Scheduling of high-speed rail traffic based on discrete-time movement model*

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In this paper, a new simulation approach for solving the mixed train scheduling problem on the high-speed double-track rail line is presented. Based on the discrete-time movement model, we propose control strategies for mixed train movement with different speeds on a high-speed double-track rail line, including braking strategy, priority rule, travelling strategy, and departing rule. A new detailed algorithm is also presented based on the proposed control strategies for mixed train movement. Moreover, we analyze the dynamic properties of rail traffic flow on a high-speed rail line. Using our proposed method, we can effectively simulate the mixed train schedule on a rail line. The numerical results demonstrate that an appropriate decrease of the departure interval can enhance the capacity, and a suitable increase of the distance between two adjacent stations can enhance the average speed. Meanwhile, the capacity and the average speed will be increased by appropriately enhancing the ratio of faster train number to slower train number from 1.

Keywords: train control, discrete-time model, simulation, rail traffic flow

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1. Introduction

The aim of train control is to make a decision of the speed and the location. An effective real-time control can increase both the transport capacity of the rail line and the average speed of trains. Nowadays, the construction of high-speed rail in China has entered into a period of rapid development. It is expected that the mileage of high-speed railway in China will be more than 18 thousand kilometers in 2020, accounting for more than half of the total mileage of the world's high-speed rail. Besides, the expanded population causes a higher demand for the capacity of the rail line. Thus the research of train control on the high-speed railway has drawn great attention.

Many models and methods were proposed for time-efficient or energy-efficient scheduling of trains.[1–10] Szpigel[11] used the branch-and-bound method to solve the train scheduling problem which minimizes the total transit time. Sauder and Westerman[12] implicitly enumerated the whole possible options and selected the minimum one. Higgins *et al.*[13] developed a model for the train scheduling problem on a single-track rail line. However, the proposed model was a complex nonlinear mixed-integer programming. Dorfman and Medanic[14] established a discrete-event model to simulate the scheduling problem on a railway network, in which travel-advance strategy (TAS) was offered to determine the speed and the location of train. They also proposed a capacity check method to avoid the deadlock. The proposed method was more effective than the nonlinear programming methods. Li and Gao[15] proposed a simulation approach applied to double-track rail networks based on the network model. In their paper, stations and section tracks were treated as nodes and edges. Li and Gao[16] presented a cellular automaton model for the simulation of the train movement on the rail line. Li *et al.*[17] developed a new cellular automaton model of scheduling problem by considering the loss of energy. Li *et al.*[18] presented a simulation of train scheduling based on global information of train. They proposed a new control method to avoid deadlock, which had looser constraints than capacity check in Ref. [14]. In their approach, the speed varied in sections when different strategies were adopted. Yang *et al.*[19] proposed a simulation method based on a discrete-time model under stochastic disturbance. Wang *et al.*[20] analyzed the properties of the rail traffic flow under the moving block system and presented the simulation of train scheduling based on the CA model. Many other scholars also proposed different improved models with considerations of varied real world problems under the moving block system.[21–24]

The main contributions of this paper are as follows. Firstly, we propose a new approach to simulating the train movement on a high-speed rail line based on the discrete-time movement model and present efficient control strategies. Secondly, a new detailed algorithm based on the control strategies is designed for less travel time. Thirdly, we analyze the properties of the rail traffic flow under the influences of the ratio of faster train number to slower train number, departure intervals, distance between two adjacent stations, and accelerations. Fi-

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nally, the simulation results demonstrate that an appropriate decrease of the departure interval can enhance the capacity, and a suitable increase of the distance between two adjacent stations can increase the average speed. The variation of the number ratio for mixed trains has a little influence on the average speed of faster trains but exerts a great effect on slower trains. Meanwhile, the capacity and the average speed will be increased by appropriately increasing the number ratio of faster trains to slower trains from 1.

The rest of this paper is organized as follows. In Section 2, the discrete-time movement model is presented and a new simulation approach based on discrete-time movement model is introduced. Moreover, the train control strategies are proposed and a detailed algorithm of simulation is designed. Our method is applied to a high-speed double-track rail line and the numerical results of simulation are analyzed in Section 3. The conclusion of this research is given in Section 4.

2. A simulation method for scheduling problem

2.1. Discrete-time movement model

In this research, we assume that the speeds of trains are varied on the rail line during the train movement. Additionally, the following assumptions are made in this research.

(i) All trains are running on a double-track line and travelling towards the same direction;

(ii) The route of each train is fixed, so is the original departure time of each train. The arrival time is free and depends on the train control strategy;

(iii) There are two types of trains running on the line: the maximum speed of faster trains is 300 km/h and the maximum speed of slower trains is 240 km/h. Slower trains can be overtaken by faster trains at any station;

(iv) The capacity of the platform in each station is unlimited. All the trains have to dwell at each station at least for two minutes;

(v) The minimal headway of train is defined as a constant.

