Conserved charges of Kerr–Ads spacetime using Poincaré gauge version*

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(Received 26 April 2013; revised manuscript received 22 May 2013)

The process of covariant conserved charge of gravitational theory, which is covariant under general coordinate and local Lorentz transformations, has been applied to many tetrad fields, which reproduce Kerr-Ads spacetime, to calculate their conserved charges. It is shown that this process gives an infinite value of the conserved charges for Kerr–Ads spacetime. Therefore, the method of "regularization through relocalization", i.e., modification of the Lagrangian of the gravitational field through a total derivative, is used. This method gaves a finite and a consistent result of total energy and angular momentum for Kerr–Ads spacetime.

Keywords: Kerr–Ads spacetimes, total conserved charges, Poincaré gauge version

PACS: 04.20.Cv, 04.50.Kd, 04.90.+e **DOI:** 10.1088/1674-1056/22/12/120401

1. Introduction

Energy–momentum tensor, $T^{\mu\nu}$, can be viewed as the corner stone for conserved quantity, which is related to the symmetry of the spacetime. Using Noether's theory and translation invariance one can obtain a tensor $T^{\mu\nu}$ which can characterize the density of energy–momentum and describes a conserved energy–momentum due to its verifying of the differential conservation law ${T^{\mu\nu}}_{,\nu} = 0$ where comma means ordinary differentiation. This is not single, because one can gather an arbitrary "curl" that relocates the zero of the energy–momentum tensor. However, the gravitational reply through Einstein's field equations $G^{\mu\nu} = -\kappa T^{\mu\nu}$ detects the total energy–momentum density for matter and gravitation fields, and consequently sectors the ordinary uncertainty in the energy–momentum terms. The interaction field is trivial only if the total energy–momentum vanishes. Therefore, it is cynic that the usual identification of the sharing to the total energy–momentum from the interaction field itself has shown to be so elusive. This is because of the construction in which Einstein's theory of general relativity (GR) built in "Riemannian geometry" from which one can construct a single metric and a single connection. This geometry is the basic reason responsible for the problem of giving a regular formula of energy besides the equivalence principle of GR theory. Applying the Lagrange–Noether method, one can deduce the conserved currents which emerge from the covariance of the ordinary action under transformations of the fields. Yet, in Riemannian geometry, which GR theory built in, one cannot detect symmetries which could be applied to the output of the conserved energy–momentum currents. One can only calculate the energy of asymptotic flat spacetime.[1–6]

In the context of teleparallel equivalent of general relativity (TEGR) it is likely to make concrete statements linked to energy and momentum of the gravitational field. This truth is considered as the major push for dealing with TEGR theory. In the four-dimensions of the TEGR,[7] and by putting Schwinger's time gauge constraint[8] for the tetrad field by establishing ${h_{(a)}}^0 = 0$ that leads to ${h^{(0)}}_a = 0$, it is shown that the Hamiltonian and vector obliges have each one a divergence in the formula of scalar and vector densities which can be linked to the energy and momentum densities of the gravitation theory of TEGR.[9] This correspondence has been demonstrated to be compatible, and it has been shown that teleparallel theories supply a natural instrument for the investigation of gravitational energy.

Nowadays it seems that, TEGR theories are motivated for the following arguments: firstly, GR can be shown as a certain theory of teleparallelism and, thus, teleparallelism can be viewed at the very least as another point of view which could direct us to the same physics.[10] Secondly, in the framework of TEGR, one can formulate an energy–momentum tensor for the interaction field which is a true tensor under all general coordinate transformations. This was the main reason why teleparallelism was reviewed by Møller when he was analyzing the problem of formulating an energy–momentum tensor for the interaction field.[11] The philosophy was taken over by Pellegrini and Plebański who built the general Lagrangian for such theories.[12] Thirdly why these theories are interesting is that they can be viewed as gauge theories of the translation group (not the full Poincaré group) and, therefore, they give an amendments interpretation of GR.[13–20] Some researchers dealt with teleparallel theory as a fundamental part of generalized non-Riemannian theories like the

*Project partially supported by the Science Fund from the Egyptian Ministry of Scientific Research (Grant No. 24-2-12).
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Poincaré gauge theory[21–29] or metric-affine gravity.[30–35] In the framework of metric-affine gravity, a static axially symmetric analytic solution of the vacuum field equations has been obtained.[36]

The main difference between the Einstein GR theory and teleparallel theories is that it is prospective to differentiate gravitation and inertia.[37–39] This returns to the fact that inertia is in the domain of the pseudotensor behavior of the usual terms of the gravitational energy–momentum density. However, in teleparallel gravity it becomes possible to write down a tensorial expression for such density.[40] It is the aim of the present study to calculate the total conserved charges of many tetrad fields that reproduce Kerr–Ads spacetime using two methods: covariant conserved charge of gravitational theory and regularization through relocalization. We show that the value of conserved charges of Kerr–Ads are finite and have a well-known formula when the regularization through relocalization method is used.

The rest of this paper is arranged as follows. In Section 2, a review of the teleparallel gravity and total conserved charges are presented. In Section 3, we give the first tetrad field that reproduces Kerr–Ads spacetime and show that its total charges are divergent in both time and angular momentum coordinates. In Section 4, we give the regularization via relocalization method, i.e. Poincaré gauge version, and show that the conserved charges are finite and in the well-know formula. In Section 5, we give the second tetrad field that also reproduces Kerr–Ads spacetime, we calculate its total conserved charge and show that the relocalization method is also valid to give the correct value of conserved charges of Kerr–Ads spacetime. In Section 6, the third tetrad field that also gives Kerr–Ads spacetime is provided and the same analysis used in Section 4 has been applied to calculate the total conserved charges. The final section is devoted to the main results.

