A self-adaptive stochastic resonance system design and study in chaotic interference*

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The us of stochastic resonance (SR) can effectively achieve the detection of weak signal in white noise and colored noise. However, SR in chaotic interference is seldom involved. In view of the requirements for the detection of weak signal in the actual project and the relationship between the signal, chaotic interference, and nonlinear system in the bistable system, a self-adaptive SR system based on genetic algorithm is designed in this paper. It regards the output signal-to-noise ratio (SNR) as a fitness function and the system parameters are jointly encoded to gain optimal bistable system parameters, then the input signal is processed in the SR system with the optimal system parameters. Experimental results show that the system can keep the best state of SR under the condition of low input SNR, which ensures the effective detection and process of weak signal in low input SNR.

Keywords: chaotic interference, self-adaptive, genetic algorithm, optimal SR

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1. Introduction

The concept of stochastic resonance (SR) was first proposed by Benzi *et al.*[1,2] The SR can be explained as the fact that noise can enhance and maximize weak signal via a nonlinear system and restrain noise itself when signal, noise, and nonlinear systems act in synergy. From the perspective of signal detection, single Gauss white noise or colored noise cannot simulate the actual noise. In practical situations, we often encounter the interference with chaotic characteristics, such as sea clutter, the noise of marine environment, and underwater target detection. As a kind of interference, chaos can also induce SR and the weak periodic signal submerged in chaos interference can be detected by the bistable system mode.[3] In view of the unpredictability of noise and signal in practice, adaptive SR systems under the background of white noise or color noise were widely studied.[4,5] However, adaptive SR systems under the background of chaotic interference are seldom involved.

The genetic algorithm[6] has good robustness, so it is especially suitable for processing complicated multi-parameter and nonlinear problems. In this paper, structure parameters in the SR system are synchronously optimized using the genetic algorithm. The experimental results show that the weak signal can be detected effectively by the self-adaptive SR system we designed based on the genetic algorithm.

2. System model of stochastic resonance in chaotic interference

An SR system contains three indispensable elements: a nonlinear system with bistable or multi-stability, input signal and noise. Bistable systems can be described by Langevin equation:

$$\dot{x} = -V'(x) + A\sin(2\pi f_0 t) + \Gamma(t). \tag{1}$$

Here, $V(x)$ corresponds to the potential fuction of the double-well potential system[7]

$$V(x) = -\frac{a}{2}x^2 + \frac{b}{4}x^4. \tag{2}$$

Combining formula (1) and formula (2), we can obtain

$$\dot{x} = ax - bx^3 + A\sin(2\pi f_0 t) + \Gamma(t). \tag{3}$$

Expression (3) is a nonlinear bistable system described by the Langevin equation, where a and b are the structural parameters of the bistable system and they are the real numbers greater than zero. A is the amplitude of signal and f_0 is the frequency to be detected. $\Gamma(t) = r(t)$ corresponds to the interference, which here is the chaotic background generated by the Duffing oscillator.[8] Duffing chaotic systems can be described by the following equation:

$$\ddot{x}(t) + k\dot{x}(t) - x(t) + x^3(t) = r\cos wt, \tag{4}$$

where $x - x^3$ is the nonlinear restoring force, $r\cos(wt)$ denotes the periodic driving force, k is the damping ratio, r is the amplitude of the cycle driving force and w is the corresponding

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frequency. k and r are greater than zero. The output Wigner distribution of the Duffing chaotic system can be obtained by the time-frequency method as shown in the following figure.

Figure 1 shows the output energy distribution of chaotic systems, where each row is corresponding to a moment and each column is corresponding to a frequency. The output of the time-frequency plane reflects the signal energy that is mainly concentrated in the low frequency band, and within a relatively narrow range. This feature indicates that the Duffing system is a low frequency narrowband system, which shows that the Duffing chaotic system has the narrow band characteristic of chaos.

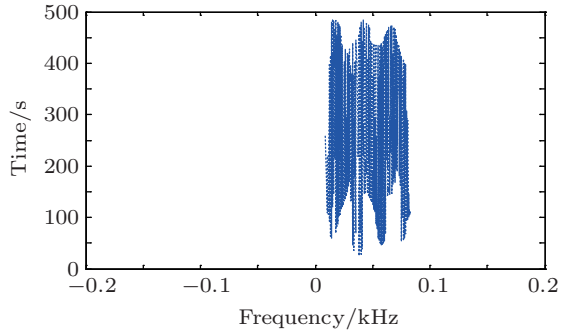


Fig. 1. (color online) Output Wigner distribution of the Duffing chaotic system.

