}$$

Linearizing Eq. (2) and using Eq. (13a), we obtain

$$\frac{\delta n_i}{n_{0i}} = \frac{\mathrm{i}(k+k_{\rm w})}{[(k+k_{\rm w})\nu_{\|i}-(\omega-\omega_{\rm w})]^2} \times\left\{\frac{e}{m\gamma_{0i}\gamma_{zi}^2}\frac{\partial\phi}{\partial z}+\frac{\mathrm{i}e}{m\gamma_{0i}^2}D_i\phi_p(z,t)\right\}, \quad i = 1,2, \tag{13b}$$

where

$$\phi_p(z,t) = -\frac{|e|}{mc^2}A_{\rm w}A_{\rm R}\,\mathrm{e}^{\mathrm{i}[(k+k_{\rm w})z-(\omega-\omega_{\rm w})t]}$$

is the ponderomotive potential,

$$\gamma_{zi}^2 = \left(1-\frac{\beta_{\|i}^2}{c^2}\right)^{-2}, \quad \beta_{\|i} = \frac{\nu_{\|i}}{c},$$

$$D_i = \gamma_{0i}\left\{\left(k_{\rm w}+\omega_{\rm w}\frac{\nu_{\|i}}{c^2}\right)\delta\nu_{\perp i}+\left(k-\omega\frac{\nu_{\|i}}{c^2}\right)\nu_{\perp 0i} - \frac{1}{c}\left[(\omega_{b1}{}^2+\omega_{b2}{}^2)\beta_{\|i}-(\omega_{b1}{}^2\beta_{\|1}+\omega_{b2}{}^2\beta_{\|2})\right] \times\nu_{\perp 0i}\delta\nu_{\perp i}\left(\frac{1}{\omega_{\rm w}+k_{\rm w}\nu_{\|i}}-\frac{1}{\omega-k\nu_{\|i}}\right)\right\}, \quad i = 1,2. \tag{13c}$$

Substituting Eq. (13b) into the Poisson equation, Eq. (3), gives

$$\frac{\partial\phi}{\partial z} = \frac{2\mathrm{i}}{\varepsilon_z}\sum_{j=1}^{2}\eta_j, \quad j = 1,2, \tag{14}$$

where

$$\varepsilon_z = 1-\sum_{i=1}^{2}\frac{2\omega_{bi}^2}{\gamma_{0i}\gamma_{zi}^2[(k+k_{\rm w})\nu_{\|i}-(\omega-\omega_{\rm w})]^2},$$

$$\eta_j = \frac{\omega_{bi}^2D_i\phi_p(z,t)}{\gamma_{0i}^2[(k+k_{\rm w})\nu_{\|i}-(\omega-\omega_{\rm w})]^2}, \quad i = 1,2.$$

Inserting Eq. (14) into Eq. (13b) leads to the following form:

$$\frac{\delta n_i}{n_{0i}} = \frac{\mathrm{i}(k+k_{\rm w})}{[(k+k_{\rm w})\nu_{\|i}-(\omega-\omega_{\rm w})]^2} \times\left\{\frac{\mathrm{i}e}{m\gamma_{0i}\gamma_{zi}^2\varepsilon_z}\sum_{j=1}^{2}\left(\frac{2\omega_{bj}^2D_j\phi_p(z,t)}{\gamma_{0j}^2[(k+k_{\rm w})\nu_{\|j}-(\omega-\omega_{\rm w})]^2}\right) +\frac{\mathrm{i}eD_j\phi_p(z,t)}{\gamma_{0j}^2m}\right\}, \quad i = 1,2. \tag{15}$$

Substituting Eqs. (5), (9), (11), and (15) into Eq. (4) yields the following dispersion relation:

$$\left[\hat{\omega}^2-\hat{k}^2-2\sum_{i=1}^{2}\hat{\omega}_{bi}^2\delta\nu_{\perp i}\right]\left[T_1^2T_2^2-\frac{2\omega_{b1}^2T_2^2}{\gamma_{01}\gamma_{z1}^2}-\frac{2\omega_{b2}^2T_1^2}{\gamma_{02}\gamma_{z2}^2}\right]$$

