The squeezing dynamics of two independent atoms by detuning in two non-Markovian environments*

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The squeezing dynamics of two independent two-level atoms off-resonantly coupled to two non-Markovian reservoirs is studied by the time-convolutionless master-equation approach. We find that the squeezing of two atoms is dependent on both detuning and the non-Markovian effect. Our results show that, in the non-Markovian regime, the bigger the detuning and the stronger the non-Markovian effect are, the larger the strength of the squeezing is. And the squeezing of two atoms can be effectively protected for a long time when both the non-Markovian effect and detuning are present simultaneously. The physical mechanism is that the detuning not only can promote the feedback of information from the environment into the atomic system but also can greatly suppress the atomic decay in the non-Markovian regime.

Keywords: non-Markovian effect, detuning, squeezing, two-level atom

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1. Introduction

Since the Heisenberg uncertainty relation is proposed by Heisenberg[1] and then generalized by Robertson,[2] the uncertainty relation is applied widely in areas both inside and outside quantum mechanics, such as entanglement detection,[3–9] quantum cryptography,[10] and signal processing.[11] For an atomic system, the spin or angular momentum system has often been regarded as squeezed if the uncertainty of one spin component, say $(\triangle S_1)^2$ or $(\triangle S_2)^2$, is smaller than $|\langle S_3\rangle|/2$.[12,13] Recently, atomic squeezing has attracted considerable attention since it can be used to enhance the sensitivity of gravity-wave detectors[14] and may play a critical role in other high precision applications, such as high-resolution spectroscopy,[15] high-precision measurements,[16] atomic clocks,[17] and optical communications.[18] The squeezing effect is also useful in tiny mechanical detection of a superconducting nano-mechanical resonator, both theoretically[19,20] and experimentally.[21] Some related work on atomic ensemble is also done.[22]

On the other hand, it is well known that real quantum systems unavoidably interact with their environments,[23] then how to maintain the squeezing of open quantum systems for a long enough time to allow the performances of quantum measurements and quantum communications naturally becomes of great interest especially when it comes to a realistic description of experimental situations. Generally, the environment is classified as Markovian without memory and non-Markovian with memory. Compared with the former, non-Markovian systems appear to be more fundamental in many physical situations, such as in photonic band-gap materials,[24,25] high-Q cavity systems,[26] solid state systems,[27,28] and spin baths.[29,30] The typicality of non-Markovian environments is that they can feed back part of the information into the systems due to the obvious memory effects. For this reason, more and more attention has been paid to the influences of non-Markovian effects on quantum information processes.

Recently, some schemes have been proposed to study the influence of the non-Markovian effects on the atomic squeezing. The squeezing of a two-level atom coupled to a non-Markovian reservoir is studied[31] by the exact solution method. And spin squeezing of an ensemble of N independent spin-1/2 particles under non-Markovian channels is also investigated by the Hierarchy equation method.[32] In this paper, we will focus our attention on the squeezing of two independent two-level atoms off-resonantly coupled to two non-Markovian reservoirs by the time-convolutionless master-equation approach. We show that the squeezing of two atoms is dependent on both the detuning and the non-Markovian effect, and can be effectively protected for a long time by the detuning in non-Markovian regimes. These results provide a potential method to extend the available time of the atomic squeezing, which would be useful in the high precision applications.

The paper is organized as follows. In Section 2, we introduce the physics model of two independent two-level atoms off-resonantly coupled to two non-Markovian reservoirs. In Section 3, we analyze the squeezing dynamics of two-atoms system. Then we discuss in Section 4 the influences of the non-Markovian effects and the detunings on the two-atomic

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squeezing and show that the two-atomic squeezing could be effectively protected for a long time by the detunings in non-Markovian regimes. Finally, a brief summary is presented in Section 5.