Symbols and parameters used in this research are listed in Table 1.

To simulate the train movement, we discretize the time, where the increment of time is denoted by Δt. The discrete-time movement model is then formulated as follows:

$$\begin{cases} X_i(t+\Delta t) - X_j(t+\Delta t) \geq f_{\min}, \\ X_i(t+\Delta t) = X_i(t) + V_i(t+\Delta t)\Delta t, \\ V_i(t+\Delta t) = V_i(t) + a\Delta t, \\ t, t+\Delta t \in [0, T], \end{cases} \tag{1}$$

where train i is the previous train and train j is the following train, $X_i(t+\Delta t)$ ($X_j(t+\Delta t)$) is the position of train i (j) at simulation time $t+\Delta t$, $V_i(t+\Delta t)$ is the speed of train i at simulation time $t+\Delta t$.

In this paper, we use model (1) to determine the physical characteristics of trains in different times, including speed and position, etc. In our method, the underlying dynamics of the proposed model is governed by the control strategies applied at discrete time steps. In the moving block system, the operation of the focal train i mainly depends on the distance between train i and its front train k. The safety headway between adjacent trains generally consists of the braking distance and the minimal headway. The safety headway of a train is guaranteed by checking the distance between adjacent trains in real time and then controlling the focal train i on the basis of its actual headway. On the journey of train i, the relevant operations include the departing operation from a station, travelling operation, entering and dwelling operations at a station, braking operation, and overtaking operation.

Table 1. Symbols and parameters.

Symbol	Description
i, j, k	train indices
$V_{\max 1}$	maximum speed of slower train
$V_{\max 2}$	maximum speed of faster train
D_m	position of station m
$f_{\min}$	minimal headway of trains
$V_i(t)$	speed of train i at time t
$\bar{V}_i(t+\Delta t)$	temporary speed of train i in the next period
$X_i(t)$	position of train i at time t
X_c	brake distance of train
Δt	time step
g	distance between train i and the forward station
c	distance between train i and the forward train
a	acceleration
r	ratio of faster trains to slower trains
e	fixed dwelling time at station
d	departure time interval
L	total length of line
$h(1,p)$	The state of position p. $h(1,p) = 1$ if position p is occupied by a train i; otherwise $h(1,p) = 0$.
$\overline{h}(1,p)$	temporary state of position p
$V_p(t)$	speed of train i at position p at simulate time t
$\overline{V}_p(t+1)$	temporary speed of train i at position p at simulate time $t+1$
D	position of the nearest frontal station
b	dwelling time of the focal train i at the current time
t_p	running time of train i from its location p to the next station

Therefore, we need to compute the braking distance in real time before controlling the movement of the focal train i, then we should choose what kind of operation can be applied at this simulate time, which are given as follows:

Step 1 If train i is stopping at an intermediate station and will depart from the station, then perform the departing rules;

Step 2 If train i is far enough from its previous train k or the station ahead, then use the travelling strategy;

Step 3 If it is the time for train i to brake for keeping safety distance from the front train k (the actual distance is less than or equal to the safe headway) or entering the station, then perform the braking operation in the next simulation time;

Step 4 If train i has a lower priority than its following train j, then use overtaking rule.

To simulate the movements of trains, it is necessary for us to design the detailed control strategies.

2.2. Travel control strategies and algorithm

In our model, each train is planned to depart from the original station per 10 minutes. There are two different types of trains departing from the original station in a constant interval. The detailed control strategies for the movements of trains in the discrete-time movement model are designed as follows:

2.2.1. Braking operation

Here, two cases are taken into consideration when the focal train i is running behind train k in a section and entering into a station as follows:

Case 1 Train i is running behind train k in a section.

Train i is not a leading train and train k is running in front of train i. The distance between them are not more than the safety distance which includes the brake distance of train i and the minimal headway of trains (that is, $c \leq \max(X_c + f_{\min}, f_{\min})$). The speed and location of train i will be determined according to the following expression:

$$\begin{cases} \bar{V}_i(t+\Delta t) = \max(0, V_i(t) - a), \\ V_i(t+\Delta t) = \min(\bar{V}_i(t+\Delta t), \sqrt{2a(c - f_{\min}) + V_k(t)^2}), \\ X_i(t+\Delta t) = X_i(t) + V_i(t+\Delta t). \end{cases} \tag{2}$$

In the above formulas (2), train k is the nearest train in front of the train i, $V_i(t)$ is the speed of train i at the current time, $V_k(t)$ is the speed of train k at the current time, $\bar{V}_i(t+\Delta t)$ presents the temporary speed of train i at the simulate time $t+\Delta t$, X_c is the brake distance of train i and $X_c = (V_i(t)^2 - V_k(t)^2)/2a$.