We first give the notations which will be used in this paper. In this study, Latin indices $i, j, \ldots$ refer to local holonomic spacetime coordinates while Greek indices $\alpha, \beta, \ldots$ label (co)frame components are used. Particular frame components are denoted by hats, $\hat{0}$, $\hat{1}$, etc. As usual, the exterior product is denoted by $\wedge$, while the interior product of a vector ξ and a p-form Ψ is denoted by $\xi\rfloor\Psi$. The vector basis dual to the frame 1-forms ϑ^α is denoted by e_α and they satisfy $e_\alpha\rfloor\vartheta^\beta = \delta_\alpha{}^\beta$. Using local coordinates x^i, we have $\vartheta^\alpha = h_i^\alpha \mathrm{d}x^i$ and $e_\alpha = h_\alpha^i \partial_i$ where h_i^α and h_α^i are the covariant and contravariant components of the tetrad field. We define the volume 4-form by

$$\eta \overset{\text{def.}}{=} \vartheta^{\hat{0}} \wedge \vartheta^{\hat{1}} \wedge \vartheta^{\hat{2}} \wedge \vartheta^{\hat{3}}. \tag{1}$$

Furthermore, with the help of the interior product we define

$$\eta_\alpha \overset{\text{def.}}{=} e_\alpha\rfloor\eta = \frac{1}{3!}\varepsilon_{\alpha\beta\gamma\delta}\ \vartheta^\beta \wedge \vartheta^\gamma \wedge \vartheta^\delta = {}^*\vartheta_\alpha,$$

where $\varepsilon_{\alpha\beta\gamma\delta}$ completely antisymmetric with $\varepsilon_{0123} = 1$. Here $*$ refers to Hodge dual operator.

$$\eta_{\alpha\beta} \overset{\text{def.}}{=} e_\beta\rfloor\eta_\alpha = \frac{1}{2!}\varepsilon_{\alpha\beta\gamma\delta}\ \vartheta^\gamma \wedge \vartheta^\delta = {}^*\left(\vartheta_\alpha \wedge \vartheta_\beta\right),$$

$$\eta_{\alpha\beta\gamma} \overset{\text{def.}}{=} e_\gamma\rfloor\eta_{\alpha\beta} = \frac{1}{1!}\varepsilon_{\alpha\beta\gamma\delta}\ \vartheta^\delta = {}^*\left(\vartheta_\alpha \wedge \vartheta_\beta \wedge \vartheta_\gamma\right),$$

which are bases for 3-, 2- and 1-forms respectively. Finally,

$$\begin{aligned}\eta_{\alpha\beta\mu\nu} &\overset{\text{def.}}{=} e_\nu\rfloor\eta_{\alpha\beta\mu} = e_\nu\rfloor e_\mu\rfloor e_\beta\rfloor e_\alpha\rfloor\eta \\ &= {}^*\left(\vartheta_\alpha \wedge \vartheta_\beta \wedge \vartheta_\mu \wedge \vartheta_\nu\right) = \varepsilon_{\alpha\beta\gamma\delta}\end{aligned}$$

is the Levi–Civita tensor density. The η-forms satisfy the useful identities:

$$\begin{aligned}&\vartheta^\beta \wedge \eta_\alpha \overset{\text{def.}}{=} \delta_\alpha^\beta \eta, \quad \vartheta^\beta \wedge \eta_{\mu\nu} \overset{\text{def.}}{=} \delta_\nu^\beta \eta_\mu - \delta_\mu^\beta \eta_\nu, \\ &\vartheta^\beta \wedge \eta_{\alpha\mu\nu} \overset{\text{def.}}{=} \delta_\alpha^\beta \eta_{\mu\nu} + \delta_\mu^\beta \eta_{\nu\alpha} + \delta_\nu^\beta \eta_{\alpha\mu}, \\ &\vartheta^\beta \wedge \eta_{\alpha\gamma\mu\nu} \overset{\text{def.}}{=} \delta_\nu^\beta \eta_{\alpha\gamma\mu} - \delta_\mu^\beta \eta_{\alpha\gamma\nu} + \delta_\gamma^\beta \eta_{\alpha\mu\nu} - \delta_\alpha^\beta \eta_{\gamma\mu\nu}.\end{aligned} \tag{2}$$

The line element $\mathrm{d}s^2 \overset{\text{def.}}{=} g_{\alpha\beta}\vartheta^\alpha \otimes \vartheta^\beta$ is defined by the spacetime metric $g_{\alpha\beta}$.

2. Brief review of teleparallel gravity and conserved currents

The Einstein–Cartan theory described by the Hilbert–Einstein Lagrangian plus a cosmological term is given by:[41]

$$V(\vartheta^\alpha, R^\alpha{}_\beta) = -\frac{1}{2\kappa}\left(R^{\alpha\beta} \wedge \eta_{\alpha\beta} - 2\Lambda\eta - \boldsymbol{F}^2\right), \tag{3}$$

where κ is the gravitational coupling constant and Λ is the cosmological constant (with a dimension of the inverse length square). The Maxwell tensor has the form:

$$F_{\mu\nu} = A_{[\mu,\nu]}. \tag{4}$$

Hereafter, we will denote the symmetric part by ( ) and the antisymmetric part [ ]. Variation of Eq. (3) with respect to the coframe ϑ^α and $R^\alpha{}_\beta$ gives the canonical energy–momentum and rotational gauge field momentum respectively in the form:[41,42]

$$\begin{aligned}E_\alpha &\overset{\text{def.}}{=} -\frac{\partial V}{\partial\vartheta^\alpha} = -\frac{1}{2\kappa}\left(R^{\beta\gamma} \wedge \eta_{\alpha\beta\gamma} - 2\Lambda\eta_\alpha\right), \\ H_{\alpha\beta} &\overset{\text{def.}}{=} -\frac{\partial V}{\partial R_{\alpha\beta}} = \frac{1}{2\kappa}\eta_{\alpha\beta}.\end{aligned} \tag{5}$$

Moreover, the translational momentum and the spin 3-forms have the form

$$H_\alpha \overset{\text{def.}}{=} -\frac{\partial V}{\partial T^\alpha} = 0, \quad E_{\alpha\beta} \overset{\text{def.}}{=} -\vartheta^{[\alpha} \wedge H^{\beta]} = 0. \tag{6}$$

The minimal coupling of matter is assumed so that

$$\frac{\partial L_{\text{matter}}}{\partial T^\alpha} = 0$$

and

$$\frac{\partial L_{\text{matter}}}{\partial R^\alpha_\beta} = 0.$$

The covariant current of gravitational field is described by[41]

$$\begin{aligned}&\mathcal{J}[\xi] = \frac{1}{2\kappa}\mathrm{d}\left\{{}^*\left[\mathrm{d}k + \xi\rfloor\left(\vartheta^\alpha \wedge T_\alpha\right)\right]\right\}, \\ &k = \xi_\alpha\vartheta^\alpha, \quad \xi^\alpha = \xi\rfloor\vartheta^\alpha,\end{aligned} \tag{7}$$

where ξ is an arbitrary vector field $\xi = \xi^i \partial_i$ and ξ^i are the components with four constant parameters ξ^0, ξ^1, ξ^2, and ξ^3. For solutions of spinless matter or in vacuum, the torsion vanishes, $T^\alpha = 0$, and hence the total conserved charges take the form

$$\mathcal{Q}[\xi] = \frac{1}{2\kappa}\int_{\partial S} {}^*\mathrm{d}k. \tag{8}$$