2. The model

Our model is a system consisting of two independent two-level atoms off-resonantly coupled to two non-Markovian reservoirs with zero temperature.[33] The total Hamiltonian that describes such a system can be written as ($\hbar = 1$)

$$H = H_0 + H_{\rm I}, \tag{1}$$

where

$$H_0 = \omega_0 \sum_{j=1}^{2} S_j^z + \sum_n \omega_{n,1} b_{n,1}^\dagger b_{n,1} + \sum_m \omega_{m,2} b_{m,2}^\dagger b_{m,2} \tag{2}$$

is the free Hamiltonian of the combined system. ω_0 is the transition frequency of two atoms, S_j^z is the inversion operators describing the j-th atom ($j = 1, 2$), while the index $n(m)$ labels the field modes of the 1-th (2-th) reservoir with frequencies $\omega_{n,1}$ ($\omega_{m,2}$), $b_{n,1}^\dagger$ ($b_{m,2}^\dagger$), and $b_{n,1}$ ($b_{m,2}$) are the modes' creation and annihilation operators of the 1-th (2-th) reservoir. The interaction Hamiltonian $H_{\rm I}$ is given by

$$H_{\rm I} = \sum_n g_{n,1} b_{n,1} S_1^+ + \sum_m g_{m,2} b_{m,2} S_2^+ + {\rm h.c.}, \tag{3}$$

where $g_{n,1}$ ($g_{m,2}$) is the coupling constant between the 1-th (2-th) atom and its corresponding reservoir, S_j^+ and S_j^- ($j = 1, 2$) are the upward and downward operators of the j-th atom, respectively. In the interaction picture defined by H_0, the interaction Hamiltonian reads

$$H_{\rm I}(t) = \sum_{j=1}^{2} \left(S_j^+ \sum_n g_{n,j} b_{n,j} {\rm e}^{{\rm i}(\omega_0 - \omega_{n,j})t} + S_j^- \sum_n g_{n,j}^* b_{n,j}^\dagger {\rm e}^{-{\rm i}(\omega_0 - \omega_{n,j})t} \right). \tag{4}$$

In the second order approximation the master equation for the system composed by the pair of uncoupled atoms,[23] described by the density operator $\rho_S(t)$, has the following form

$$\frac{{\rm d}}{{\rm d}t}\rho_{\rm S}(t) = -\int_0^t {\rm d}\tau {\rm Tr}_B([H_{\rm I}(t), [H_{\rm I}(\tau), \rho_S(t) \bigotimes \rho_B]]), \tag{5}$$

with the environment state ρ_B.

And we have supposed that $\rho(t) = \rho_S(t) \bigotimes \rho_B$ and ${\rm Tr}_B([H_{\rm I}(t), \rho_S(0) \bigotimes \rho_B]) = 0$, we write Eq. (5) as

$$\frac{{\rm d}}{{\rm d}t}\rho_{\rm S}(t) = L^{(1)}\rho_S(t) + L^{(2)}\rho_S(t), \tag{6}$$

where $L^{(j)}$ ($j = 1, 2$) is the Liouville superoperator[34] associated to the interaction Hamiltonian (4) and it is defined by

$$L^{(j)}\rho_{\rm S}(t) = B_j(t)[S_j^- \rho_{\rm S}(t), S_j^+] + B_j^*(t)[S_j^-, \rho_{\rm S}(t) S_j^+] + A_j^*(t)[S_j^+ \rho_{\rm S}(t), S_j^-] + A_j(t)[S_j^+, \rho_{\rm S}(t) S_j^-]. \tag{7}$$