Case 2 Train i is entering into a station.

The distance between train i and its frontal station is no more than the brake distance of train i (that is, $g \leq V_i(t)^2/2a$). The speed and location of train i will be determined by the following equations:

$$\begin{cases} \bar{V}_i(t+\Delta t) = \max(0, V_i(t) - a), \\ V_i(t+\Delta t) = \min(\bar{V}_i(t+\Delta t), \sqrt{2ag}), \\ X_i(t+\Delta t) = X_i(t) + V_i(t+\Delta t). \end{cases} \tag{3}$$

In the above formulas, g is the distance between train i and the nearest frontal station. X_c is the brake distance of train i and here $X_c = V_i(t)^2/2a$.

2.2.2. Overtaking rules

If train j travels following the focal train i, and one of the following three situations occurs:

a) Train i and train j are running in the same section and train i is travelling in the vicinity of the frontal station m;

b) Train i is stopping at station m and train j is running behind train i;

c) Train i and train j are dwelling at the same station m.

Then we should calculate t_i and t_j to determine the priority of trains. $t_i(t_j)$ represents the time that train i (j) spends for arriving at the next station $m+1$.

If situation a) occurs, then

$$\begin{cases} t_i = (D_{m+1} - X_i)/V_{\max i} + e, \\ t_j = (D_{m+1} - X_j)/V_{\max j} + e. \end{cases} \tag{4}$$

If situation b) occurs, then

$$\begin{cases} t_i = (D_{m+1} - X_i)/V_{\max i}, \\ t_j = (D_{m+1} - X_j)/V_{\max j} + e. \end{cases} \tag{5}$$

If situation c) occurs, then

$$\begin{cases} t_i = (D_{m+1} - X_i)/V_{\max i}, \\ t_j = (D_{m+1} - X_j)/V_{\max j}. \end{cases} \tag{6}$$

The calculations show that train j will have a higher priority than train i if $t_i \geq t_j$. In this case, train i has to decelerate and enter into its frontal station m or continue dwelling at station m until train j leaves station m.

2.2.3. Travelling strategy

If train i is the leading train or running far enough away from its previous train k and the station ahead (that is $c > \max(X_c + f_{\min}, f_{\min}), g > V_i(t)^2/2a$), it will travel with acceleration or maintain the same speed

$$\begin{cases} V_i(t+\Delta t) = \min(V_{\max i}, V_i(t) + a), \\ X_i(t+\Delta t) = X_i(t) + V_i(t+\Delta t). \end{cases} \tag{7}$$

Here, the distance between train i and the front train k is denoted as c. The distance between train i and the frontal station m is represented with g. They can be given as follows:

$$\begin{cases} c = X_k(t) - X_i(t), \\ g = D_m - X_i(t). \end{cases} \tag{8}$$

2.2.4. Departing rules

We can consider two cases when the focal train i is a slower train or a faster train as follows:

Case 1 Train i is a slower train.

i) If train i has not stayed at station for the fixed dwelling time d, then it has to continue staying at the station

$$\begin{cases} V_i(t+\Delta t) = 0, \\ X_i(t+\Delta t) = X_i(t). \end{cases} \tag{9}$$

ii) If the fixed dwelling time is meet, but there is a train k in the next section and $c \leq \max(X_c + f_{\min}, f_{\min})$, then train i should stay at the station, where $X_c = (V_i(t)^2 - V_k(t)^2)/2a$.

$$\begin{cases} V_i(t+\Delta t) = 0, \\ X_i(t+\Delta t) = X_i(t). \end{cases} \tag{10}$$

iii) If the fixed dwelling time is met and the distance between the two adjacent trains is larger than the safety distance ($c > \max(X_c + f_{\min}, f_{\min})$), but the train j running behind train i has a higher priority than train i, then train i has to stay at the station

$$\begin{cases} V_i(t+\Delta t) = 0, \\ X_i(t+\Delta t) = X_i(t). \end{cases} \tag{11}$$

iv) Otherwise, train i can leave the station

$$\begin{cases} V_i(t+\Delta t) = \min(V_{\max 1}, V_i(t)+a), \\ X_i(t+\Delta t) = X_i(t) + V_i(t+\Delta t). \end{cases} \tag{12}$$

Case 2 Train i is a faster one.

If train i is a faster train, there is no need to consider the influence of priority.