This invariant conserved quantity, $\mathcal{Q}[\xi]$, is the same as Komar formula.[43]

3. First Kerr–Ads spacetime

The covariant form of the first Kerr–Ads tetrad field, $(h^\alpha{}_i)_1$, having axial symmetry in Boyer–Lindquist coordinates (t, r, θ, ϕ), can be written as

$$\left(h^\alpha{}_i\right)_1 = \begin{pmatrix} \mathcal{A}_1 & 0 & 0 & \mathcal{A}_2 \\ 0 & \mathcal{A}_3 & 0 & 0 \\ 0 & 0 & \mathcal{A}_4 & 0 \\ 0 & 0 & 0 & -\mathcal{A}_5 \sin\theta \end{pmatrix}, \tag{9}$$

where $\mathcal{A}_i$, $i = 1, \ldots, 5$ are functions of r, θ and have the form

$$\mathcal{A}_1 = \sqrt{\frac{\mathcal{B}_1}{\mathcal{A}}}, \quad \mathcal{A}_2 = \frac{\mathcal{C}}{\sqrt{\mathcal{A}\mathcal{B}_1}}, \quad \mathcal{A}_3 = \sqrt{\frac{\mathcal{A}}{\mathcal{B}}}, \quad \mathcal{A}_4 = \sqrt{\frac{\mathcal{A}}{\mathcal{B}_2}},$$
$$\mathcal{A}_5 = \sqrt{\frac{\mathcal{A}\mathcal{B}\mathcal{B}_2}{\mathcal{B}_1}}, \quad \mathcal{A} = r^2 + a^2\cos^2\theta,$$
$$\mathcal{B} = (r^2+a^2)\left(1 - \frac{r^2\Lambda}{3}\right) - 2Mr, \quad \mathcal{B}_1 = \mathcal{B} - a^2\mathcal{B}_2\sin^2\theta,$$
$$\mathcal{B}_2 = 1 + \frac{a^2\Lambda\cos^2\theta}{3}, \quad \mathcal{C} = a\Omega\sin^2\theta\left(\mathcal{A}\Lambda[r^2+a^2] + 6Mr\right),$$
$$\Omega = 1 + \frac{a^2\Lambda}{3}. \tag{10}$$

M, a, and Λ are the mass, the rotation, and the cosmological constant respectively.[44] We consider a non-asymptotically flat spacetime in this paper, and impose the boundary condition that for $r \to \infty$ and $\Lambda \to 0$ the tetrad (9) approaches the tetrad of Minkowski spacetime, in Cartesian coordinate, i.e., $O_{\alpha\beta} = \mathrm{diag}(+1, -1, -1, -1)$. The metric tensor $g_{ij} \overset{\text{def.}}{=} O_{\mu\nu}h^\mu{}_i h^\nu{}_j$ associated with the tetrad field (9) has the form

$$\mathrm{d}s^2 = \mathcal{A}_1{}^2\mathrm{d}t^2 - \mathcal{A}_3{}^2\mathrm{d}r^2 - \mathcal{A}_4{}^2\mathrm{d}\theta^2 - \left(\mathcal{A}_5{}^2\sin^2\theta - \mathcal{A}_2{}^2\right)\mathrm{d}\phi^2 - \mathcal{A}_2\mathcal{A}_5\,\mathrm{d}t\,\mathrm{d}\phi, \tag{11}$$

which is the Kerr–Ads spacetime written in Boyer–Lindquist coordinates.[37,38] The previously obtained solutions, Schwarzschild and Kerr spacetimes can be generated as special solutions of tetrad (9), using Eq. (10), by putting $a = 0$ and $\Lambda = 0$, $\Lambda = 0$ respectively.[45,46]

The coframe ϑ^δ of tetrad (9) has the form

$$\vartheta_1{}^{\hat{0}} = \mathcal{A}_1\,\mathrm{d}t + \mathcal{A}_2\,\mathrm{d}\phi, \quad \vartheta_1{}^{\hat{1}} = \mathcal{A}_3\,\mathrm{d}r, \quad \vartheta_1{}^{\hat{2}} = \mathcal{A}_4\,\mathrm{d}\theta,$$
$$\vartheta_1{}^{\hat{3}} = \sin\theta\mathcal{A}_5\,\mathrm{d}\phi. \tag{12}$$

Using Eq. (12) in Eq. (7) we get

$$k = \mathcal{A}_1(\mathcal{A}_1\xi_0 + \mathcal{A}_2\xi_3)\,\mathrm{d}t - \mathcal{A}_3\xi_1\,\mathrm{d}r - \mathcal{A}_4\,\mathrm{d}\theta - \left([\mathcal{A}_5{}^2\sin^2\theta - \mathcal{A}_2{}^2]\xi_3 + \mathcal{A}_1\mathcal{A}_2\xi_0\right)\mathrm{d}\phi. \tag{13}$$

The total derivative of Eq. (13) has the form

$$\begin{aligned}\mathrm{d}k = {} & \left(\left[2\mathcal{A}_5\frac{\partial\mathcal{A}_5}{\partial\theta}\sin^2\theta + 2\mathcal{A}_2\frac{\partial\mathcal{A}_2}{\partial\theta} - \mathcal{A}_5{}^2\sin 2\theta\right]\xi_3 \right. \\ & \left. + \left[\mathcal{A}_1\frac{\partial\mathcal{A}_2}{\partial\theta} + \mathcal{A}_2\frac{\partial\mathcal{A}_1}{\partial\theta}\right]\xi_0\right)(\mathrm{d}\theta\wedge\mathrm{d}\phi), \\ & \left(\left[2\mathcal{A}_5\frac{\partial\mathcal{A}_5}{\partial\theta}\sin^2\theta - 2\mathcal{A}_2\frac{\partial\mathcal{A}_2}{\partial\theta}\right]\xi_3 \right. \\ & \left. - \left[\mathcal{A}_1\frac{\partial\mathcal{A}_2}{\partial\theta} + \mathcal{A}_2\frac{\partial\mathcal{A}_1}{\partial\theta}\right]\xi_0\right)(\mathrm{d}r\wedge\mathrm{d}\phi) \\ & - \left(\left[\mathcal{A}_1\frac{\partial\mathcal{A}_2}{\partial\theta} + \mathcal{A}_2\frac{\partial\mathcal{A}_1}{\partial\theta}\right]\xi_3 + 2\mathcal{A}_1\frac{\partial\mathcal{A}_1}{\partial\theta}\xi_0\right) \\ & \times\left[(\mathrm{d}r\wedge\mathrm{d}t) + (\mathrm{d}\theta\wedge\mathrm{d}t)\right].\end{aligned} \tag{14}$$

