Simulating train movement in an urban railway based on an improved car-following model*

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Based on the optimal velocity car-following model, in this paper, we propose an improved model for simulating train movement in an urban railway in which the regenerative energy of a train is considered. Here a new additional term is introduced into a traditional car-following model. Our aim is to analyze and discuss the dynamic characteristics of the train movement when the regenerative energy is utilized by the electric locomotive. The simulation results indicate that the improved car-following model is suitable for simulating the train movement. Further, some qualitative relationships between regenerative energy and dynamic characteristics of a train are investigated, such as the measurement data of regenerative energy presents a power-law distribution. Our results are useful for optimizing the design and plan of urban railway systems.

Keywords: urban railway system, train movement, regenerative braking energy

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1. Introduction

In urban railways, regenerative braking mode not only saves energy, reduces pollution, but also has no abrasive wear. Moreover, compared with other braking modes, the regenerative braking does not need to add a new component, especially, when train moves with high velocity, so the regenerative braking mode is an ideal braking mode for saving train energy. How is the regenerative braking energy utilized as much as possible? Recently, such a problem has attracted more and more attention.

Optimizing the train timetable is one way to make full use of regenerative energy. Using such a method, the regenerative braking energy can be delivered to traction trains directly with less energy loss. In this field, Nag and Pal[1] presented the first timetable model in 2004. Here they considered a fixed level of regenerative braking energy. Lately, Nasri *et al.*[2] improved the timetable model of train movement, in which the regenerative energy can be completely utilized. Yang *et al.*[3] proposed a cooperative scheduling method to maximize the overlapping time between the accelerating and the braking of successive trains, so that the regenerative energy from the braking train can directly be utilized by the accelerating train.

Rational design and an optimal control strategy of storage devices are other ways to utilize the regenerative braking energy. In recent years, many studies have been reported. Foiadelli *et al.*[4] programmed a kind of software which can simulate the electric power flow and optimize the storage device size and control. Lannuzzi and Tricoli[5] proposed a real time control strategy which is based on the simplified mathematical model of both the electrical drive and the electrical railway network. In Ref. [6], light railway vehicles which are based on the tracking of the actual train speed were recommended to use supercapacitor control. Here different sizes of the storage device were selected according to storage capacity and braking resistance power. In Ref. [7], a power distribution control strategy for an on-board supercapacitor energy storage system was proposed. It considers matching characteristic between curves of the vehicles and supercapacitors.

So far, about the utilization of regenerative braking energy, few studies have been reported from the view point of controlling the train movement. Here a kind of simulation model is necessary for controlling train velocity. The car-following model provides a choice to solve such a problem. In past decades, the car-following model has been used to describe various traffic phenomena, including the early car-following models[8–10] and their improvements.[11–14] Some extended models considered not only the leading vehicle but also a few other vehicles ahead of the leading vehicle.[15,16] In the car-following model, realistic driver behavior and detailed vehicle characteristics are included so that it is suitable for simulating the vehicle's movement in complex conditions. Recently, the car-following model is used to analyze and discuss the problem of energy consumption of a vehicle. For example, Shi and Xue[17] studied the energy consumption in several typical car-following traffic models where the effects of many-neighbor and non-locality and relative velocity are considered.

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Based on the optimal velocity model, Mo *et al.*[18] proposed an extended model to investigate the energy consumption of cars in road traffic.

In the present work, an improved optimal velocity car-following model is proposed to simulate train movement, with the regenerative braking energy taken into consideration. Here an additional term is introduced into the traditional optimal velocity car-following model. The rest of this paper is organized as follows. In Section 2, the multiple car-following model is introduced. The fundamental principle of regenerative braking energy is outlined in Section 3. Our new model is proposed in Section 4. The simulation results are shown in Section 5. Finally, the conclusions are drawn from the present study in Section 6.

2. Multiple car-following model

In the classic car-following model,[8] the velocity of the n-th vehicle is affected by the velocity of the $(n+1)$-th vehicle. When the velocity of the leading $(n+1)$-th vehicle changes, the velocity of the following n-th vehicle also changes. The n-th vehicle tends to move with the same velocity as that of the $(n+1)$-th vehicle.

As is well known, if the space gap between two successive vehicles is changed, the following vehicle will accelerate or decelerate. This means that in order to avoid collision between two successive vehicles, the minimum safety distance must be kept. Because of such a reason, in 1953, based on the classic car-following model, Pipes[8] derived a new equation model. For a more realistic reason, Newell[10] proposed a kind of optimal velocity car-following model. In 1995, Bando *et al.*[11] proposed a famous optimal velocity car-following model. This model in which the position of the n-th vehicle is used as an input is formulated mathematically in terms of second-order differential equations. Lately, based on such a model, many studies have been conducted to simulate various traffic phenomena.