1) If train i has not stayed at the station for the fixed dwelling time d, then it has to continue staying at the station

$$\begin{cases} V_i(t+\Delta t) = 0, \\ X_i(t+\Delta t) = X_i(t). \end{cases} \tag{13}$$

2) If the fixed dwelling time is met, but there is a train k in the next section and $c \le \max(X_c + f_{\min}, f_{\min})$, then train i should stay at the station, where $X_c = (V_i(t)^2 - V_k(t)^2)/2a$,

$$\begin{cases} V_i(t+\Delta t) = 0, \\ X_i(t+\Delta t) = X_i(t). \end{cases} \tag{14}$$

3) Otherwise, train i can leave the station

$$\begin{cases} V_i(t+\Delta t) = \min(V_{\max 2}, V_i(t)+a), \\ X_i(t+\Delta t) = X_i(t) + V_i(t+\Delta t). \end{cases} \tag{15}$$

Based on the proposed train control strategies, the detailed algorithm of train scheduling on the high-speed rail line is presented as follows.

Algorithm

Step 1 (Initialization) set the time $t = 0$, $b = 0$.

Step 2 (Initialization) set the position $p = 0$, the speed $V_p(0) = 0$.

Step 3 If $h(1,p) = 1$ (i.e. there is a train i at position p), go to step 4; otherwise, go to step 6.

Step 4 Set $k = 1$, $g = D - p$.

Step 4.1 If $p + k \le L$, $h(1, p+k) = 1$ or $p + k > L$, let $c = \min(k, g)$, go to step 5; otherwise, go to step 4.2.

Step 4.2 If $p + k \le L$, $h(1, p+k) \ne 1$, let $k = k + 1$, go to step 4.1.

Step 5 Determine the speed and location of the focal train i.

Step 5.1. If train i is a faster train.

Case 1 If $p \ne D$ and $c < g$.

1a) If $c \le \max(X_c + f_{\min}, f_{\min})$, let $\overline{V}_p(t+1) = \max(0, V_p(t) - a)$, then $V_p(t+1) = \min(\overline{V}_p(t+1), \sqrt{2a(c - f_{\min}) + V_{p+k}(t)^2})$. here $X_c = \left(V_p(t)^2 - V_{p+k}(t)^2\right)/2a$;

1b) If $c > \max(X_c + f_{\min}, f_{\min})$, let $V_p(t+1) = \min(V_{\max 2}, V_p(t) + a)$.

Case 2 If $p \ne D$ and $c = g$.

2a) If $g \le V_p(t)^2/2a$, let $V_p(t+1) = \min(\overline{V}_p(t+1), \sqrt{2ag})$, where $\overline{V}_p(t+1) = \max(0, V_p(t) - a)$;

2b) If $g > V_p(t)^2/2a$, let $V_p(t+1) = \min(V_{\max 2}, V_p(t) + a)$.

Case 3 If $p = D$.

3a) If $b \le e$, let $V_p(t+1) = 0$, and $b = b + 1$;

3b) If $b > e$ and $c \le \max(X_c + f_{\min}, f_{\min})$, let $V_p(t+1) = 0$, and $b = b + 1$;

3c) If $b > e$ and $c > \max(X_c + f_{\min}, f_{\min})$, let $V_p(t+1) = \min(V_{\max 2}, V_p(t) + a)$, $b = 0$.

Then go to step 5.3.

Step 5.2 If train i is a slower train.

Case 1 If $p \ne D$ and $c < g$.

1a)′ If $c \le \max(X_c + f_{\min}, f_{\min})$, let $\overline{V}_p(t+1) = \max(0, V_p(t) - a)$, then let $V_p(t+1) = \min(\overline{V}_p(t+1), \sqrt{2a(c - f_{\min}) + V_{p+k}(t)^2})$;

1b)′ If $c > \max(X_c + f_{\min}, f_{\min})$, let $V_p(t+1) = \min(V_{\max 1}, V_p(t) + a)$.

Case 2 If $p \ne D$ and $c = g$.

2a)′ If $g \le V_p(t)^2/2a$, let $V_p(t+1) = \min(\overline{V}_p(t+1), \sqrt{2ag})$, where $\overline{V}_p(t+1) = \max(0, V_p(t) - a)$;

2b)′ If $g > V_p(t)^2/2a$, let $V_p(t+1) = \min(V_{\max 1}, V_p(t) + a)$.

Case 3 If $p = D$.

3a)′ If $b \le e$, $V_p(t+1) = 0$, $b = b + 1$;

3b)′ If $b > e$ and $c \le \max(X_c + f_{\min}, f_{\min})$, let $V_p(t+1) = 0$, $b = b + 1$;

3c)′ If $b > e$, $c > \max(X_c + f_{\min}, f_{\min})$ and $t_p \ge t_{p-j}$, where $h(1, p-j) = 1$, then let $V_p(t+1) = 0$, $b = b + 1$;

3d)′ If $b > e$, $c > \max(X_c + f_{\min}, f_{\min})$ and $t_p < t_{p-j}$, where $h(1, p-j) = 1$, then let $V_p(t+1) = \min(V_{\max 1}, V_p(t) + a)$, $b = 0$.

Then go to step 5.3.