When all parameters of the Duffing chaotic system configuration are appropriate, with the parameter $k = 0.5$, $r = 0.8$, and $w = 1$ taken in this article, the system is in a chaos motion state. A large number of studies have shown that when the amplitude of drivng force r is constant, no matter what value the driving frequency W takes, the threshold of Duffing systems from the chaotic state to the periodic state is basically unchanged. That is to say, as long as we set the appropriate damping ratio and amplitude, the Duffing system will stay in a chaotic state of motion.

Since the chaotic signals generated by driving force at different frequencies have similar characteristics of the movement, we choose chaos generated by driving force when $w = 1$ as noise to study the SR under the chaotic background.

So we obtain the final bistable system equation in chaotic interference as follows:

$$\dot{x} = ax - bx^3 + A\sin(2\pi f_0 t) + dr(\mathrm{t}), \tag{5}$$

where d is the amplitude of the chaotic signal. We take $s(t) = A\sin(2\pi f_0 t) + dr(t)$ as the input signal of the nonlinear bistable system.

According to the Wiener–Khintchine theorem, we can obtain the output power spectrum expression of the signal with the additive noise as follows:

$$S(f) = \int_{-\infty}^{\infty} R(\tau)\,\mathrm{e}^{-\mathrm{j}w\tau}\,\mathrm{d}\tau, \tag{6}$$

where

$$R(\tau) = \int_{-\infty}^{\infty} s(t)s(t-\tau)\,\mathrm{d}t$$

is the autocorrelation function of the noisy signal. The SR state can be judged by the output SNR, i.e.,

$$\mathrm{SNR} = 10\lg\frac{S}{N} = 10\lg\frac{S(\omega_0)}{N(\omega_0)}, \tag{7}$$

where $S(\omega_0)$ is the amplitude of the output power spectrum $Y(\omega_0)$ at the input signal frequency ω_0. Namely, $S(\omega_0) = |Y(\omega_0)|^2$, and $N(\omega_0)$ is the background noise spectrum of the input signal frequency at ω_0.

So we can obtain the output SNR expression of the bistable system through the adiabatic approximation as follows:

$$\mathrm{SNR}_{\mathrm{out}} = \frac{S}{N} \approx \frac{\sqrt{2}A^2a^2}{d^2b}\exp\left(-\frac{a^2}{2bd}\right), \tag{8}$$

where a and b are system parameters, d is the variance of the additive noise, here it is the chaotic noise intensity. As can be seen from the above equation, when the chaotic disturbance intensity d is a constant, we can change the output SNR of the system by adjusting system parameters a, b. It is known that when the system is in the best SR state, the output SNR will reach the maximum, which provides a theoretical basis for the adaptive SR system. In this paper, we regard the output SNR as the fitness function and the system parameters are jointly encoded to gain optimal bistable system parameters, which make the system reach the best state of SR.

3. Design of the adaptive SR system

In a traditional adaptive SR system a single system parameter is often used as an optimizing object on the condition that other parameters are kept constant, to study the influence on the system output. But in this optimizing way, the synergistic effect between the system parameters is ignored. In this paper, SR system parameters a and b are optimized synchronously by the good multi-parameter optimization ability of the genetic algorithm. The flow chart is shown in Fig. 3.

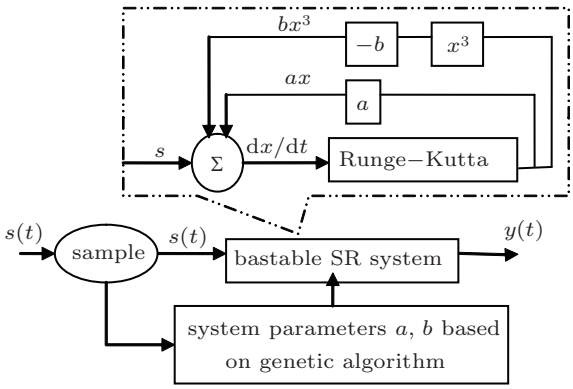


Fig. 2. Block diagram of an adaptive SR system based on a genetic algorithm.

Detailed steps are described as follows.