Using Eq. (12) we get

$$\mathrm{d}t = \frac{\vartheta_1{}^0\mathcal{A}_5\sin\theta - \vartheta_1{}^3}{\sin\theta\mathcal{A}_5\mathcal{A}_1}, \quad \mathrm{d}r = \frac{\vartheta_1{}^1}{\mathcal{A}_3}, \quad \mathrm{d}\theta = \frac{\vartheta_1{}^2}{\mathcal{A}_4},$$
$$\mathrm{d}\phi = \frac{\vartheta_1{}^3}{\mathcal{A}_5\sin\theta}. \tag{15}$$

Using Eq. (15) in Eq. (14) and apply the Hodge-dual to $\mathrm{d}k$ we finally get the total conserved charge in the form

$$\mathcal{Q}[\xi] = \left(12\Omega M - 4r\Lambda[\Omega r^2 - a\{1 + a\Omega\}]\right)\frac{\xi_0}{3} + \Omega\left(24M - 12r + 4\Lambda r^3 - 24aM\Omega\right)\xi_3. \tag{16}$$

Equation (16) shows that the total conserved charge of tetrad (9) using Eq. (8) is divergent in both coordinates, time and angular momentum coordinates. Therefore, equation (8) needs a regularization.

4. Regularization through relocalization of the conserved charge

Equation (8) is invariant under general coordinate and local Lorentz transformations. It is shown in Ref. [41] that in addition to the local coordinate and the local Lorentz freedom, there is another ambiguity in the definition of the conserved quantities which originates in the fact that the field equations always allow for a relocalization of the gravitational field momenta. Thus, the conserved currents and the values of the total conserved charges can be changed by means of the relocalization of a translational and rotational momenta. A relocalization produced by the change of the gravitational field Lagrangian by a total derivative is given by

$$V' = V + \mathrm{d}\Phi, \quad \Phi = \Phi(\vartheta^\alpha, \Gamma_\alpha{}^\beta, T^\alpha, R_\alpha{}^\beta). \tag{17}$$

The second term appears in the Lagrangian, i.e., $\mathrm{d}\Phi$, changes only the boundary part of the action, leaving the field equations unchanged (for more details see Ref. [41] and references therein). It is shown that total conserved quantities can be regularized by means of a relocalization of the gravitational field momenta. It is shown that the most active procedure which can solve the unacceptable result derived in Eq. (16) is the use of relocalization generated by a boundary term in the action. The

invariant conserved charge using the relocalization method has the form:[41]

$$\mathcal{Q}[\xi] = -\frac{3}{4\kappa\lambda}\int_{\partial S}\eta_{\alpha\beta\mu\nu}\Xi^{\alpha\beta}W^{\mu\nu}, \tag{18}$$

where $W^{\mu\nu}$ is the Weyl 2-form

$$W^{\alpha\beta} = \frac{1}{2}C_{\mu\nu}{}^{\alpha\beta}\vartheta^{\mu}\wedge\vartheta^{\nu}, \tag{19}$$

with $C_{\mu\nu}{}^{\alpha\beta} = h_{\mu}{}^{i}h_{\nu}{}^{j}h^{\alpha}{}_{k}h^{\beta}{}_{l}C_{ij}{}^{kl}$ being the Weyl tensor. $\Xi^{\alpha\beta}$ is defined as

$$\Xi_{\alpha\beta} = \frac{1}{2}e_{\beta}\rfloor e_{\alpha}\rfloor \mathrm{d}k. \tag{20}$$

Now we are going to calculate the non-vanishing components of Weyl tensor (These calculations are checked using Maple software[15])

$$C_{0303} = -C_{0330} = -C_{3030} = C_{3003} = -C_{1212} = -C_{1221} = -C_{2112} = C_{2121} = \frac{Mr[3a^2\cos^2\theta - r^2]}{\mathcal{A}^3},$$

$$C_{0321} = -C_{0312} = C_{3012} = -C_{3021} = C_{1230} = -C_{1203} = -C_{2130} = C_{2103} = \frac{Ma\cos\theta[a^2\cos^2\theta - 3r^2]}{\mathcal{A}^3},$$

$$C_{0202} = -C_{0220} = C_{2020} = -C_{2002} = -C_{1313} = C_{1331} = C_{3113} = -C_{3131}$$
$$= \frac{Mr[r^2 - 3a^2\cos^2\theta]\Big(\frac{1}{3}\Lambda a^4\cos^4\theta + a^2\Big(1-\frac{a^2\Lambda}{3}\Big)\cos^2\theta + \frac{a^2}{2}\Big(\frac{r^2\Lambda}{3}-3\Big) + r\Big[\frac{r^3\Lambda}{6} - \frac{r}{2} + 2M\Big]\Big)}{\mathcal{A}^3\Big(-\frac{1}{6}\Lambda a^4\cos^4\theta - \frac{a^2}{2}\Big(1-\frac{a^2\Lambda}{3}\Big)\cos^2\theta + r\Big(\frac{r^3\Lambda}{6} + \frac{ra^2\Lambda}{6} - \frac{r}{2} + M\Big)\Big)},$$

$$C_{0201} = -C_{0210} = C_{1020} = -C_{1002} = C_{1323} = -C_{1332} = C_{2313} = -C_{2331} = C_{2010} = -C_{2001} = C_{3123}$$
$$= -C_{3132} = C_{3213} = -C_{3231} = -\frac{9Ma^2\sqrt{\mathcal{B}_2\mathcal{B}}\sin 2\theta\Big[r^2 - \frac{a^2\cos^2\theta}{3}\Big]}{\mathcal{A}^3\Big(-\frac{1}{3}\Lambda a^4\cos^4\theta - a^2\Big(1-\frac{a^2\Lambda}{3}\Big)\cos^2\theta + r\Big(\frac{r^3\Lambda}{3} + \frac{ra^2\Lambda}{3} - r + 2M\Big)\Big)},$$