$$= 2(\hat{k}+1)\hat{\Omega}_{\rm w}^2\left[\hat{\omega}_{b1}^2\nu_{\perp 01}\left\{\frac{2}{\gamma_{01}\gamma_{z1}^2}\frac{\hat{\omega}_{b2}^2D_2}{\gamma_{02}^2}-\frac{2}{\gamma_{02}\gamma_{z2}^2}\frac{\hat{\omega}_{b2}^2D_1}{\gamma_{01}^2}+\frac{D_1T_2^2}{\gamma_{01}^2}\right\}\right.$$

$$+\hat{\omega}_{b2}^2 \nu_{\perp 02}\left\{\frac{2}{\gamma_{02}\gamma_{z2}^2}\frac{\hat{\omega}_{b1}^2 D_1}{\gamma_{01}^2}-\frac{2}{\gamma_{01}\gamma_{z1}^2}\frac{\hat{\omega}_{b1}^2 D_2}{\gamma_{02}^2}+\frac{D_2 T_1^2}{\gamma_{02}^2}\right\}\Big], \quad (16)$$

where $\hat{\omega}=\omega/ck_{\rm w}$, $\hat{k}=k/k_{\rm w}$, and $T_i^2=[(\hat{k}+1)\beta_{\|i}-(\hat{\omega}-\beta_{\rm p})]^2$. Using the FEL resonance condition, $\omega=\omega_{\rm w}+(k+k_{\rm w})v_z$, the axial phase velocity becomes $v_{\rm ph}=(\omega-\omega_{\rm w})/(k+k_{\rm w})$.

Under the condition of resonance for backward wave pumping,

$$k=\frac{(1+\beta_{zi})^2}{(1-\beta_{zi}^2)}k_{\rm w}, \quad i=1,2,$$

we can obtain

$$\omega_{\rm w}+k_{\rm w}v_{zi}=\omega-kv_{zi}, \quad i=1,2.$$

Under these assumptions, equation (16) can be simplified as follows:

$$\begin{aligned}&\Big[\hat{\omega}^2-\hat{k}^2-2\sum_{i=1}^{2}\hat{\omega}_{bi}^2\delta\nu_{\perp i}\Big]\Big[T_1^2T_2^2-\frac{2\omega_{b1}^2T_2^2}{\gamma_{01}\gamma_{z1}^2}-\frac{2\omega_{b2}^2T_1^2}{\gamma_{02}\gamma_{z2}^2}\Big]\\&=4(\hat{k}+1)\hat{\Omega}_{\rm w}^2\Big[\hat{\omega}_{b1}^2\nu_{\perp 01}\Big\{\frac{2}{\gamma_{01}\gamma_{z1}^2}\frac{\hat{\omega}_{b2}^2(1+\beta_{z2})\lambda_2}{\gamma_{02}^2}\\&-\frac{2}{\gamma_{02}\gamma_{z2}^2}\frac{\hat{\omega}_{b2}^2(1+\beta_{z1})\lambda_1}{\gamma_{01}^2}+\frac{T_2^2(1+\beta_{z1})\lambda_1}{\gamma_{01}^2}\Big\}\\&+\hat{\omega}_{b2}^2\nu_{\perp 02}\Big\{\frac{2}{\gamma_{02}\gamma_{z2}^2}\frac{\hat{\omega}_{b1}^2(1+\beta_{z1})\lambda_1}{\gamma_{01}^2}-\frac{2}{\gamma_{01}\gamma_{z1}^2}\frac{\hat{\omega}_{b1}^2(1+\beta_{z2})\lambda_2}{\gamma_{02}^2}\\&+\frac{T_1^2(1+\beta_{z2})\lambda_2}{\gamma_{02}^2}\Big\}\Big], \quad (17)\end{aligned}$$

where, $\lambda_i=\gamma_{0i}v_{0\perp i}$.