Step 1 ***Coding*** The parameters a and b to be optimized in this paper are positive real numbers. They are jointly coded in the binary form after setting the system parameter search range and accuracy. The search interval of a is $[A_{\min}, A_{\max}]$ and that of b is $[B_{\min}, B_{\max}]$. Setting the search precision $\varepsilon = 0.001$, so the corresponding coding lengths m and n can be determined from the following formulas:

$$\begin{cases} 2^m - 1 = (A_{\max} - A_{\min})/\varepsilon, \\ 2^n - 1 = (B_{\max} - B_{\min})/\varepsilon. \end{cases} \tag{9}$$

Then the corresponding codeword $l_m l_{m-1} l_{m-2} \ldots l_1 k_n k_{n-1} k_{n-2} \ldots k_1$ can be obtained through the binary coding.

Step 2 ***Decoding*** The corresponding decoding formulas are

$$\begin{cases} a = A_{\min} + \left(\sum\limits_{i=1}^{m} c_i 2^{i-1} \right) \dfrac{A_{\max} - A_{\min}}{2^m - 1}, \\ b = B_{\min} + \left(\sum\limits_{j=1}^{n} c_j 2^{j-1} \right) \dfrac{B_{\max} - B_{\min}}{2^n - 1}. \end{cases} \tag{10}$$

The decimal number of a and b can be acquired through the decoding formula.

Step 3 ***Initialization*** Set the population size, select individual randomly, and initialize the group. The size of the population is 200 in this paper.

Step 4 ***Fitness evaluation*** In this paper, system output SNR is regarded as the fitness function of the genetic algorithm, namely: $\mathrm{SNR}_{\mathrm{out}} = f(a,b)$. The system output signal can be obtained by solving Eq. (4) with the fourth-order Runge–Kutta algorithm. Then calculate the power spectrum, so as to obtain the individual fitness. The Runge–Kutta algorithm is shown below[9]

$$\begin{cases} k_1 = ay(n) - by(n)^3 + s(n), \\ k_2 = a\,(y(n) + k_1/2) - b(x(n) + k_1/2)^3 + s(n), \\ k_3 = a(y(n) + k_2/2) - b(y(n) + k_2/2)^3 + s(n+1), \\ k_4 = a(y(n) + k_3) - b(y(n) + k_3)^3 + s(n+1), \\ y(n+1) = y(n) + h/6(k_1 + 2k_2 + 2k_3 + k_4), \end{cases} \tag{11}$$

where h denotes the integration step, s the input signal, and y the output signal.

Step 5 ***Selection and genetic*** This step includes three substeps: selection, crossover, and mutation. In this paper, we take the conditional best reserved mechanism to select superior individuals, and with a certain probability of crossover (one-point crossover) and mutation (alleles) to produce a new generation of group.

Step 6 ***Termination*** Repeat Steps 3–5 until the set iterative condition is met and terminate the cycle. Then output the current optimal solutions that are the optimal system parameters. In this article, there are two termination conditions: 1) the difference of fitness value between the two generations is less than the preset accuracy, and 2) the evolution algebra is greater than the presetting threshold value. When one of them is satisfied, the iterative stops.

4. Numerical experiments and system performance analysis

We suppose sample frequency $f_s = 10$ Hz, chaotic interference amplitude $d = 0.42$, and Runge–Kutta integration step $h = 0.1$. In the genetic algorithm, we set the system parameters a and b to be in the corresponding search ranges [0.01, 5] and [0.01, 10], the crossover rate $P_c = 0.8$, the mutation rate $P_m = 0.08$, population scale to be 100, and the largest iteration number to be 100.

Let the periodic signals with frequency $f_0 = 0.01$, 0.015, 0.025, 0.03 input the adaptive system separately. We obtain the corresponding optimal parameters, which are listed in Table 1.

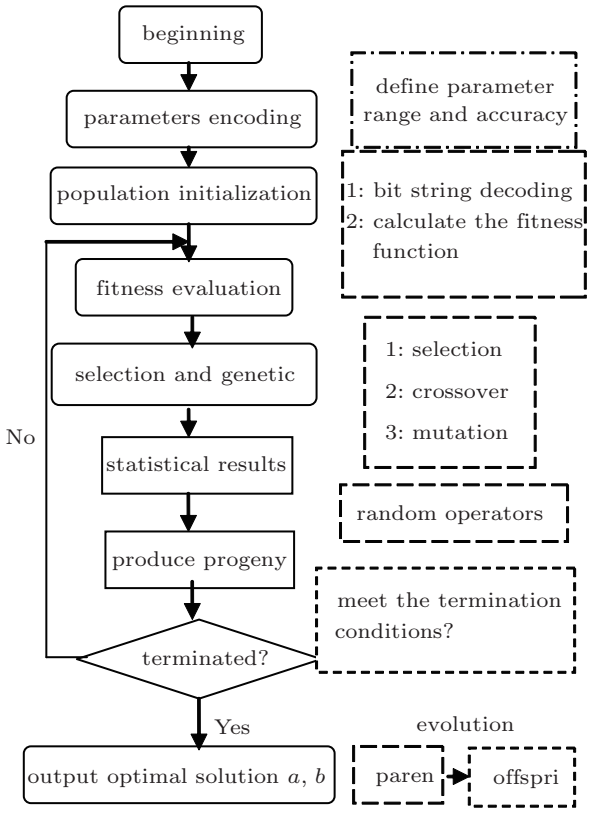


Fig. 3. Flow chart of genetic algorithms to determine the optimal parameters.