The quantities $A_j(t)$ and $B_j(t)$ are the correlation functions and are given by

$$A_j(t) = {\rm i} \sum_n |g_{n,j}|^2 \langle b_{n,j}^\dagger b_{n,j} \rangle_{B_j} \frac{1 - {\rm e}^{{\rm i}(\omega_0 - \omega_{n,j})t}}{\omega_0 - \omega_{n,j}}, \tag{8}$$

$$B_j(t) = {\rm i} \sum_n |g_{n,j}|^2 \langle b_{n,j} b_{n,j}^\dagger \rangle_{B_j} \frac{1 - {\rm e}^{{\rm i}(\omega_0 - \omega_{n,j})t}}{\omega_0 - \omega_{n,j}}, \tag{9}$$

where $\langle b_{n,j}^\dagger b_{n,j} \rangle_{B_j} = {\rm Tr}_{B_j}((b_{n,j}^\dagger b_{n,j})\rho_{B_j})$ and $\langle b_{n,j} b_{n,j}^\dagger \rangle_{B_j} = {\rm Tr}_{B_j}((b_{n,j} b_{n,j}^\dagger)\rho_{B_j})$.

Assuming that the two reservoirs are initially prepared in a thermal state with $T = 0$, the correlation functions reduce to

$$A_j(t) = 0, \quad B_j(t) = {\rm i} \sum_n |g_{n,j}|^2 \frac{1 - {\rm e}^{{\rm i}(\omega_0 - \omega_{n,j})t}}{\omega_0 - \omega_{n,j}}. \tag{10}$$

We take a Lorentzian spectral density of the j-th ($j = 1$, 2) reservoir[23,35]

$$J(\omega_j) = \frac{1}{2\pi} \frac{\gamma_{0j} \lambda_j^2}{(\omega_0 - \omega_j - \delta_j)^2 + \lambda_j^2}, \tag{11}$$

where δ_j is the detuning between ω_0 and the center frequency ω_j of the j-th reservoir. And the parameter λ_j defines the spectral width of the coupling, which is connected to the reservoir correlation time τ_{R_j}=λ_j^{-1} and the parameter γ_{0j} is related to the relaxation time scale τ_{S_j} by the relation τ_{S_j}=γ_{0j}^{-1}.

Utilizing the Lorentzian spectral density $J(\omega_j)$, we can obtain the correlation functions

$$A_j(t) = 0, \quad B_j(t) = \frac{\gamma_{0j}\lambda_j}{2(\lambda_j - {\rm i}\delta_j)}[1 - {\rm e}^{({\rm i}\delta_j - \lambda_j)t}]. \tag{12}$$

3. The squeezing dynamics of a two-atoms system

To analyze the squeezing properties of the two-level atoms system, we define the collective operators S_1, S_2, and S_3

$$\begin{aligned} S_1 &= \frac{S_+ + S_-}{2}, \\ S_2 &= \frac{S_+ - S_-}{2{\rm i}}, \\ S_3 &= S_1^z + S_2^z, \end{aligned} \tag{13}$$

where $S_+ = S_1^+ + S_2^+$, $S_- = S_1^- + S_2^-$, and $[S_1, S_2] = {\rm i}S_3$. The Heisenberg uncertainty relation[12,13] is given by

$$(\triangle S_1)^2 \cdot (\triangle S_2)^2 \geq \frac{1}{4}\langle S_3 \rangle^2, \tag{14}$$

where $(\triangle S_j)^2 = \langle S_j^2 \rangle - \langle S_j \rangle^2$ ($j = 1, 2$). Consequently, the fluctuation in the component S_j of the collective operators is said to be squeezed if S_j satisfies the condition

$$(\triangle S_j)^2 < \frac{1}{2}|\langle S_3 \rangle| \tag{15}$$

or

$$F_j = (\triangle S_j)^2 - \frac{1}{2}|\langle S_3\rangle| < 0. \quad (16)$$

Let us assume that after time t the density matrix of the two-atoms system has the following form

$$\rho(t) = \begin{pmatrix} \rho_{11}(t) & \rho_{12}(t) & \rho_{13}(t) & \rho_{14}(t) \\ \rho_{21}(t) & \rho_{22}(t) & \rho_{23}(t) & \rho_{24}(t) \\ \rho_{31}(t) & \rho_{32}(t) & \rho_{33}(t) & \rho_{34}(t) \\ \rho_{41}(t) & \rho_{42}(t) & \rho_{43}(t) & \rho_{44}(t) \end{pmatrix}. \quad (17)$$

We can get $\langle S_j\rangle = \mathrm{Tr}(S_j\rho(t))$ and $\langle S_j^2\rangle = \mathrm{Tr}(S_j^2\rho(t))$ by solving the master equation (6). Then we insert $\langle S_j\rangle$ and $\langle S_j^2\rangle$ into the equation (15) or (16) and can analyze numerically the squeezing properties of the two-atomic system.