$$C_{0232} = -C_{0223} = C_{2320} = -C_{2302} = C_{3202} = -C_{3220} = C_{2023} = -C_{2032} = -C_{1310} = C_{1301} = C_{0131} = -C_{0113} = -C_{1013}$$
$$= C_{1031} = -C_{3110} = C_{3101} = \frac{3Mar\sin\theta[3a^2\cos^2\theta - r^2]\sqrt{\mathcal{B}\mathcal{B}_2}}{\mathcal{A}^3\Big(-\frac{1}{3}\Lambda a^4\cos^4\theta - a^2\Big(1-\frac{a^2\Lambda}{3}\Big)\cos^2\theta + r\Big(\frac{r^3\Lambda}{3} + \frac{ra^2\Lambda}{3} - r + 2M\Big)\Big)},$$

$$C_{0231} = -C_{0213} = C_{1320} = -C_{1302} = C_{2013} = -C_{2031} = -C_{3120} = C_{3102}$$
$$= \frac{3Ma\cos\theta\Big[r^2 - \frac{a^2\cos^2\theta}{3}\Big]\Big(\frac{1}{3}\Lambda a^4\cos^4\theta + a^2\Big(1-\frac{a^2\Lambda}{3}\Big)\cos^2\theta + \frac{a^2}{2}\Big(\frac{r^2\Lambda}{3}-3\Big) + r\Big(\frac{r^3\Lambda}{6} - \frac{r}{2} + M\Big)\Big)}{\mathcal{A}^3\Big(-\frac{1}{6}\Lambda a^4\cos^4\theta - \frac{a^2}{2}\Big(1-\frac{a^2\Lambda}{3}\Big)\cos^2\theta + r\Big(\frac{r^3\Lambda}{6} + \frac{ra^2\Lambda}{6} - \frac{r}{2} + M\Big)\Big)},$$

$$C_{0102} = -C_{0120} = C_{2010} = -C_{2001} = C_{1020} = -C_{1002}$$
$$= -\frac{9Ma^2\mathcal{B}_2\sin 2\theta\Big[r^2 - \frac{a^2\cos^2\theta}{3}\Big]\Big[\frac{a^2}{2}\Big(\frac{1}{3}\Lambda r^2 - 1\Big) + r\Big(\frac{r^3\Lambda}{6} - \frac{r}{2} + M\Big)\Big]}{\mathcal{A}^3\sqrt{\mathcal{B}}\Big(-\frac{1}{3}\Lambda a^4\cos^4\theta - a^2\Big(1-\frac{a^2\Lambda}{3}\Big)\cos^2\theta + r\Big(\frac{r^3\Lambda}{3} + \frac{ra^2\Lambda}{3} - r + 2M\Big)\Big)},$$

$$C_{0321} = -C_{0312} = C_{3012} = -C_{3021} = \frac{Ma\cos\theta[a^2\cos^2\theta - 3r^2]}{\mathcal{A}^3},$$

$$C_{0101} = -C_{0110} = -C_{1010} = C_{1001} = C_{2323} = -C_{2332} = -C_{3223} = C_{3232}$$
$$= \frac{Mr[r^2 - 3a^2\cos^2\theta]\Big(\frac{1}{6}\Lambda a^4\cos^4\theta + \frac{a^2}{2}\Big(1-\frac{a^2\Lambda}{3}\Big)\cos^2\theta + \frac{a^2}{2}\Big(\frac{2r^2\Lambda}{3}-3\Big) + r\Big(\frac{r^3\Lambda}{3} - r + 2M\Big)\Big)}{\mathcal{A}^3\Big(-\frac{1}{6}\Lambda a^4\cos^4\theta - \frac{a^2}{2}\Big(1-\frac{a^2\Lambda}{3}\Big)\cos^2\theta + r\Big(\frac{r^3\Lambda}{6} + \frac{ra^2\Lambda}{6} - \frac{r}{2} + M\Big)\Big)},$$

$$C_{0132} = -C_{0123} = C_{2310} = -C_{2301} = C_{3201} = -C_{3210} = -C_{1032} = C_{1023} = C_{2310}$$
$$= -C_{2301}\frac{3Ma\cos\theta\Big[r^2 - \frac{a^2\cos^2\theta}{3}\Big]\Big(\frac{1}{6}\Lambda a^4\cos^4\theta + \frac{a^2}{2}\Big(1-\frac{a^2\Lambda}{3}\Big)\cos^2\theta + \frac{a^2}{2}\Big(\frac{2r^2\Lambda}{3}-3\Big) + r\Big(\frac{r^3\Lambda}{3} - r + 2M\Big)\Big)}{\mathcal{A}^3\Big(-\frac{1}{6}\Lambda a^4\cos^4\theta - \frac{a^2}{2}\Big(1-\frac{a^2\Lambda}{3}\Big)\cos^2\theta + r\Big(\frac{r^3\Lambda}{6} + \frac{ra^2\Lambda}{6} - \frac{r}{2} + M\Big)\Big)}. \tag{21}$$

The non-vanishing components of $\Xi^{\alpha\beta}$ up to $O(1/r^3)$ have the form

$$\Xi_{01} \approx -\frac{(r^3\lambda - 3M)(\xi_0 - a\Omega\sin^2\theta\xi_3)}{3r^2} + O\Big(\frac{1}{r^3}\Big), \quad \Xi_{13} \approx -\frac{\mathcal{B}_2\sin\theta(2\xi_0\Lambda ar + \xi_3\Omega[2r^3\Lambda - \Lambda ra^2 - 3r + 6M])}{\sqrt{\Lambda}} + O\Big(\frac{1}{r^3}\Big),$$

$$\Xi_{23} \approx \Big(\frac{a\Lambda\xi_0}{3} - \Omega^2\xi_3\Big)\cos\theta + O\Big(\frac{1}{r^3}\Big). \tag{22}$$

Using Eqs. (21) and (22) we get

$$\eta_{\alpha\beta\mu\nu}\Xi^{\alpha\beta}W^{\mu\nu}=\frac{4M\Omega\Lambda\sin\theta[2\xi_0-3\xi_3 a\Omega\sin^2\theta]}{3}. \quad (23)$$

Substituting Eq. (23) into Eq. (18) we finally get

$$\mathcal{Q}[\xi]=M\Omega\xi_0-Ma\Omega^2\xi_3, \quad (24)$$

which is the conserved quantities of Kerr–Ads.[38,41]