Table 1. Optimized parameters when different frequency signals pass through the adaptive system.

Input signal frequency \ Optimal parameter	a	b
$f_0 = 0.01$	0.69	0.42
$f_0 = 0.015$	0.69	0.37
$f_0 = 0.025$	0.69	0.35
$f_0 = 0.03$	0.69	0.33

Take $f_0 = 0.01$ in Table 1 for example, we justify the reliability of the adaptive system that we use to test SR state in chaotic interference. We take Runge–Kutta algorithm and spectrum analysis with 8000 sampling points using the optimal system parameters $a = 0.69$, $b = 0.42$ (from chart 1). Figures 4 and 5 show the simulation results in time domain and frequency domain.

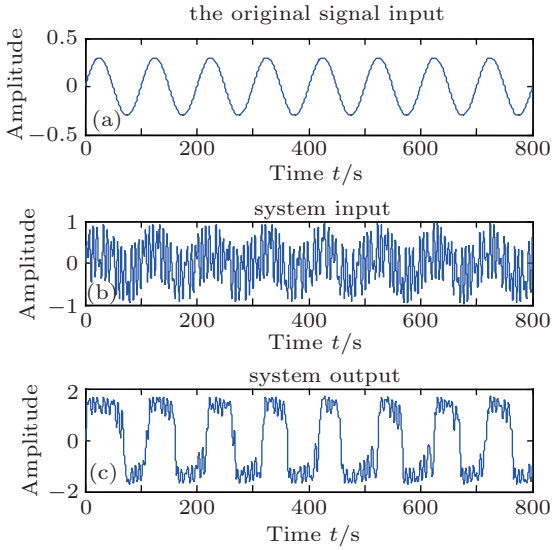


Fig. 4. (color online) Variations of amplitudes of the time-domain waveforms of original input (a), input signal with chaotic interference (b), and system output signal (c), with time.

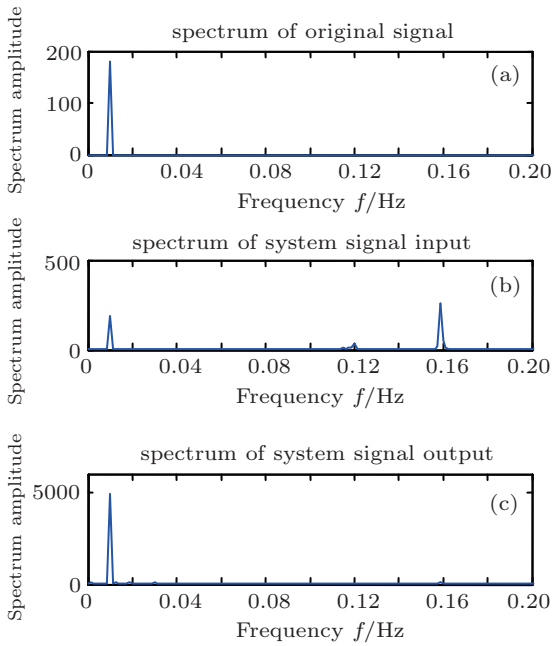


Fig. 5. (color online) Variations of waveforms of original input signal (a), input signal with chaotic interference (b), and system output signal (c), with time.

From Fig. 4, it can be seen that chaotic interference is in effective periodic motion as the input periodic signal, and the amplitude of the output signal is increased by nearly 2 times. Also from Fig. 5, we can see the input signal in the chaotic interference has two spectral peaks. The left peak is the spectrum of the weak periodic signal to be detected and the right is the spectrum of the interference (here is the chaotic characteristics interference). Best SR takes place when the input passes through the adaptive system and most of the energy is transferred from the chaotic interference to the input periodic signal. So the interference signal is weakened obviously, the periodic signal to be detected is enhanced greatly. $a = 0.69$, $b = 0.42$ are the result of genetic operators which regard output SNR as the fitness function, therefore, the output SNR reaches a maximum at this time. It can be seen from the table, the chaotic noise has been coordinately maximized, which further shows the system in the best SR state. This is consistent with the previous analysis.