This dispersion relation for a single beam can be obtained by setting $\hat{\omega}_{b2}=0$ in the above equation. In this procedure, first we divide Eq. (17) by $T_1^2T_2^2$, then setting $\hat{\omega}_{b2}=0$ and performing some straightforward algebra, we obtain

$$\begin{aligned}&\Big[\hat{\omega}^2-\hat{k}^2-2\hat{\omega}_{b1}^2\delta\nu_{\perp 1}\Big]\Big[T_1^2-\frac{2\omega_{b1}^2T_1^2}{\gamma_{01}\gamma_{z1}^2}\Big]\\&=4(\hat{k}+1)\hat{\Omega}_{\rm w}^2\frac{\hat{\omega}_{b1}^2\lambda_1^2(1+\beta_{z1})}{\gamma_{01}^3}. \quad (18)\end{aligned}$$

This is the dispersion relation for a single beam. In the special case used in Eq. (1), $\hat{k}\gg 1$ ($k\gg k_{\rm w}$) and $\beta_{z1}\approx 1$, equation (18) becomes

$$\Big[\hat{\omega}^2-\hat{k}^2-2\hat{\omega}_{b1}^2\delta\nu_{\perp 1}\Big]\Big[T_1^2-\frac{2\omega_{b1}^2T_1^2}{\gamma_{01}\gamma_{z1}^2}\Big]=8\hat{k}\hat{\Omega}_{\rm w}^2\frac{\hat{\omega}_{b1}^2\lambda_1^2}{\gamma_{01}^3}. \quad (19)$$

3. Numerical results and discussion

The steady-state relativistic two-stream electron trajectories and the stabilities of groups in the cases with and without self-fields in EMW and axial guide field have been investigated numerically by Saviz *et al.*,[10] and Mehdian and Saviz.[11] The normalized axial velocities of two beams, β_{z1} and β_{z2}, in the cases with self-fields and without self-fields are shown in Figs. 1(a) and 1(b) and Figs. 2(a) and 2(b), respectively. As seen in these figures, the adding of self-fields can increase the group trajectories, from seven groups to nine groups.

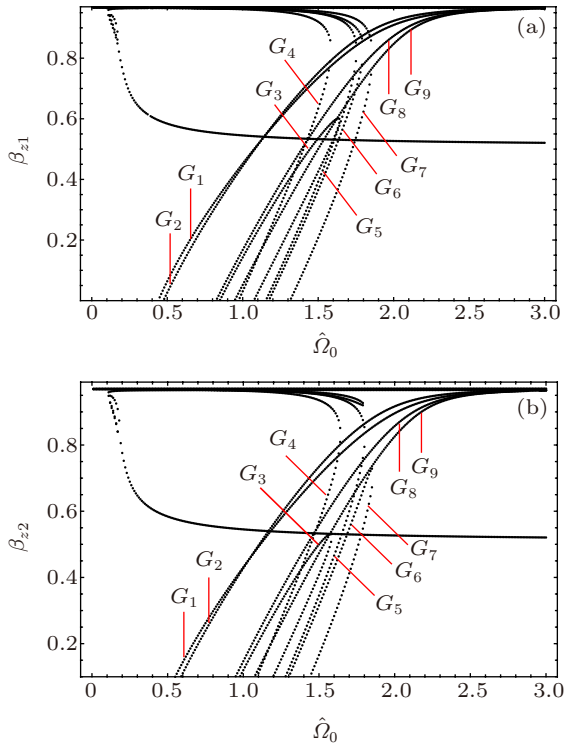


Fig. 1. (color online) Graphs of the normalized axial velocities (a) β_{z1} and (b) β_{z2} of steady-state orbits versus normalized cyclotron frequency ($\hat{\Omega}_0$) in the case of two-streams and self-fields effect. The chosen parameters are $\hat{\Omega}_{\rm w}=0.2$ and $\hat{\omega}_{b1}=\hat{\omega}_{b2}=0.212$.