5. Second Kerr−Ads spacetime

The covariant form of the second Kerr–Ads tetrad field, $\left(h^{\alpha}{}_{i}\right)_2$, in Boyer–Lindquist coordinates (t,r,θ,ϕ), can be written as

$$\left(h^{\alpha}{}_{i}\right)_2=\begin{pmatrix}\mathcal{A}_6 & 0 & 0 & 0\\ 0 & \mathcal{A}_3 & 0 & 0\\ 0 & 0 & \mathcal{A}_4 & 0\\ \mathcal{A}_7 & 0 & 0 & \mathcal{A}_8\end{pmatrix}, \quad (25)$$

where $\mathcal{A}_6$, $\mathcal{A}_7$, and $\mathcal{A}_8$ are functions of r and θ that have the form

$$\begin{aligned}\mathcal{A}_6 &= \sqrt{\frac{\mathcal{A}^2\mathcal{B}\mathcal{B}_2}{\mathcal{A}(\mathcal{B}_2(r^2+a^2)^2-\mathcal{B}a^2\sin^2\theta)}},\\ \mathcal{A}_7 &= \frac{-a\sin\theta(\mathcal{B}_2(r^2+a^2)-\mathcal{B})}{\sqrt{\mathcal{A}(\mathcal{B}_2(r^2+a^2)^2-\mathcal{B}a^2\sin^2\theta)}},\\ \mathcal{A}_8 &= \frac{\sqrt{\mathcal{A}(\mathcal{B}_2(r^2+a^2)^2-\mathcal{B}a^2\sin^2\theta)}\Omega\sin\theta}{\mathcal{A}}. \end{aligned} \quad (26)$$

The metric tensor of tetrad (25) has the same form of tetrad (9) which is the Kerr–Ads spacetime.

The two tetrad fields (9) and (25) are related through local Lorentz transformation that has the form

$$\left(h^{\alpha}{}_{i}\right)_1=\left(\Lambda^{\alpha}{}_{\gamma}\right)\left(h^{\gamma}{}_{i}\right)_2, \quad (27)$$

where $(\Lambda^{\alpha}{}_{\gamma})$ has the form

$$\left(\Lambda^{\alpha}{}_{\gamma}\right)=\begin{pmatrix}\mathcal{A}_9 & 0 & 0 & -\dfrac{a\sin\theta(3+\Lambda[2r^2+a^2\cos^2\theta])}{2r^2\sqrt{3\Lambda\Omega}}\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ \dfrac{a\sin\theta(3+\Lambda[2r^2+a^2\cos^2\theta])}{2r^2\sqrt{3\Lambda\Omega}} & 0 & 0 & \mathcal{A}_{10}\end{pmatrix}, \quad (28)$$

where $\mathcal{A}_7$ and $\mathcal{A}_8$ are given by

$$\begin{aligned}\mathcal{A}_9 &= \frac{a^4\Lambda\cos^4\theta-a^2\cos^2\theta[a^2\Lambda-2r^2\Lambda-3]-3a^2+6r^2}{6r^2\sqrt{\mathcal{B}_2\Omega}},\\ \mathcal{A}_{10} &= \frac{a^4\Lambda\cos^4\theta+a^2\cos^2\theta[a^2\Lambda+2r^2\Lambda+9]-a^2[2a^2\Lambda+4r^2\Lambda+9]-6r^2}{6r^2\sqrt{\mathcal{B}_2\Omega}}. \end{aligned} \quad (29)$$

The coframe of tetrad (25) has the form

$$\begin{aligned}&\vartheta_2{}^{\hat{0}}=\mathcal{A}_6\,\mathrm{d}t, \quad \vartheta_2{}^{\hat{1}}=\mathcal{A}_3\,\mathrm{d}r,\\ &\vartheta_2{}^{\hat{2}}=\mathcal{A}_4\,\mathrm{d}\theta, \quad \vartheta_2{}^{\hat{3}}=\mathcal{A}_7\,\mathrm{d}t+\mathcal{A}_8\,\mathrm{d}\phi. \end{aligned} \quad (30)$$

Using Eq. (30) in Eq. (7) we get

$$\begin{aligned}k =\ & ([\mathcal{A}_6{}^2-\mathcal{A}_7{}^2]\xi_0-\mathcal{A}_7\mathcal{A}_8\xi_3)\,\mathrm{d}t-\mathcal{A}_3\xi_1\,\mathrm{d}r-\mathcal{A}_4\,\mathrm{d}\theta\\ &-\mathcal{A}_8[\mathcal{A}_7\xi_0+\mathcal{A}_8\xi_3]\,\mathrm{d}\phi. \end{aligned} \quad (31)$$

The total derivative of Eq. (31) has the form

$$\begin{aligned}\mathrm{d}k =\ & -\left(\frac{\partial(\mathcal{A}_7\mathcal{A}_8)}{\partial\theta}\xi_0+2\mathcal{A}_8\frac{\partial\mathcal{A}_8}{\partial\theta}\xi_3\right)(\mathrm{d}\theta\wedge\mathrm{d}\phi)\\ &-\left(\frac{\partial(\mathcal{A}_7\mathcal{A}_8)}{\partial r}\xi_0+2\mathcal{A}_8\frac{\partial\mathcal{A}_8}{\partial r}\xi_3\right)(\mathrm{d}r\wedge\mathrm{d}\phi)\\ &-\left(2\left[\mathcal{A}_7\frac{\partial\mathcal{A}_7}{\partial r}-\mathcal{A}_6\frac{\partial\mathcal{A}_6}{\partial r}\right]\xi_0+\frac{\partial(\mathcal{A}_7\mathcal{A}_8)}{\partial r}\xi_3\right)(\mathrm{d}r\wedge\mathrm{d}t)\\ &-\left(2\left[\mathcal{A}_7\frac{\partial\mathcal{A}_7}{\partial\theta}-\mathcal{A}_6\frac{\partial\mathcal{A}_6}{\partial\theta}\right]\xi_0+\frac{\partial(\mathcal{A}_7\mathcal{A}_8)}{\partial\theta}\xi_3\right)(\mathrm{d}\theta\wedge\mathrm{d}t)\\ &+\left(2\mathcal{A}_3\frac{\partial\mathcal{A}_3}{\partial\theta}\xi_1-\mathcal{A}_4\frac{\partial\mathcal{A}_4}{\partial r}\xi_2\right)(\mathrm{d}r\wedge\mathrm{d}\theta). \end{aligned} \quad (32)$$

Using Eq. (30) we get

$$\mathrm{d}t=\frac{\vartheta_2{}^{\hat{0}}}{\mathcal{A}_6},\quad \mathrm{d}r=\frac{\vartheta_2{}^{\hat{1}}}{\mathcal{A}_3},\quad \mathrm{d}\theta=\frac{\vartheta_2{}^{\hat{2}}}{\mathcal{A}_4},\quad \mathrm{d}\phi=\frac{\vartheta_2{}^{\hat{3}}\mathcal{A}_6-\mathcal{A}_7\vartheta_2{}^{\hat{0}}}{\mathcal{A}_6\mathcal{A}_8}. \quad (33)$$