In reality, the velocity of the n-th vehicle is determined by its leading $(n+1)$-th vehicle and a few other vehicles ahead of the leading $(n+1)$-th vehicle. That is, the driver of the n-th vehicle considers the combinational effect of its leading vehicles, and then controls the velocity of the n-th vehicle. The effect is referred to as effect of "next-nearest neighbor". The "next-nearest neighbor" about follower-the-leader mode can be described as a linear second-order differential equation,[15]

$$\begin{aligned}\ddot{x}_n(t+T) &= \psi^{(1)}[\dot{x}_{n+1}(t)-\dot{x}_n(t)] \\ &\quad + \psi^{(2)}[\dot{x}_{n+2}(t)-\dot{x}_{n+1}(t)]. \end{aligned} \tag{1}$$

Here $\psi^{(1)}$ and $\psi^{(2)}$ are two phenomenological response coefficients, and T is the length of simulation time step.

Because the combinational effect of the leading vehicles is considered, the effect of multi-vehicle interactions should be added to the optimal velocity car-following model. In 1999, Lenz *et al.*[16] proposed a modified model which is written as follows:

$$\ddot{x}_n = \sum \tau_j \left[V^{\text{opt}}\left(\frac{x_{n+j}-x_n}{j}\right) - v_n \right]. \tag{2}$$

Here τ_j is the sensitivity coefficient, and V^{opt} is a function of universal optimal velocity model.

3. Principle of the utilization of regenerating braking energy

The function of the traction drive system of a train is to convert electrical energy into mechanical energy which is used to make the train accelerate, or convert mechanical energy into electric energy which is fed into the power grid. When the train accelerates, three-phase alternating current (AC) from overhead catenary system is supplied to the traction motor through a traction drive system. In general, the braking period of a train has two stages: step 1 is about the dynamic braking in which the traction motor is considered as a generator, and step 2 is about the regenerative braking in which a traction drive system converts three-phase AC from the traction motor into single-phase and feeds braking energy to the overhead catenary system.

In the braking period of the train, the traction motor becomes a generator. In this process, the kinetic energy of the train is converted into electric energy. Regenerative braking mode is that electrical energy which is converted from kinetic energy is fed into the power grid, so that other trains can utilize the regenerative energy. The regenerative braking capability of trains depends on two factors: (i) the rate power of the traction motor and (ii) the line voltage of the power grid.

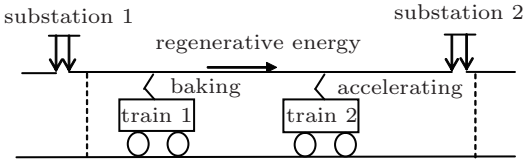


Fig. 1. Transmission process of regenerative braking energy.

Usually, a railway track section is divided into several parts in which each part is controlled by one substation. In the urban railway traffic, the regenerative energy belonging to different substations cannot be used with each other. The reason is that the traction power supply system employs the power supply section. Catenary which is between adjacent substations is insulated, only the regenerative energy belonging to the same substation can be used. Figure 1 shows the transmission principle of regenerative braking energy. In Fig. 1, trains 1 and 2 are located at a part of the section which is controlled by the same substation. In this case, the regenerative energy

which is produced by train 1 can be directly utilized by train 2 with the help of the power grid.

4. Improved car-following model

According to the fundamental principle of the transmission of the regenerative energy (see Fig. 1), the regenerative energy from the braking trains can be used by the accelerating trains only when braking trains and accelerating trains are in the same substation region. On a single track section, we suppose that within one substation region, there are K braking trains and N accelerating trains. Because the regenerative braking is a mode in which the kinetic energy of a train is converted and stored. So in this case, the kinetic energy of the train is taken into account. All braking trains produce regenerative energy that can be calculated from the following expressions:

$$E_{\mathrm{r}} = \sum_{j=1}^{K} e_j, \tag{3}$$

$$e_j = \frac{1}{2} M_j (v'^2_j - v^2_j), \tag{4}$$

where E_{r} is the total regenerative energy within a given substation region, and e_j is the braking energy of the j-th vehicle in such a region. In the braking process of a train, v'_j is the terminal velocity of the j-th vehicle, and v_j is the initial velocity of the j-th vehicle during its braking. M_j is the mass of the j-th vehicle. The regenerative energy can be used by all accelerating trains in the same substation region. Considering that the regenerative energy can be utilized averagely by all accelerating trains, the accelerating rate of each accelerating train, which is driven by utilizing the regenerative energy, can be derived as follows:

$$\ddot{x}'_i = -v_i + \sqrt{v_i^2 + \frac{1}{M_i N} \sum_j M_j (v'^2_j - v^2_j)}. \tag{5}$$