Step 5.3 Let $\overline{h}(1, p + V_p(t+1)) = 1$, $\overline{V}_{p+v_p(t+1)}(t+1) = V_P(t+1)$. Go to step 6.

Step 6 If $p < L$, let $p = p + 1$, go to step 3; otherwise, go to step 7;

Step 7 Update the state of the system:

$$h(1,p) = 0, \quad h(1, p + V_p(t+1)) = \overline{h}(1, p + V_p(t+1));$$

$$V_p(t+1) = 0, \quad V_{p+v_p(t+1)}(t+1) = \overline{V}_{p+v_p(t+1)}(t+1).$$

Then, let $t = t + 1$, go to step 2.

3. Simulation and analysis

3.1. Simulation on a railway line

We simulate the train schedule in the high-speed double-track rail line. A rail line with 5 sections is defined with a total length of 480 km. Each section has a length of 80 km. In this simulation, two types of trains are considered. Each group of the trains has its own maximum speed: $V_{\max 1} = 240$ km/h = 4 km/min; $V_{\max 2} = 300$ km/h = 5 km/min. $V_{\max 1}$ denotes the speed of the slower trains and $V_{\max 2}$ denotes the speed of the faster ones, which are roughly corresponding to the real situation of the Beijing–Shanghai High-speed Rail Line. It is assumed that the number of tracks in the station are sufficient enough, which means that more than one train can be in the same station at the same time. With the ratio of mixed trains $r = 2$ (the number ratio of faster trains to slower trains), slower trains and faster trains depart from the original station per 10 min. Trains have to reduce their speeds when approaching the stations and increase speed when departing from stations. All the trains have to stop at the station at least for 2 minutes to meet the demand of passengers getting on and off.

According to the above-mentioned approach, we simulate the schedule of the trains from simulation time 80 minutes to 200 minutes as shown in Fig. 1. The time is displayed on the horizontal axis, while the distance (position) is displayed on the vertical axis. The empty dot lines represent the positions of slower trains and the solid dot lines represent the positions of faster trains. The black horizontal lines denote the positions of stations. In Fig. 1, the train dispatched at 120 min stops at the first intermediate station waiting for its following trains, and this slower train is overtaken by its following two faster trains. The slower train shows a longer waiting time than the faster trains, which is because the slower train always has a lower priority and has to be overtaken.

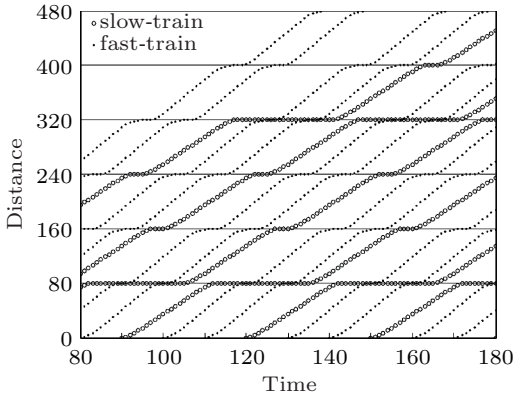


Fig. 1. Schedule of the mixed trains.

3.2. Analysis of the capacity

The capacity of the line in this paper is defined as the number of trains arrived at the terminal station in a given time. As shown in Fig. 2(a), the basic trend is that the capacity of the line decreases with the increase of departure interval, so does the overall trend of the average travel time. The reason is that the conflicts between trains will increase when the service interval is smaller, and that the smaller departure interval will lead to a high train density which can delay the travel time. However, the curve showing the average travel time has some slight fluctuations, such as the fluctuation from $d = 13$ min to $d = 15$ min. We can find the reason from the schedule of trains (see Fig. 3 and Table 2). In the condition of $d = 11$ min, the slower trains have to wait for the following two faster trains at both station 1 and station 5; while in the condition of $d = 13$ min, the slower trains only have to wait for the following faster trains at stations 2 and 3. So the average travel time with $d = 13$ min is shorter than that with $d = 11$ min. In the condition of $d = 15$ min, the dispatching order of each station is the same as in the condition of $d = 13$ min. But train 2 departs from station 2 at the 56th minute in the condition of $d = 13$ min, while it departs at the 58th minute in the condition of $d = 15$ min. As a result, in the condition of $d = 15$ min, train 1 has to wait for two more minutes at station 2, which makes the slower trains wait for a longer time, thereby leading to a longer average travel time. The same reason can account for the other fluctuations.