4. Results and discussion

Based on the physical model we examined in Section 2, we investigate how the types of environments and the detunings affect the squeezing properties of the two-atomic system. We consider two atoms are initially prepared in the superposition state

$$|\psi_s(0)\rangle = \cos\left(\frac{\theta}{2}\right)|--\rangle + \sin\left(\frac{\theta}{2}\right)\mathrm{e}^{\mathrm{i}\varphi}|++\rangle. \quad (18)$$

And let $\omega_1 = \omega_2$, $\lambda_1 = \lambda_2$, $\gamma_{01} = \gamma_{02}$, $\delta_1 = \delta_2$. Following the above calculations, we can numerically obtain the time evolutions of the squeezing factor F_2 in both Markovian ($\gamma_0 < \lambda/2$) and non-Markovian regimes ($\gamma_0 > \lambda/2$) in the detuning.

Figures 1 shows the squeezing evolution of two atoms as functions of time for different detunings in the non-Markovian regime. It can be seen that the squeezing of two atoms with and without detuning shows qualitative different behaviors in the non-Markovian regimes. When $\delta = 0$ (i.e. without detuning), the component S_2 is initially squeezed but loses quickly this feature shown as the dashed line in Fig. 1. Obviously, in many cases the time of the atomic squeezing is too short to be used in the high-precision measurements. However, when $\delta = 0.4\gamma_0$ (i.e. with small detuning), the oscillation in squeezing appears. The initially atomic squeezing will quickly disappear but the squeezing is again revived in a very short time, shown as the dotted line in Fig. 1. When $\delta = 40\gamma_0$ (i.e. with big detuning), the squeezing of the component S_2 can be effectively preserved for a long time shown as the solid line in Fig. 1. Therefore the squeezing of two atoms is dependent on the detuning in the non-Markovian regime. The bigger the detuning is, the larger the strength of the squeezing is, and the squeezing of two atoms can be effectively preserved for a long time.

Figure 2 shows the squeezing evolution of two atoms as functions of time for the Markovian regime and the non-Markovian regimes in the detuning. It can be seen that the squeezing behaviors of two atoms have essential differences in different reservoirs. When $\lambda = 10\gamma_0$ (i.e. in the Markovian regime), the component S_2 is initially squeezed but loses quickly this feature shown as the dashed line in Fig. 2. However, when $\lambda = 0.1\gamma_0$ (i.e. with small non-Markovian effect), the oscillation in squeezing appears. The initially atomic squeezing will also quickly disappear but the squeezing is again revived in a very short time shown as the dotted line in Fig. 2. When $\lambda = 0.001\gamma_0$ (i.e. with the strong non-Markovian effect), the squeezing of the component S_2 can be effectively preserved for a long time, shown as the solid line in Fig. 2. Therefore the squeezing of two atoms is dependent on the characteristics of reservoirs in the detuning. The stronger the non-Markovian effect is, the larger the strength of the squeezing is, and the two-atomic squeezing can also be effectively preserved for a long time.