Substituting Eq. (33) into Eq. (32) and applying the Hodge-dual to $\mathrm{d}k$ we finally get the total conserved charge in the form

$$\mathcal{Q}[\xi]=-\left([r^2+a^2]r\Lambda-3M\right)\frac{\Omega\xi_0}{6}-\Omega^2Ma\xi_3, \quad (34)$$

which is divergent in the time coordinate. Therefore, we use the relocalization method to recalculate the total conserved charge of tetrad (25). Therefore, we need to calculate the Weyl tensor whose non-vanishing components have the form

$C_{0303} = -C_{0330} = -C_{3030} = C_{3003} = -C_{1212} = -C_{1221} = -C_{2112} = C_{2121}$ are the same value as given by Eq. (21),

$C_{0321} = -C_{0312} = C_{3012} = -C_{3021} = C_{1230} = -C_{1203} = -C_{2130} = C_{2103}$ are the same value as given by Eq. (21),

$$C_{0202} = -C_{0220} = C_{2020} = -C_{2002} = -C_{1313} = C_{1331} = C_{3113} = -C_{3131} = \left(-\frac{Mr[r^2-3a^2\cos^2\theta]}{\mathcal{A}^3}\right)$$
$$\times\left(\frac{[\Lambda a^4+(4r^2\Lambda-6)a^2+3r^4\Lambda-6r^2+12Mr]a^2\cos^2\theta+a^4(9-2r^2\Lambda)-ra^2(2r^3\Lambda-12r+12M)+3r^4}{\left([-\Lambda a^6-(r^2\Lambda+3)a^4-(3r^2-6Mr)a^2]\cos^2\theta-a^4r^2\Lambda-\Lambda r^4a^2-3r^2a^2-6Ma^2-3r^4\right)}\right),$$

$$C_{0201} = -C_{0210} = C_{1020} = -C_{1002} = C_{1323} = -C_{1332} = C_{2313} = -C_{2331} = C_{2010} = -C_{2001} = C_{3123}$$
$$= C_{3213} = -C_{3231} = C_{0102} = -C_{0120} = C_{2010} = -C_{2001} = C_{1020} = -C_{1002}$$
$$= -C_{3132} - \frac{9M\sqrt{\mathcal{B}_2\mathcal{B}}\sin 2\theta[3r^4+3r^2a^2-[r^2+a^2]a^2\cos^2\theta]}{\left([-\Lambda a^6-(r^2\Lambda+3)a^4-(3r^2-6Mr)a^2]\cos^2\theta-a^4r^2\Lambda-\Lambda r^4a^2-3r^2a^2-6Ma^2-3r^4\right)},$$

$$C_{0232} = -C_{0223} = C_{2320} = -C_{2302} = C_{3202} = -C_{3220} = C_{2023} = -C_{2032} = -C_{1310} = C_{1301} = C_{0131}$$
$$= -C_{0113} = -C_{1013} = C_{1031} = -C_{3110} = C_{3101}$$
$$= \frac{9Mar\sin\theta[r^4+r^2a^2-3[r^2+a^2]a^2\cos^2\theta]\sqrt{\mathcal{B}\mathcal{B}_2}}{\left([-\Lambda a^6-(r^2\Lambda+3)a^4-(3r^2-6Mr)a^2]\cos^2\theta-a^4r^2\Lambda-\Lambda r^4a^2-3r^2a^2-6Ma^2-3r^4\right)},$$

$$C_{0231} = -C_{0213} = C_{1320} = -C_{1302} = C_{2013} = -C_{2031} = -C_{3120} = C_{3102} = -\left(\frac{3Ma\cos\theta[r^2-a^2\cos^2\theta/3]}{\mathcal{A}^3}\right)$$
$$\times\left(\frac{[\Lambda a^4+(4r^2\Lambda-6)a^2-3r^4\Lambda-6r^2+12Mr^2]a^2\cos^2\theta-a^4[2r^2\Lambda-9]-(2\Lambda r^3-12r+12M)ra^2+3r^4}{\left([-\Lambda a^6-(r^2\Lambda+3)a^4-(3r^2-6Mr)a^2]\cos^2\theta-a^4r^2\Lambda-\Lambda r^4a^2-3r^2a^2-6Ma^2-3r^4\right)}\right),$$

$C_{0321} = -C_{0312} = C_{3012} = -C_{3021} =$ are the same value as given by Eq. (21),

$$C_{0101} = -C_{0110} = -C_{1010} = C_{1001} = C_{2323} = -C_{2332} = -C_{3223} = C_{3232} = \left(\frac{Mr[r^2-3a^2\cos^2\theta]}{\mathcal{A}^3}\right)$$
$$\times\left(\frac{[2\Lambda a^6+(5r^2\Lambda-3)a^4-(3r^2-3r^4\Lambda-6Mr)a^2]\cos^2\theta-a^4r^2\Lambda+9a^4-\Lambda r^4a^2+15r^2a^2-6Mra^2+6r^4}{\left([-\Lambda a^6-(r^2\Lambda+3)a^4-(3r^2-6Mr)a^2]\cos^2\theta-a^4r^2\Lambda-\Lambda r^4a^2-3r^2a^2-6Ma^2-3r^4\right)}\right),$$

$$C_{0132} = -C_{0123} = C_{2310} = -C_{2301} = C_{3201} = -C_{3210} = -C_{1032} = C_{1023} = C_{2310} = -C_{2301}\left(\frac{3Ma\cos\theta[r^2-a^2\cos^2\theta/3]}{\mathcal{A}^3}\right)$$
$$\times\left(\frac{[2\Lambda a^6+(5r^2\Lambda-3)a^4-(3r^2-3r^4\Lambda-6Mr)a^2]\cos^2\theta-a^4r^2\Lambda+9a^4-\Lambda r^4a^2+15r^2a^2-6Mra^2+6r^4}{\left([-\Lambda a^6-(r^2\Lambda+3)a^4-(3r^2-6Mr)a^2]\cos^2\theta-a^4r^2\Lambda-\Lambda r^4a^2-3r^2a^2-6Ma^2-3r^4\right)}\right). \quad (35)$$