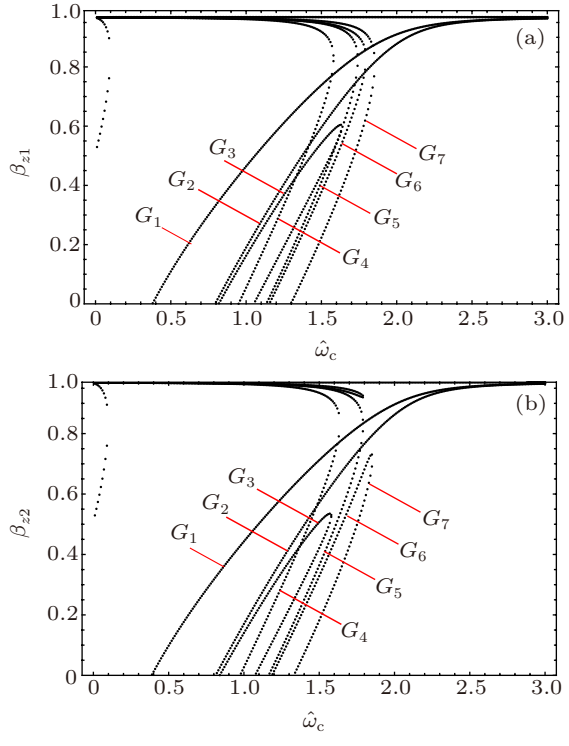


Fig. 2. (color online) Graphs of the normalized axial velocities (a) β_{z1} and (b) β_{z2} of steady-state orbits versus normalized cyclotron frequency ($\hat{\Omega}_0$) in the case of two-streams and without self-fields effect. The chosen parameters are $\hat{\Omega}_{\rm w}=0.2$ and $\hat{\omega}_{b1}=\hat{\omega}_{b2}=0.212$.

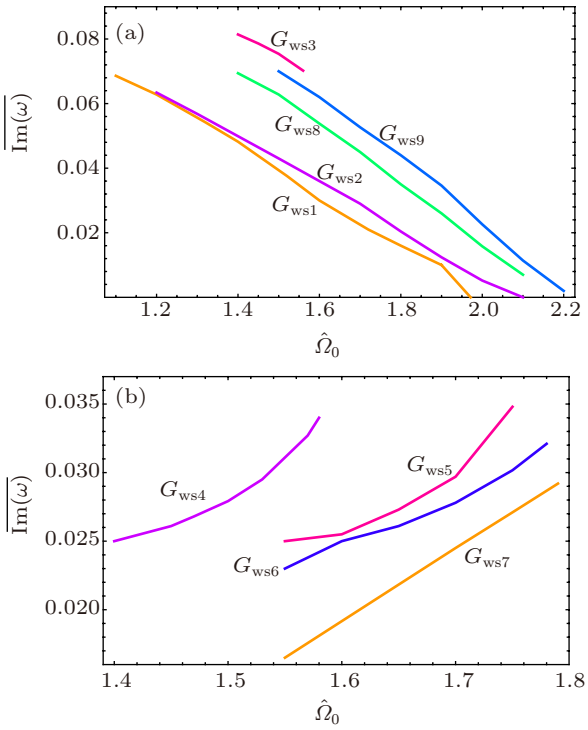


Fig. 3. (color online) Plots of the normalized maximum growth rate $\overline{\mathrm{Im}(\omega)}$ versus normalized cyclotron frequency $(\hat{\Omega}_0)$ for (a) groups (G_1–G_3 and G_8–G_9) and (b) groups (G_4–G_7). The chosen parameters are $\hat{\Omega}_\mathrm{w} = 0.2$ and $\hat{\omega}_{b1}$=$\hat{\omega}_{b2}$= 0.212.

The numerical solution of the dispersion relation is obtained by employing Eq. (16). The energy factors of electron beams are taken to be $\gamma_{01} = 4$ and $\gamma_{02} = 4.1$, the normalized magnetic wiggler and the plasma frequencies of the two beams are chosen to be $\hat{\Omega}_\mathrm{w}$= 0.2, $\hat{\omega}_{b1} = \hat{\omega}_{b2}$= 0, respectively. The plots of normalized maximum growth rate, $\overline{\mathrm{Im}(\omega)} = (\mathrm{Im}\omega/ck_\mathrm{w})_\mathrm{max}$, versus normalized cyclotron frequency $(\hat{\Omega}_0)$ for groups (G_1–G_3 and G_8–G_9) and (G_4–G_7) are shown in Figs. 3(a) and 3(b), respectively. Figure 3(a) shows that the maximum growth rates of groups (G_1–G_3 and G_8–G_9) are similar and decrease with the cyclotron frequency increasing. The behaviors for the maximum growth rate of groups (G_1–G_2 and G_8–G_9) are that $G_{ws1} < G_{ws2} < G_{ws8} < G_{ws9}$ (G_{wsi} denotes the normalized maximum growth rate for group i with self-fields).

Contrary to Fig. 3(a), the normalized maximum growth rates of groups (G_4–G_7) presented in Fig. 3(b) follow an increasing trend with $\hat{\Omega}_0$ increasing. The order of these normalized maximum growth rates is $G_{ws4} > G_{ws5} > G_{ws6} > G_{ws7}$.