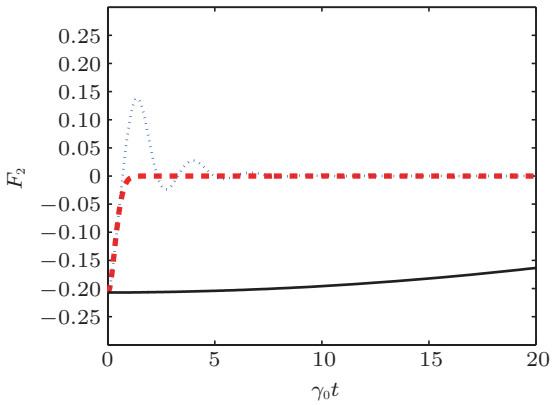


Fig. 1. (color online) The dynamics of squeezing factors F_2 as a function of scaled time $\gamma_0 t$ for different δ in the non-Markovian regime ($\lambda = 0.1\gamma_0$): $\delta = 0$ (dashed line), $\delta = 0.4\gamma_0$ (dotted line), $\delta = 40\gamma_0$ (solid line). Other parameters are $\theta = \pi/4$, $\varphi = 0$.

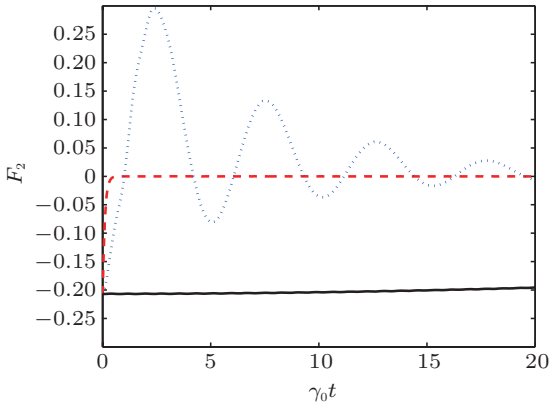


Fig. 2. (color online) The dynamics of squeezing factors F_2 as a function of scaled time $\gamma_0 t$ for different λ in the detuning ($\delta = 0.8\gamma_0$): $\lambda = 10\gamma_0$ (dashed line), $\lambda = 0.1\gamma_0$ (dotted line), $\lambda = 0.001\gamma_0$ (solid line). Other parameters are $\theta = \pi/4$, $\varphi = 0$.

Comparing Figs. 1 and 2, we find that the squeezing of two atoms is dependent on both the detuning and the non-Markovian effect. In order to understand the physical mechanism of the squeezing dynamics of two atoms, we investigate the influences of the detuning and the non-Markovian effect

on the correlation function. In terms of Eq. (12), we rewrite the correlation function as

$$B(t) = \frac{\gamma_0\lambda}{2(\lambda - \mathrm{i}\delta)}[1 - \mathrm{e}^{(\mathrm{i}\delta-\lambda)t}]. \tag{19}$$

In the non-Markovian regime, the effects of the detuning δ on the correlation function $B(t)$ are shown in Fig. 3. When $\delta = 0$, the correlation function $B(t)$ is always positive and quickly reaches a stationary value as shown in Fig. 3(a). Thus, the squeezing information will very quickly outflow from the atomic system so that the squeezing of two atoms quickly vanishes, it is just the case shown as the dashed line in Fig. 1. Nevertheless, when $\delta \neq 0$, the oscillation in the correlation function appears, and with the increase of detuning, the amplitude of the correlation function will reduce as shown in Figs. 3(b) and 3(c). Negative values of the correlation function can be interpreted as the feedback of information from the environment into the atomic system due to the memory effects of the structured reservoir. The reduction of the amplitude of the correlation function means the decrease of the atomic decay rate. Therefore the squeezing information will be well protected by the detuning in the non-Markovian regime, which is quite coincident with the results shown as the dotted line and the solid line in Fig. 1.