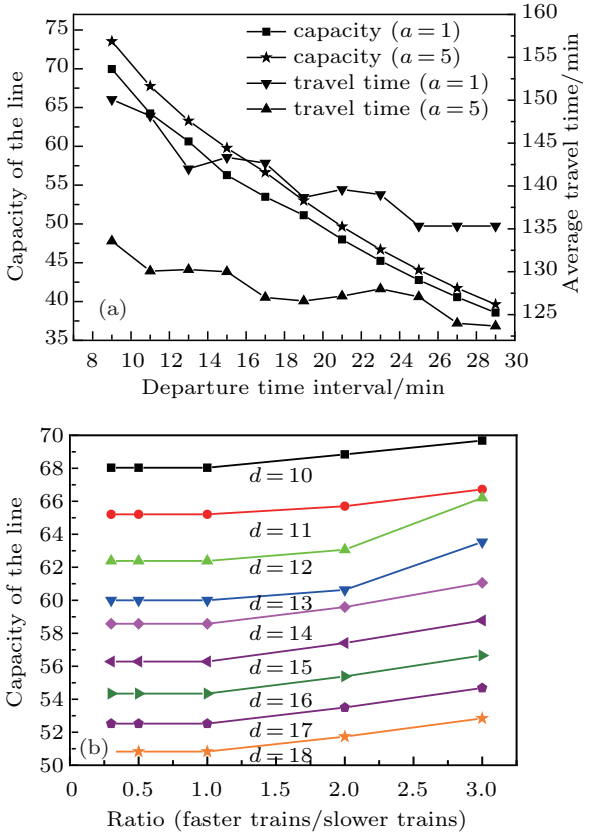


Fig. 2. (color online) Capacities of the line varying with (a) departure time interval and (b) number ratio for mixed trains.

Table 2. Departure times with (a) $d = 11$ min, (b) $d = 13$ min, and (c) $d = 15$ min.

Table 2(a)

	Starting station	Station 1	Station 2	Station 3	Station 4	Station 5	Terminal station
Train 1	0	47	72	97	122	168	191
Train 2	11	32	54	76	98	120	140
Train 3	22	43	65	87	109	131	154
Train 4	33	80	105	130	155	201	224
Train 5	44	65	87	109	131	153	173
Train 6	55	76	98	120	142	164	184
Train 7	66	113	138	163	188	234	257
Train 8	77	98	120	142	164	186	206
Train 9	88	109	131	153	175	197	217
Train 10	99	146	171	196	221	246	269
Train 11	110	131	153	175	197	219	239
Train 12	121	142	164	186	208	230	250

Table 2(b)

	Starting station	Station 1	Station 2	Station 3	Station 4	Station 5	Terminal station
Train 1	0	25	60	95	120	145	168
Train 2	13	34	56	78	100	122	142
Train 3	26	47	69	91	113	135	155
Train 4	39	64	99	134	159	184	207
Train 5	52	73	95	117	139	161	181
Train 6	65	86	108	130	152	174	194
Train 7	78	103	138	173	198	223	246
Train 8	91	112	134	156	178	200	220
Train 9	104	125	147	169	191	213	233
Train 10	117	142	177	212	237	262	285
Train 11	130	151	173	195	217	239	259
Train 12	143	164	186	208	230	252	272

Table 2(c)

	Starting station	Station 1	Station 2	Station 3	Station 4	Station 5	Terminal station
Train 1	0	25	62	99	124	149	172
Train 2	15	36	58	80	102	124	144
Train 3	30	51	73	95	117	139	159
Train 4	45	70	107	144	169	194	217
Train 5	60	81	103	125	147	169	189
Train 6	75	96	118	140	162	184	204
Train 7	90	115	152	189	214	239	262
Train 8	105	126	148	170	192	214	234
Train 9	120	141	163	185	207	229	249
Train 10	135	160	197	234	259	284	307
Train 11	150	171	193	215	237	259	279
Train 12	165	186	208	230	252	274	294

For a fixed value of departure interval, the average travel time of trains with $a = 5$ is shorter than that with $a = 1$, which is more obvious when the departure interval is small. It indicates that the accelerating or the decelerating can influence the travel time, because the train can take less time to reach its maximum speed with a bigger acceleration, which can reduce the accelerating time. Accelerating time is part of the travel time, so a bigger acceleration means a shorter travel time.

In Fig. 2(b), when $r > 1$, the general trend is that the capacity of the line increases with the increase of the ratio; when $0 < r < 1$, the capacity of the line does not change distinctly. What causes the phenomenon is that the mixed running of different trains influences each other badly when the numbers of the trains are almost equal, which is corresponding to the situation in real railway traffic. In addition, when $r > 1$, the increase of r means the decrease of slower trains, which causes less conflicts, less waiting time, and less average travel time. When $0 < r < 1$, the increase of r means the increase in the number of faster trains, which causes more conflicts for slower trains (i.e. there is an increasing possibility for slower trains to be overtaken by faster trains) but less average travel time.

