Drift-wave fluctuation in an inviscid tokamak plasma*

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In order to describe the characterization of resistive drift-wave fluctuation in a tokamak plasma, a coupled inviscid two-dimensional Hasegawa–Wakatani model is investigated. Two groups of new analytic solutions with and without phase shift between the fluctuant density and the fluctuant potential are obtained by using the special function transformation method. It is demonstrated that the fluctuant potential shares similar spatio–temporal variations with the density. It is found from the solutions without phase shift that the effect of the diffusion and adiabaticity on the fluctuant density is quite complex, and that the fluctuation may be controlled through the adiabaticity and diffusion. By using the typical parameters in the quasi-adiabatic regime in the solutions with phase shift, it is shown that the density gradient becomes larger as the contours become dense toward the plasma edge and the contours have irregular structures, which reveal the nonuniform distribution in the tokamak edge.

Keywords: tokamak plasma, Hasegawa–Wakatani model, drift-wave fluctuation

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1. Introduction

In the investigation of the turbulent drift wave in tokamak plasma, some experimental results have revealed a strong density fluctuation near the plasma edge with a strong magnetic field.[1–5] Additionally, to explain qualitatively the stationary spectrum of the strong turbulence in magnetized nonuniform tokamak plasma, Hasegawa and Mima proposed first a nonlinear model for the quasi-two-dimensional low frequency turbulence.[6,7] Based on that model, many other models for strongly turbulent drift waves have been discussed.[8–10] Especially, to describe the high level of observed turbulence near the plasma edge, Hasegawa and Wakatani derived the model mode-coupling equations based on collisional drift waves, which were solved numerically.[11,12]

Later, by employing the Hasegawa–Wakatani (HW) model, the turbulent drift wave has been extensively studied. Density and potential fluctuations were simulated in the limit of small viscosity, and the transfer of energy and vorticity was directly computed.[13] The simulation of bifurcation phenomena showed a regime where, after an initial transient, the drift wave turbulence was suppressed through zonal flow generation.[14] Decaying and driven cases of impurity advection were simulated by using structure function analysis.[15] The more complex situations involving a magnetic shear[16] and the mechanism of self-sustainment in the collisional drift wave turbulence[17] were studied. The presence and nature of the coherent structures were found to depend on the adiabatic parameter.[18] The above researches have been carried out with numerical simulation methods.

In this paper, a coupled HW model without viscosity in tokamak plasma, which gives the coupling relation between the nonuniform density and the electrostatic potential through an adiabatic parameter, is employed to study the fluctuant density and the potential in Section 2. In Section 3, applying a kind of special transformation method,[19,20] we search for new analytic solutions with and without phase shift between the fluctuant density and the potential. In Section 4, considering that the solutions with phase shift are more realistic, we apply the typical parameters of tokamak plasma in the quasi-adiabatic regime in the solutions and analyze and discuss the character of drift-wave fluctuation in detail. The final section is devoted to a summary.

2. Model equations

For a magnetized turbulent tokamak plasma, we suppose that the constant magnetic field is $\boldsymbol{B} = B_0\boldsymbol{z}$; the nonuniform equilibrium plasma density is $n_0 = n_0(x)$; the fluctuant components of the plasma density and the electrostatic potential are n_1 and ϕ_1, respectively; the total number density is N, and $N = n_0 + n_1 > 0$. The cold ion temperature T_i and the isothermal electron temperature T_e are considered, $T_\mathrm{i} \ll T_\mathrm{e} \equiv T$. The temperature gradient effect is neglected. Therefore, the resistive drift-wave turbulence is described by the two-dimensional (2D) HW equations[11,12,15]

$$\left(\frac{\partial}{\partial t} - D\nabla^2\right)n + \frac{\partial\phi}{\partial y} + C(n-\phi) = [n,\phi], \qquad (1)$$

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$$\left(\frac{\partial}{\partial t}-\nu\nabla^2\right)\nabla^2\phi+C(n-\phi)=[\nabla^2\phi,\phi], \tag{2}$$

where D is the diffusion coefficient; C is the adiabatic parameter related to the parallel electron conductivity and is defined as $C=T_{\rm e}k_z^2/e^2n_0\eta\omega_{\rm ci}$; ν is the viscosity coefficient and is defined as $\nu=\mu/(\rho_{\rm s}^2\omega_{\rm ci})$, $\mu=3T_{\rm i}\gamma_{\rm ii}/(10m_{\rm i}\omega_{\rm ci}^2)$, η is the electron resistivity, $\rho_{\rm s}$ is the hybrid Larmor radius, $\gamma_{\rm ii}$ is the collision frequency between ions, and $\omega_{\rm ci}$ is the ion cyclotron frequency. The model describes a slab region of the plasma edge in a toroidal device. For a group of typical tokamak parameters in the quasi-adiabatic regime, $C\sim1$, $D\sim0.01$, and $\nu\sim10^{-4}$, the viscosity is very small.[12,13]