The non-vanishing components of $\Xi^{\alpha\beta}$ up to $O(1/r^3)$ have the form

$$\Xi_{01} \approx -\frac{\xi_0 r\Lambda\mathcal{B}_2}{3\Omega}+O\left(\frac{1}{r^3}\right), \quad \Xi_{02} \approx -\frac{\xi_0 a^2\sin\theta\cos\theta\Lambda^{3/2}}{3^{3/2}\sqrt{\Omega}}+O\left(\frac{1}{r^3}\right),$$
$$\Xi_{13} \approx -\frac{r\sin\theta\sqrt{\Lambda}(a[a\Omega\xi_3-\xi_0]\Lambda+3\Omega\xi_3)}{3^{3/2}\sqrt{\Lambda}}+O\left(\frac{1}{r^3}\right), \quad \Xi_{23} \approx -\frac{\cos\theta\sqrt{\mathcal{B}_2}(a[a\Omega\xi_3-\xi_0]\Lambda+3\Omega\xi_3)}{3\sqrt{\Lambda}}+O\left(\frac{1}{r^3}\right). \quad (36)$$

Using Eqs. (35) and (36) we get the same value of Eq. (23) which in turn gives the value of the conserved quantities of Kerr–Ads as that given by Eq. (24).[38,41]

6. Third Kerr–Ads spacetime

The covariant form of the third Kerr–Ads spacetime, $\left(h^{\alpha}{}_{i}\right)_3$, in Boyer–Lindquist coordinates, can be written as

$$\left(h^{\alpha}{}_{i}\right)_3 = \begin{pmatrix} \sqrt{\frac{\mathcal{B}}{\mathcal{A}}} & 0 & 0 & -\Omega a\sin^2\theta\sqrt{\frac{\mathcal{B}}{\mathcal{A}}} \\ -a\sin\theta\sin\phi\sqrt{\frac{\mathcal{B}_2}{\mathcal{A}}} & \sin\theta\cos\phi\sqrt{\frac{\mathcal{A}}{\mathcal{B}}} & \cos\theta\cos\phi\sqrt{\frac{\mathcal{A}}{\mathcal{B}_2}} & \Omega(r^2+a^2)\sin\theta\sin\phi\sqrt{\frac{\mathcal{B}_2}{\mathcal{A}}} \\ a\sin\theta\cos\phi\sqrt{\frac{\mathcal{B}_2}{\mathcal{A}}} & \sin\theta\sin\phi\sqrt{\frac{\mathcal{A}}{\mathcal{B}}} & \cos\theta\sin\phi\sqrt{\frac{\mathcal{A}}{\mathcal{B}_2}} & \Omega(r^2+a^2)\sin\theta\cos\phi\sqrt{\frac{\mathcal{B}_2}{\mathcal{A}}} \\ 0 & \cos\theta\sqrt{\frac{\mathcal{A}}{\mathcal{B}}} & -\sin\theta\sqrt{\frac{\mathcal{A}}{\mathcal{B}_2}} & 0 \end{pmatrix}. \quad (37)$$

Tetrad field (37) has the same associated metric of tetrad (9), i.e., Kerr–Ads spacetime. Tetrad field (37) is related to tetrad (9) through the relation $(h^{\alpha}{}_{i})_1 = (\Lambda_1{}^{\alpha}{}_{\gamma})(h^{\gamma}{}_{i})_3$ where $(\Lambda_1{}^{\alpha}{}_{\gamma})$ is the local Lorentz transformation given by

$$\left(\Lambda^{\alpha}{}_{\gamma}\right)_1 = \begin{pmatrix} \sqrt{\frac{\mathcal{B}}{\mathcal{B}_1}} & a\sin\theta\sin\phi\sqrt{\frac{\mathcal{B}_2}{\mathcal{B}_1}} & -a\sin\theta\cos\phi\sqrt{\frac{\mathcal{B}_2}{\mathcal{B}_1}} & 0 \\ 0 & -\sin\theta\cos\phi & -\sin\theta\sin\phi & -\cos\theta \\ 0 & \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ a\sin\theta\sqrt{\frac{\mathcal{B}_2}{\mathcal{B}_1}} & \sin\phi\sqrt{\frac{\mathcal{B}}{\mathcal{B}_1}} & \cos\phi\sqrt{\frac{\mathcal{B}}{\mathcal{B}_1}} & 0 \end{pmatrix}. \tag{38}$$

Following the same techniques used in Section 4 to calculate the total conserved charges of tetrad field (37) we finally get the same value of total conserved charges given by Eq. (24).

7. Discussion and conclusion

The calculation of conserved charges of black holes is a rich problem, for which many methods are known. Until recently, there was some controversy on the proper definition of conserved charges in asymptotically anti-de Sitter (Ads) spacetimes. This paper provided systematic and explicit calculations of the energy and angular momentum of asymptotically AdS spacetime in dimension $D = 4$.

The results of the present study can be made concise as follows:

(i) Many tetrad fields have been used to calculate their conserved charges. All these tetrad fields reproduced the same spacetime, i.e. Kerr–Ads metric which is asymptotically AdS spacetime. From this metric one can generate Kerr, Schwarzschild–Ads, and Schwarzschild spacetimes by putting $\Lambda = 0$, $a = 0$, and $\Lambda = 0$ and finally $a = 0$ respectively.

(ii) We have calculated the conserved quantities of the first tetrad field given by Eq. (9) using the covariant conserved charge method which is invariant under general coordinate and local Lorentz transformations and unfortunately we get divergent quantities for both energy and angular momentum. Therefore, we have used the regularization via recolization method to recalculate the conserved charges of the first tetrad field. This method gave a finite and known formula of the conserved quantities of Kerr–Ads spacetime. From these conserved quantities one can see that the values of energy and angular momentum depend on the cosmological constant Λ and when it is vanishing we obtain the conserved quantities of Kerr spacetime.[47]

(iii) A second tetrad field, which is related to the first tetrad field through local Lorentz transformation, has been used to calculate its conserved charges. Unfortunately the conserved quantities associated to this tetrad, when one utilizes the covariant conserved charge method, are not acceptable, i.e., the energy is divergent! Therefore, we have used the regularization via recolization method to recalculate the conserved charges of the second tetrad and a consistent result is derived.

(iv) A third tetrad field is provided and is linked to the first tetrad through local Lorentz transformation. The total conserved charge of this tetrad can be calculated using the same procedure applied for the first tetrad and the second tetrad, and a finite known form of the conserved quantities of Kerr–Ads spacetime can be derived.

(v) A general conclusion is that regularization via recolization method seems to be a powerful method in the calculations of total conserved charges for Kerr–Ads spacetime that is asymptotically the Ads spacetime.

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