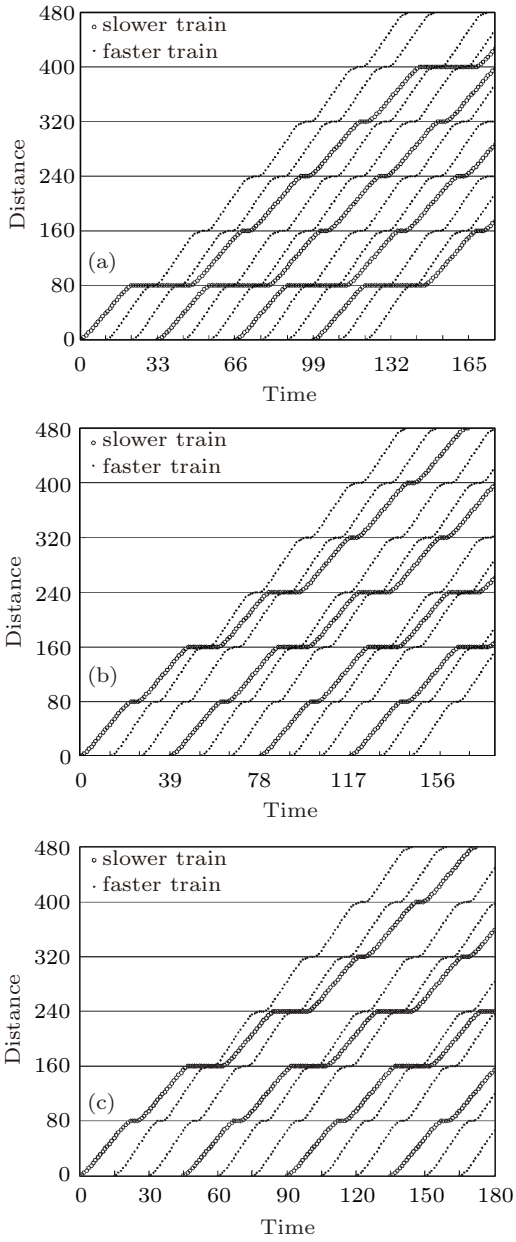


Fig. 3. Schedules of the trains with (a) $d = 11$ min, (b) $d = 13$ min, and (c) $d = 15$ min.

3.3. Analysis of average speed

What we can see from Fig. 4(a) is that the average speed of a train decreases with the increase in the number of intermediate stations. Because the larger number of intermediate stations means the less distance between two adjacent stations, the times for accelerating and decelerating would be increased, which leads to the decrease of the average speed. With the number of intermediate stations fixed, the average speed of trains increases with the increase of the number ratio of the mixed trains (see Fig. 5(a)). Because the increase of the ratio represents the increasing number of faster trains, which causes the shorter average travel time and the faster average speed.

The average speed of faster trains is slowed down by increasing the number of intermediate stations in Fig. 4(b). If the number of intermediate stations is fixed (see Fig. 5(b)), we can observe that the average speed of faster trains shows no distinct change with respect to the number ratio of mixed trains, because the faster trains always have the priority to overtake the slower trains.

In Fig. 4(c), the average speed of a slower train is also reduced with the increasing of the number of intermediate stations. Fixing the number of the intermediate stations (see Fig. 5(c)), the average speed of a slower train generally decreases with the increase of the number ratio of mixed trains. The explanation is that the increase of the number ratio means the increasing number of faster trains, which makes the slower trains face more constraints such as more chances to be overtaken. So the slower trains have to wait for a longer time, which leads to a longer total travel time.