The plots of the normalized wave number $\hat{k} = (k/k_\mathrm{w})$ corresponding to the maximum growth rate versus $\hat{\Omega}_0$ are shown in Fig. 4. As shown in Fig. 4, the order of normalized frequencies for groups (G_1–G_2 and G_8–G_9) are $\hat{k}_{ws2} > \hat{k}_{ws8} > \hat{k}_{ws9} > \hat{k}_{ws1}$ ($\hat{k}_{wsi}$ refers to the normalized wave number for groups i with self-fields). The normalized wave numbers for groups (G_4–G_7) are in reverse order to the normalized maximum growth rates, i.e., $\hat{k}_{ws7} > \hat{k}_{ws6} > \hat{k}_{ws5} > \hat{k}_{ws4}$.

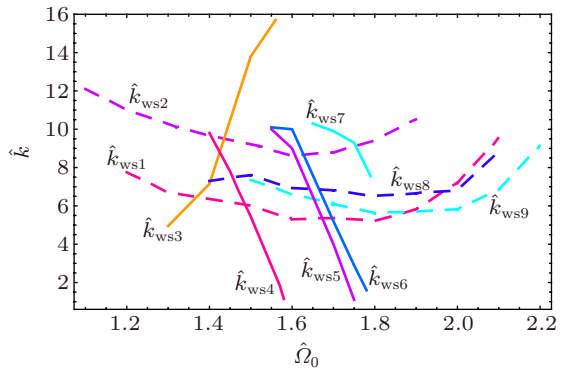


Fig. 4. (color online) Plots of the normalized maximum wave number $\hat{k}$ versus normalized cyclotron frequency $(\hat{\Omega}_0)$ for groups G_1–G_9. The chosen parameters are $\hat{\Omega}_\mathrm{w} = 0.2$ and $\hat{\omega}_{b1}$=$\hat{\omega}_{b2}$= 0.212.

The comparisons of normalized maximum growth rate for the two-streams with and without self-fields are given in Figs. 5(a) and 5(b). In these figures, the solid lines belong to the self-field, G_ws, and the dashed lines refer to the case without self-fields, G_wos. As seen in Fig. 5(a), the normalized maximum growth rates in the presence of self-fields for groups 8 and 9 (G_{ws8} and G_{ws9}) are greater than those for without self-fields (G_{wos1} and G_{wos2}). Figure 5(b) shows that $G_{ws4} > G_{wos4}$, $G_{ws5} > G_{wos5}$, $G_{ws6} > G_{wos6}$, and $G_{ws7} < G_{wos7}$.

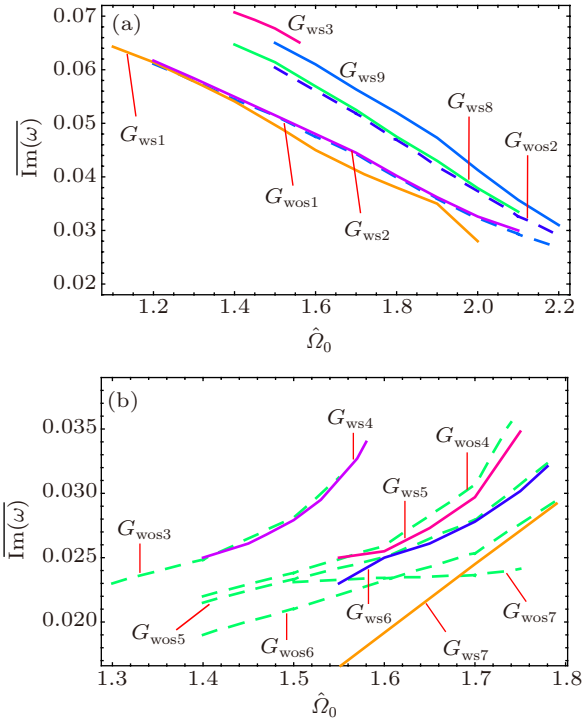


Fig. 5. (color online) (a) Graphs of the normalized maximum growth rate versus normalized cyclotron frequency $(\hat{\Omega}_0)$ for two streams with self-fields effect, groups G_1–G_3 and G_8 and G_9 (solid line), and two streams without self-fields effect for groups G_1 and G_2 (dashed line); (b) maximum growth rate versus normalized cyclotron frequency $\hat{\Omega}_0$ for two streams with self-fields effect for groups G_4–G_7 (solid line) and two streams without self-fields effect for groups G_4–G_7 (dashed line). The chosen parameters are $\hat{\Omega}_\mathrm{w} = 0.2$ and $\hat{\omega}_{b1}$=$\hat{\omega}_{b2}$= 0.212.