In this paper, the viscous term is negligible for simplification, then the HW equations can be written in the form

$$\left(\frac{\partial}{\partial t}-D\nabla^2\right)n+\frac{\partial\phi}{\partial y}+C(n-\phi)=[n,\phi], \tag{3}$$

$$\frac{\partial}{\partial t}(\nabla^2\phi)+C(n-\phi)=[\nabla^2\phi,\phi], \tag{4}$$

where the dimensionless variables in Eqs. (1)–(4) are defined as

$$\frac{e\phi_1}{T_{\rm e}}\to\phi,\quad\frac{n_1}{n_0}\to n,\quad\frac{c_{\rm s}}{L_n}t\to t,\quad\frac{x}{\rho_{\rm s}}\to x,\quad\frac{y}{\rho_{\rm s}}\to y. \tag{5}$$

Here $c_{\rm s}$ is the speed of sound and the equilibrium density length scale $L_n\equiv n_0/|{\rm d}n_0/{\rm d}x|$ is a constant. The Poisson bracket is $[A,B]=(\partial A/\partial x)(\partial B/\partial y)-(\partial B/\partial x)(\partial A/\partial y)$.

The fluctuant density and the potential are cross-coupled by resistive dissipation and are proportional to the adiabatic parameter C. As is well known,[6,13,15] for $C\gg1$, in the adiabatic regime, the Hasegawa–Wakatani system reduces to the Hasegawa–Mima system. In this case, there is no phase shift between the n and ϕ fields; therefore, there is no particle transport. While for $C\ll1$, in the hydrodynamic regime, the HW system reduces to the 2D Navier–Stokes equation for the $\boldsymbol{E}\times\boldsymbol{B}$ flow vortices and a passive advection for the density fluctuation. For $C\approx1$, in the quasi-adiabatic regime, a phase shift is allowed so that the particle transport can occur.

3. Fluctuant component solutions

In order to look for the analytic solutions of Eqs. (3) and (4), we employ the following special function transformation method, which is similar to that in Refs. [19]–[21]. Assume that

$$\phi\equiv\phi(x,y,t)=b_1f(\xi)+b_2\frac{{\rm d}f(\xi)}{{\rm d}\xi}, \tag{6}$$

$$n\equiv n(x,y,t)=b_3f^2(\xi)+b_4\left(\frac{{\rm d}f(\xi)}{{\rm d}\xi}\right)^2, \tag{7}$$

where $b_i\equiv b_i(x,y)=h_i{\rm e}^{a_1x+a_2y}$ $(i=1,\ldots,4)$, $\xi\equiv\xi(x,y,t)=k_1x+k_2y+k_3t$, ϕ,n,b_i,ξ, and $f(\xi)$ are all real-valued functions, and $f(\xi)$ is given by

$$\frac{{\rm d}f(\xi)}{{\rm d}\xi}=\sqrt{c_0+c_1f(\xi)+c_2f^2(\xi)}, \tag{8}$$

with h_i $(i=1,\ldots,4)$, a_1, a_2, k_i $(i=1,2,3)$, and c_i $(i=0,1,2)$ being undetermined constants.

When $c_2<0$, one solution of Eq. (8) is

$$f(\xi)=\frac{-c_1+\sqrt{-4c_0c_2+c_1^2}\cos(\sqrt{-c_2}\xi+\alpha)}{2c_2}, \tag{9}$$

with an arbitrary constant α.

Substituting Eqs. (6)–(8) into Eqs. (3) and (4) and setting all the coefficients of $f(\xi)$ terms to zero, we obtain a group of equations. In this group of equations, by setting all the coefficients of ${\rm e}^{a_1x+a_2y}$ terms to zero, an over-determined set of equations for the unknowns is obtained. In order to find the local solutions of Eqs. (3) and (4), it is required that the fluctuant density and the potential should tend to finite values when x and y tend to zero and/or the boundary. Considering this requirement and solving the system of determined equations, we have the following solutions.

3.1. Solutions without phase shift

Solving the determinant equations leads to the following solutions:

$$b_1=h_1{\rm e}^{a_1x+a_2y},\quad b_2=0,\quad b_3=h_3{\rm e}^{a_1x+a_2y},$$

$$b_4=h_4{\rm e}^{a_1x+a_2y}, \tag{10}$$

$$a_2=\frac{1}{8}(2C-D+\sqrt{4C^2+12CD+D^2})>0, \tag{11}$$

$$a_1=\pm\frac{1}{2\sqrt{D}}\sqrt{a_2(D^2-2CD+4)-CD^2}, \tag{12}$$

$$c_0=-\frac{c_1^2k_1^2(a_1^2+a_2^2)[2D(C-a_2)(a_1^2+a_2^2)-Ca_2]}{a_1^2(C-a_2)^2}, \tag{13}$$