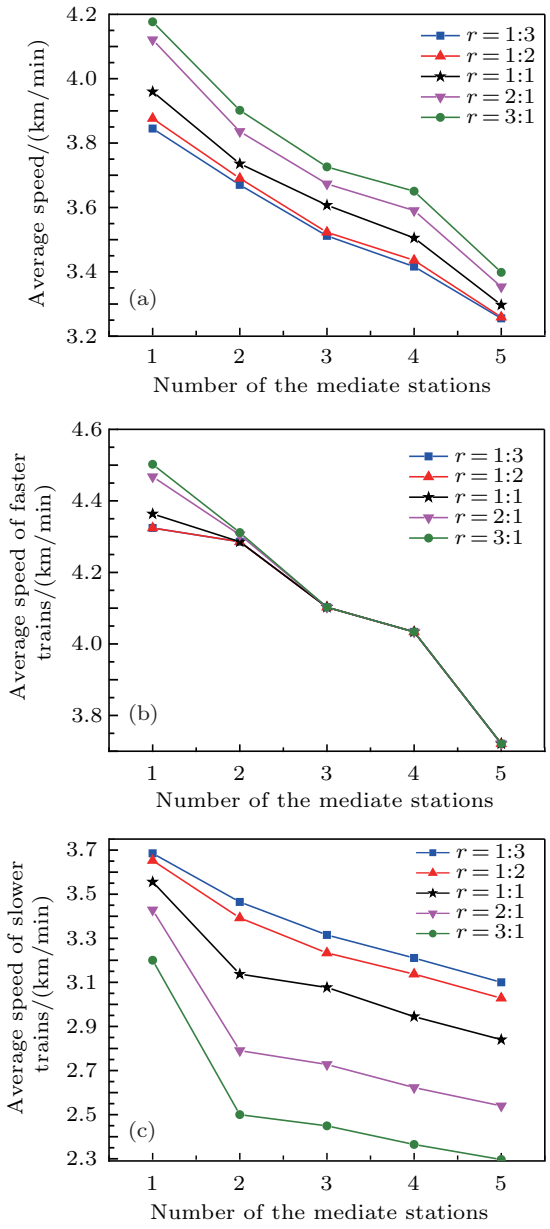


Fig. 4. (color online) Average speeds of mixed trains varying with the number of intermediate stations in different cases. (a) The average speeds of the whole trains. (b) The average speeds of faster trains. (c) The average speeds of slower trains.

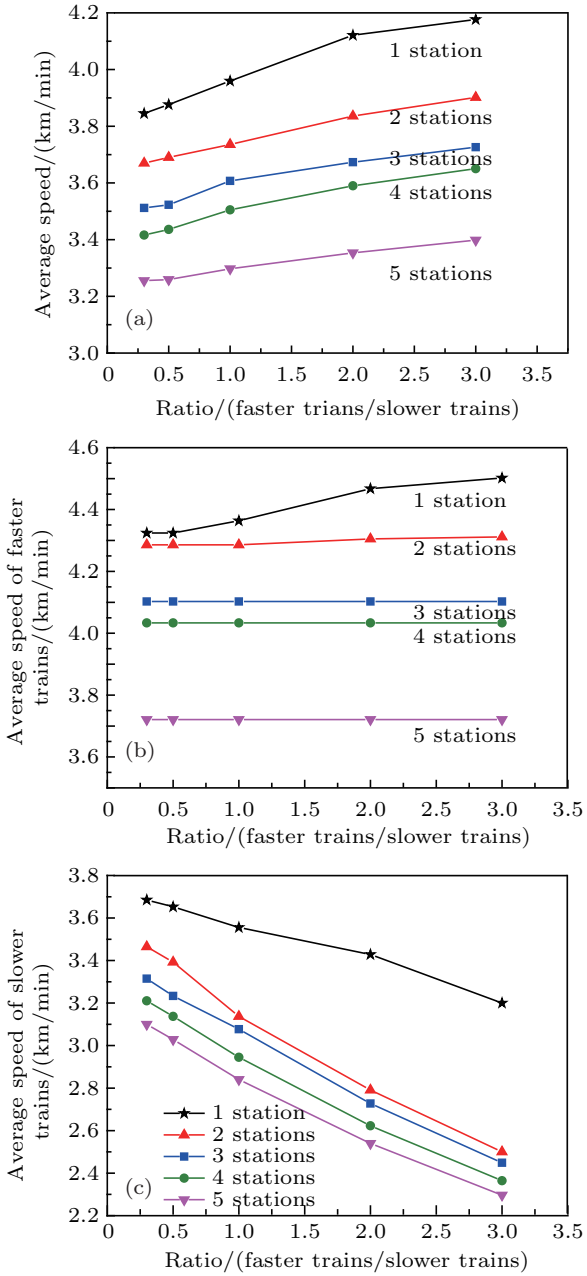


Fig. 5. (color online) Average speeds of mixed trains varying with the number ratio for mixed trains in different cases. (a) The average speeds of the whole trains. (b) The average speeds of faster trains. (c) The average speeds of slower trains.

4. Conclusions

In this paper, we propose a new approach to simulating the train movements on the high-speed rail line based on the discrete-time movement model and also present efficient control strategies of the train movements. In our method, different strategies are presented for the different situations of train movements, which is related to the actual headway and the gap between the station and the train. Based on the proposed control strategies a new detailed algorithm is designed for less travel time. The designed algorithm can be successfully used for simulating the train scheduling on the high-speed rail line.

Moreover, we analyze the properties of rail traffic flow under the influences of the number ratio of mixed trains with different speeds, departure intervals, distance between two adjacent stations and acceleration. The simulation results indicate that the capacity of the rail line will increase with departure interval decreasing. When the number ratio of mixed trains with different speeds is 1 (i.e., in the case where the number of faster trains is equal to the number of slower trains), the capacity of the rail line is badly influenced. Meanwhile, the capacity and the average speed will be increased by appropriately increasing the number ratio of mixed trains with different speeds from 1. As to the average speed, the variation of mixed ratios exerts little effect on the average speed of a faster train but a great influence on a slower train. With a reasonably large distance between adjacent stations, the average speed of a train is increased obviously.

In the further study, the network-based simulation should be carried out based on this paper. Meanwhile, we can consider more influential elements for the analysis of the properties of rail traffic flow.

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