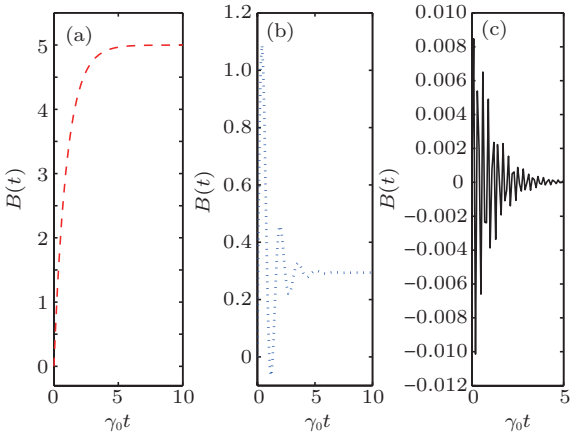


Fig. 3. (color online) The correlation function B as a function of scaled time $\gamma_0 t$ for different δ in the non-Markovian regime ($\lambda = 0.1\gamma_0$): (a) $\delta = 0$ (dashed line), (b) $\delta = 0.4\gamma_0$ (dotted line), (c) $\delta = 40\gamma_0$ (solid line). Other parameters are $\theta = \pi/4$, $\varphi = 0$.

Under the condition of the detuning, the effects of the spectral width λ on the correlation function $B(t)$ are shown in Fig. 4. When $\lambda = 10\gamma_0$, the correlation function $B(t)$ is always positive and quickly reaches a stationary value as shown in Fig. 4(a). The squeezing information will also very quickly outflow from the atomic system so that the squeezing of two atoms quickly vanishes, it is just the case shown as the dashed line in Fig. 2. Nevertheless, when $\lambda < 2\gamma_0$, the oscillation in the correlation function appears, and the strong non-Markovian effect can greatly reduce the amplitude of the correlation function as shown in Figs. 4(b) and 4(c). Therefore the squeezing information will be well protected by the non-Markovian effect in the detuning, which is just the case shown as the dotted line and the solid line in Fig. 2.

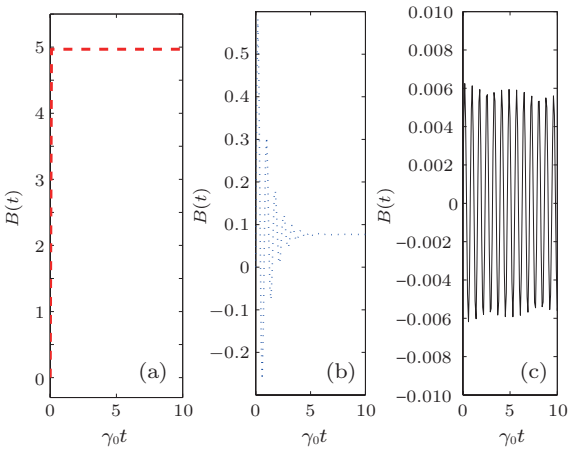


Fig. 4. (color online) The correlation function B as a function of scaled time $\gamma_0 t$ for different λ in the detuning ($\delta = 0.8\gamma_0$). (a) $\lambda = 10\gamma_0$ (dashed line), (b) $\lambda = 0.1\gamma_0$ (dotted line), (c) $\lambda = 0.001\gamma_0$ (solid line). Other parameters are $\theta = \pi/4$, $\varphi = 0$.

5. Conclusion

In conclusion, we have investigated the squeezing dynamics of two independent two-level atoms off-resonantly coupled to two non-Markovian environments and discussed the influences of the non-Markovian effect and detuning on the squeezing dynamics of two atoms. We find that the squeezing of two atoms is dependent on both the detuning and the non-Markovian effect. Our results show that, in the non-Markovian regime, the bigger the detuning and the stronger the non-Markovian effect are, the larger the strength of the squeezing is. And the squeezing of two atoms can be effectively protected for a long time when both the non-Markovian effect and detuning are present simultaneously. The physical mechanism is that the detuning not only can promote the feedback of information from the environment into the atomic system but can also greatly suppress the atomic decay in the non-Markovian regime. Our work provides a new way to indirectly control and manipulate the atomic squeezing in the high-precision measurements with the current experimental capabilities.

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