In the case where the other parameters are kept constant, we change the chaotic disturbance intensity d. We can see the changes of the optimal system parameters with d (see Fig. 6.)

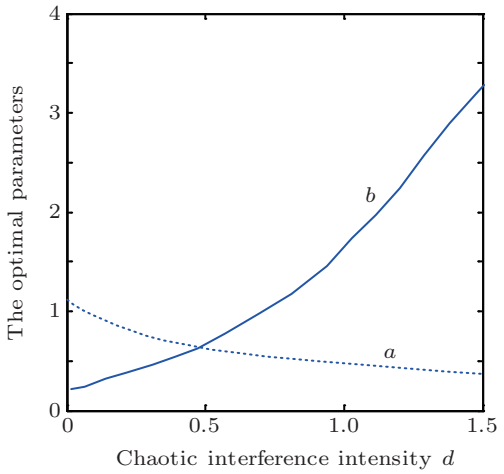


Fig. 6. (color online) Variations of optimal system parameters a and b with d.

It can be seen from Fig. 6 that the amplitude of parameter a changes smoothly and b changes greatly. Comparing the adaptive SR in this paper with the conventional adaptive SR system with a single parameter, and further studying the SR states with different values of chaotic interference signal strength d, we obtain the output SNR curves, with parameters a fixed, b changed, and b fixed, a changed respectively. These curves are shown in Fig. 7.

Comparing the adaptive SR systems in the cases of parameters a fixed, b changed and b fixed, a changed, it can be seen that when the chaotic interference intensity is small ($d < 0.48$), the performance of the system with adjusting parameter a is superior to that of the system with adjusting parameter b; When chaos disturbance intensity $d = 0.48$, the two curves have an intersection point, the system output SNRs are equal, and the optimum system parameters a and b each have a value 1; when the chaotic interference intensity is large

($d > 0.48$), the performance of the system with adjusting parameter b is superior to that of the system with adjusting parameter a. In this paper, the adaptive resonance system is designed by combining the two ways above and the parameters a and b are adjusted synchronously with the genetic operators to achieve the best state of SR. As can be seen from Fig. 7, the performance of the adaptive SR system designed in this paper is obviously better than that of the system with a adjusting single parameter. The anti-noise performance is improved by 10 dB∼25 dB on the condition that no computational complexity is increased.

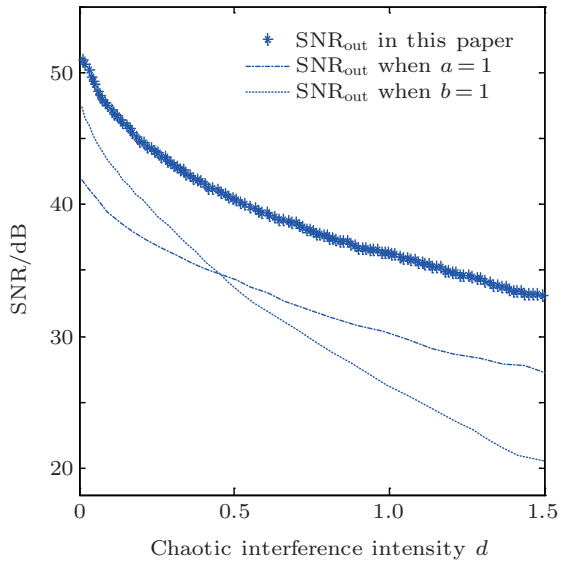


Fig. 7. (color online) Output SNR curves of the conventional adaptive SR system and adaptive SR system in this article.

5. Conclusions

In this paper, an adaptive SR system under the chaotic interference is put forward. It is based on the genetic algorithm and realizes the multi-parameter optimization. It has the following advantages.

i) Genetic algorithm using group search strategy and the information exchange between the individuals of the group. For the optimal system parameters a and b, the search does not depend on gradient information. It has strong robustness and low probability of undetection.

ii) The proposed system overcomes the disadvantages of the traditional adaptive SR algorithm only by optimizing single parameters and adjusting the parameters a and b through using genetic operators simultaneously, so as to achieve the best SR state. The anti-noise performance of the proposed system in this paper is stronger than that of the existing adaptive SR system with adjusting single parameters, and it increases about 10 dB∼25 dB.

iii) In view of the weak signal detection under the background of chaos in actual engineering, the adaptive SR system can effectively detect the weak signal we need, which is of great practical significance.

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