In our model, we consider the effect of the utilization of regenerative braking energy on the car-following model. Here a new additional term is added to a traditional optimal velocity car-following model. Then, a new formula of car-following model can be written as

$$\ddot{x}_i = f_i^{(1)}(\Delta x) + f_i^{(2)}(\Delta v). \tag{6}$$

In the above equation, $f_i^{(1)}(\Delta x)$ represents the acceleration of the i-th vehicle, and it depends on the headway distance x of the train, $f_i^{(2)}(\Delta v)$ is related to the velocity difference v between trains, and it is restricted to train braking. $f_i^{(1)}(\Delta x)$ is written as[11]

$$f_i^{(1)}(\Delta x) = \frac{1}{\tau}(V_i^{\mathrm{OPT}} - v_i), \tag{7}$$

where V^{OPT} is the optimal velocity function of train movement, $V_i^{\mathrm{OPT}} = \{\tanh[\Delta x_i(t) - S_{\mathrm{m}}] + \tanh(S_{\mathrm{m}})\} v_{\max}/2$. $v_{\max}$ is the maximum velocity of the train, and $\Delta x_i(t)$ is the distance between the i-th vehicle and the $(i+1)$-th vehicle. In the urban railway, S_{m} is called the minimum safety distance, $S_{\mathrm{m}} = \tau v_{\max} + v^2_{\max}/2d_{\mathrm{b}}$ and d_{b} represents the maximum decelerating rate. The $f_i^{(2)}(\Delta v)$ is expressed by Eq. (5) as

$$f_i^{(2)}(\Delta v) = \lambda \ddot{x}'_i, \quad \lambda = \begin{cases} 1, & \Delta x_i(t) > S_{\mathrm{m}}, \\ 0, & \Delta x_i(t) < S_{\mathrm{m}}. \end{cases}$$

We use the improved car-following model, i.e., Eq. (6), to simulate the train movement in the urban railway. At time t, the executed process of the proposed model can be described as follows.

Step 1 initial parameters $E_{\mathrm{r}} = 0$, $N_{\mathrm{r}} = 0$;

Step 2 calculating the accelerating rate $f_i^{(1)}(\Delta x)$ according to Eq. (7);

Step 3 if $f_i^{(1)} > 0$, $N_{\mathrm{r}} = N_{\mathrm{r}} + 1$;

Step 4 if $f_i^{(1)} < 0$, $E_{\mathrm{r}} = E_{\mathrm{r}} + e_j$, $\ddot{x}_i = f_i^{(1)}$;

Step 5 as $N_{\mathrm{r}} = N$, calculating the accelerating rate $f_i^{(2)}(\Delta x)$, $\ddot{x}_i = f_i^{(1)} + f_i^{(2)}$;

Step 6 updating the site and the velocity of the i-th train as

$$v_i(t+1) = v_i(t) + \ddot{x}_i(t), \quad x_i(t+1) = x_i(t) + v_i(t+1).$$

When a train goes into the station, the safety distance is taken to be zero, i.e., $S_{\mathrm{m}} = 0$. In this case, from the expression of the function $V^{\mathrm{OPT}} = \{\tanh[\Delta x_i(t) - S_{\mathrm{m}}] + \tanh(S_{\mathrm{m}})\} v_{\max}/2$, we derive the following formula: $V^{\mathrm{OPT}} = \{\tanh[\Delta x_i(t)]\} v_{\max}/2$. In general, the velocity of train movement near the station must be limited, so the formula of V^{OPT} can be modified as $V_i^{\mathrm{OPT}} = v_{\mathrm{lr}}\{\tanh[\Delta x_i(t)]\}$, where v_{lr} is called limited velocity.

In our method, the boundary condition is open. When the section from the site $x = 1$ to the site $x = S_{\mathrm{m}}$ is empty, a train with the velocity $v_i = 0$ is created at the site $x = 1$. The newborn train immediately moves according to the proposed model. At the site L, trains simply move out of the single track section. L is the length of the considered single-track section.