The normalized wave numbers for the two cases are shown in Figs. 6(a) and 6(b). As seen in Fig. 6(a), the normalized wave number in the presence of self-fields for group 2 ($\hat{k}_{\rm ws2}$) is greater than those for without self-fields ($\hat{k}_{\rm wos1}$ and $\hat{k}_{\rm wos2}$). Figure 6(b) shows that $\hat{k}_{\rm wos4} > \hat{k}_{\rm ws4}$, $\hat{k}_{\rm wos5} > \hat{k}_{\rm ws5}$, $\hat{k}_{\rm wos6} > \hat{k}_{\rm ws6}$, and $\hat{k}_{\rm wos7} > \hat{k}_{\rm ws7}$.

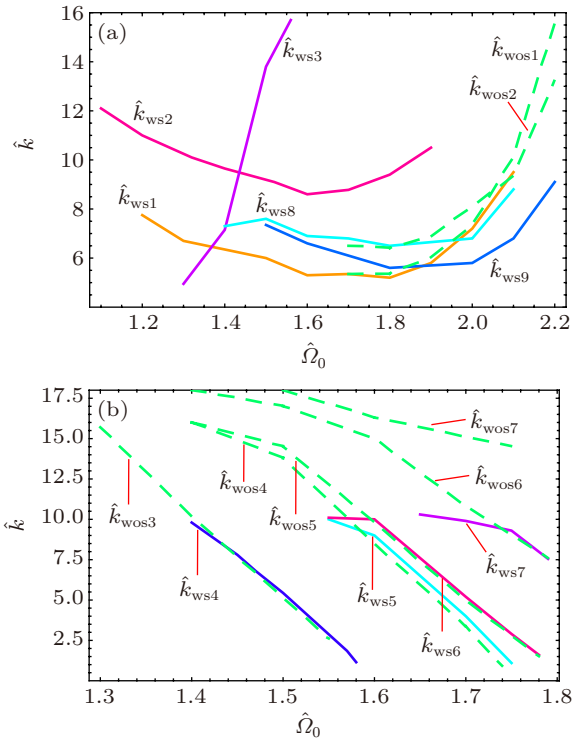


Fig. 6. (color online) (a) Graphs of the normalized wave number versus normalized cyclotron frequency ($\hat{\Omega}_0$) for two streams with self-fields effect, groups G_1–G_3 and G_8 and G_9 (solid line), and two streams without self-fields effect for groups G_1 and G_2 (dashed line); (b) maximum growth rate versus normalized cyclotron frequency ($\hat{\Omega}_0$) for two streams with self-fields effect for groups G_4–G_7 (solid line) and two streams without self-fields effect for groups G_4–G_7 (dashed line). The chosen parameters are $\hat{\Omega}_{\rm w} = 0.2$ and $\hat{\omega}_{b1}$=$\hat{\omega}_{b2} = 0.212$.

4. Conclusions

We summarize the specific results of this project as follows.

(i) The maximum growth rates of groups (G_1–G_3 and G_8–G_9) are similar and decrease with cyclotron frequency decreasing. The maximum growth rate of group 9 is greater than that of the other groups. The behavior of normalized wave number (corresponding to maximum growth rate) as a function of cyclotron frequency is reverse to maximum growth rate and group 2 has the greatest wave number.

(ii) The normalized maximum growth rates of groups (G_4–G_7) follow an increasing trend with the increase of cyclotron frequency and group 4 has the greatest maximum growth rate. The behavior of normalized wave number (corresponding to maximum growth rate) as a function of cyclotron frequency is reverse to maximum growth rate and group 7 has a greatest value.

(iii) The comparison of maximum growth rate of a two-stream FEL with and without self-fields shows that the normalized maximum growth rate for two-stream with self-field have an enhancement with respect to without self-fields except for group 7.

(iv) The comparison of normalized wave number of a two-stream FEL with and without self-fields shows that the normalized wave number for two-stream without self-field is enhanced with respect to with self-fields except for group 2.

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