5. Numerical computation

Using the improved car-following model, we simulate the train movement under the moving block condition. In this case, trains must move only the minimum safety distance that is kept among successive trains. The length of the simulated single track section is set to be $L = 1000$, and the maximum iteration time step is $T = 1000$. A station is designed at the middle site of the simulated system. At the station, all trains should stop for the same time length T_{d} and then leave. T_{d} is called the station dwell time. The maximum accelerating rate a_{d} and decelerating rate d_{b} are respectively set to be $a_{\mathrm{d}} = 1.5$, and $d_{\mathrm{b}} = 1$. M_j is set to be $M_j = 1$.

In order to verify the safety characteristic of the improved car-following model, the headway distance $D_i(t)$ of

one tracked train is measured. Figure 2 shows the distribution of the headway distance of one tracked train which departs at the time $t = 320$. In Fig. 2, the minimum safety distance S_m is indicated by the dotted line, and the simulation results are indicated by the solid line. From Fig. 2, it is clear that the simulation results are almost larger than the safety distance S_m. Even if a few of them are slightly smaller than the safety distance S_m, the tracked train would decelerate quickly so that the required minimum safety distance is met. As $t > 40$, the site of the tracked train is larger than 500. Since the boundary condition is open, when the site of the train is larger than 500, the headway distance amongst the trains is larger. This result demonstrates that the proposed car-following model provides a valid way to simulate a train movement.

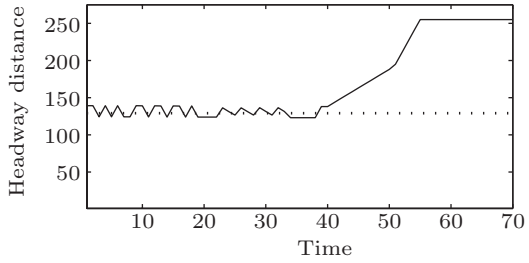


Fig. 2. Distribution of distance headway for $v_{mas} = 15$, $v_{lr} = 10$, $T_d = 5$, and $\tau = 1$.

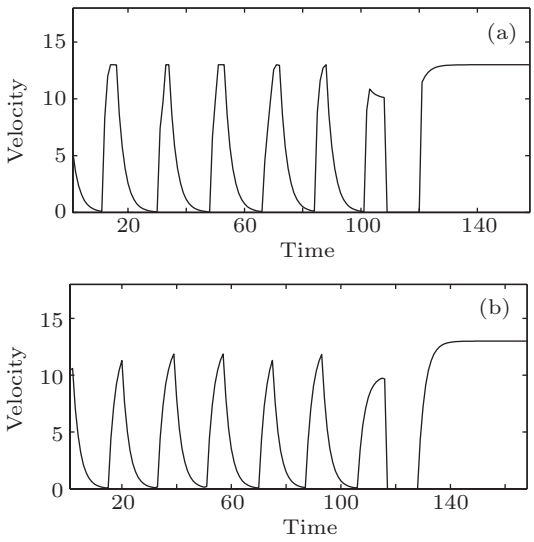


Fig. 3. Plots of v versus t for $v_{mas} = 13$, $v_{lr} = 10$, $T_d = 13$, and $\tau = 3$, in the cases with (a) and without (b) considering the regenerative energy.

In the urban railway, as one train obtains regenerative energy, its accelerating rate will increase. But, such an accelerating rate must be smaller than the maximum accelerating rate a_d. Figure 3 shows the variations of velocity with time of one tracked train. The tracked train departs from the site $x = 1$ at the time $t = 320$. Figure 3(a) denotes the velocity curve in which the regenerative braking energy is considered. Figure 3(b) represents the results in which no regenerative braking energy is utilized. From Fig. 3 it follows that the trends of these two curves are basically consistent with each other, but the average velocity calculated from Fig. 3(a) is larger than that from Fig. 3(b). Moreover, the time of the train arriving at station in Fig. 3(a) is earlier than that in Fig. 3(b). The reason is that the train can reach a larger accelerating rate by utilizing the regenerative energy in the case of Fig. 3(a).

In the process of train movement, an accelerating train can use the regenerative energy from the decelerating train. We define the utilization rate of regenerative energy P_r as $P_r = E'_{to}/E_{to}$. Here E'_{to} represents the total regenerative energy which is actually utilized by the accelerating train at all times, and E_{to} represents the total regenerative energy which is produced by the decelerating train at all time. The relationship between the utilization rate P_r of regenerative energy and the maximum velocity v_{max} of the train is measured. The measurement result is shown in Fig. 4. From Fig. 4 it obviously follows that the utilization rate P_r of the regenerative energy decreases as the maximum speed v_{max} increases. The relationship between v_{max} and P_r is linear. As $v_{max} > 30$, $P_r = 0$. The reason is that the safety stopping distance between successive trains is larger when the maximum velocity v_{max} of a train is larger. As the safety stopping distance exceeds a given value, there is no overlap time among decelerating trains and accelerating trains within the same substation. These measured results are similar to some empirical observations.

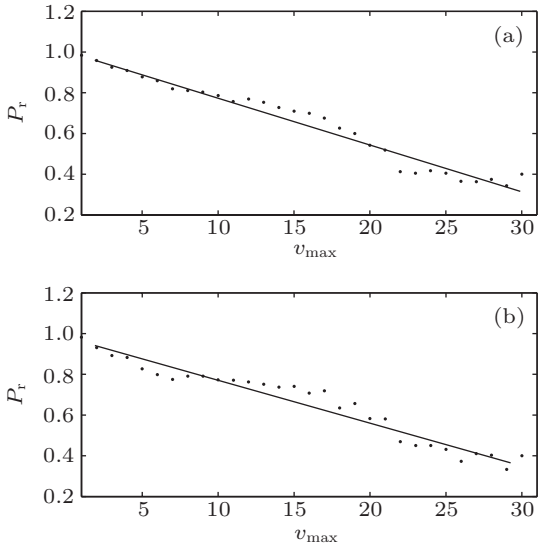


Fig. 4. Plots of P_r versus v_{max} for $T_d = 5$ and $\tau = 2$ at $v_{lr} = 0.3v_{max}$ (a) and $0.5v_{max}$ (b).

The dwell time T_d and the safety stopping distance S_m also have a great effect on the utilization of regenerative braking energy. Figure 5 shows the variations of P_r for $v_{mas} = 15$, $v_{lr} = 10$, and $\tau = 2$. From Fig. 5(a), it is clear that as the dwell time T_d is larger, the utilization rate P_r is smaller. From Fig. 5, it is concluded that as the values of T_d and S_m are smaller, the values of P_r are larger. The data shown in Fig. 5(a) presents a power law distribution, i.e., $P_r \propto T_d^{-\gamma}$, where γ is an exponent. In Fig. 5(b), we change the values of S_m artificially. The data shown in Fig. 5(b) displays a kind of linear distribution.

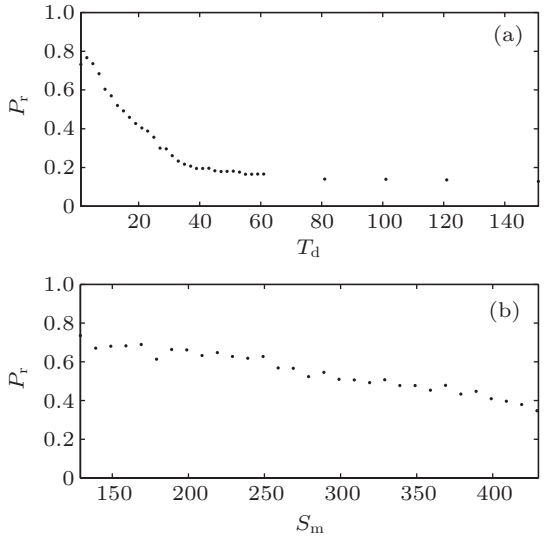


Fig. 5. (a) Plot of P_r versus T_d and (b) plot of P_r versus S_m.

6. Conclusions

In this paper, based on an optimal velocity car-following model, an improved simulation model is proposed to capture the dynamic characteristics of train movement in which the velocity difference is considered. In our study, the velocity difference is only related to the braking process of train movement so that the regenerative braking energy can be investigated. The numerical computation results demonstrate that our proposed model is not only able to well capture some major characteristics of train movement, but also suitable for simulating the train movement. Optimizing the velocity of train movement can utilize the regenerative braking energy as much as possible so that the utilization rate of regenerative energy is larger. The obtained results provide the theoretical basis of planning and designing a traction power supply in an urban railway system.

Our improved car-following model provides a new way to discuss and analyze the utilization of regenerative braking energy from the viewpoint of controlling train movement. However, the proposed model also has some disadvantages which must be considered in our future studies, such as some complex traffic environments are not considered.

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