$$c_2=-\frac{{a_1}^2}{{k_1}^2}<0, \tag{14}$$

$$h_3=\frac{h_1a_1^2(C-a_2)}{Ck_1^2c_1}, \tag{15}$$

$$h_4=\frac{h_1(C-a_2)}{Ck_1^2c_1}, \tag{16}$$

$$k_2=\frac{k_1a_2}{a_1}, \tag{17}$$

$$k_3=\frac{k_1[2D(C-a_2)(a_1^2+a_2^2)-Ca_2]}{a_1(C-a_2)}. \tag{18}$$

Thus a group of fluctuant component solutions of Eqs. (3) and (4) can be obtained as

$$n=\frac{h_1(a_2-C)}{2Cc_2}\left[\sqrt{c_1^2-4c_0c_2}\cos(\sqrt{-c_2}\xi+\alpha)+c_1^2-2c_0c_2\right]\times{\rm e}^{a_1x+a_2y}, \tag{19}$$

$$\phi=\frac{h_1}{2c_2}\left[\sqrt{c_1^2-4c_0c_2}\cos(\sqrt{-c_2}\xi+\alpha)-c_1\right]{\rm e}^{a_1x+a_2y}. \tag{20}$$

Here k_1, h_1, and c_1 are nonzero constants, and must ensure $n > -1$. In this case, it is obvious that the phases of n and ϕ are the same.

3.2. Solutions with phase shift

Solving the determinant equations results in the other solutions

$$b_1 = h_1 e^{a_1 x + a_2 y}, \quad b_2 = h_2 e^{a_1 x + a_2 y},$$
$$b_3 = h_3 e^{a_1 x + a_2 y}, \quad b_4 = h_4 e^{a_1 x + a_2 y}, \tag{21}$$
$$a_1 = \pm \frac{\sqrt{D a_2 (1 - D a_2)}}{D}, \tag{22}$$
$$\begin{aligned} c_0 = {} & c_1^2 \{ c_2 k_1^2 \left[D(a_2 + C)(a_1^2 + a_2^2) - a_2^2 \right] \\ & - a_1 a_2 c_2 k_1 k_3 - a_1^2 (a_2 - C) \left[D(a_1^2 + a_2^2) - a_2 \right] \} \\ & \times \left[-a_2^2 c_2 k_1^2 + a_1^2 (a_2 - C)^2 \right]^{-1} / 2 c_2, \end{aligned} \tag{23}$$
$$c_2 = -\frac{2(a_2 - C)(D a_2 - 1)\left(3 a_2{}^2 - C a_2 + C D\right)}{D\left(6 a_2{}^2 + 2 C a_2 + C D\right) k_1{}^2} < 0, \tag{24}$$
$$h_1 = \frac{h_4 \left(c_1{}^2 - 2 c_2 c_0 \right)}{c_1}, \tag{25}$$
$$\begin{aligned} h_2 = {} & h_4 c_1 \left[\frac{\left(2 a_2{}^2 D - a_2 + 2 D a_1{}^2\right) k_1}{(a_2 - C) a_1} - \frac{k_3}{a_2 - C} \right] \\ & + \frac{2 a_2 c_0 c_2 k_1 h_4}{(a_2 - C) a_1 c_1}, \end{aligned} \tag{26}$$
$$h_3 = -h_4 c_2, \tag{27}$$
$$k_2 = \frac{k_1 a_2}{a_1}, \tag{28}$$
$$k_3 = -\frac{C k_1}{2 a_1 \left(a_2{}^2 + a_1{}^2\right)}, \tag{29}$$

where c_1, h_4, and k_1 are nonzero constants, and must ensure $n > -1$; a_2 is limited by the conditions

$$\begin{aligned} & 120 a_2^5 - 64 C a_2^4 + 8 C (C + 5D) a_2^3 - 26 D C^2 a_2^2 \\ & + C^2 D (2C + 3D) a_2 - 2 C^3 D^2 = 0, \end{aligned} \tag{30}$$
$$D a_2 (1 - D a_2) > 0. \tag{31}$$

Therefore, another group of fluctuant component solutions of Eqs. (3) and (4) can be obtained as

$$\begin{aligned} n = {} & \frac{h_4 \left(-c_1{}^2 + c_1 \sqrt{-4 c_2 c_0 + c_1{}^2} \cos\left(\sqrt{-c_2}\xi + \alpha\right) + 2 c_2 c_0 \right)}{2 c_2} \\ & \times e^{a_1 x + a_2 y}, \end{aligned} \tag{32}$$
$$\begin{aligned} \phi = {} & -\frac{\left[h_1 c_1 - \sqrt{-4 c_2 c_0 + c_1{}^2} \cos\left(\sqrt{-c_2}\xi + \alpha + \beta\right) \right]}{2 c_2} \\ & \times e^{a_1 x + a_2 y}, \end{aligned} \tag{33}$$

where $\beta = \arctan(h_2 \sqrt{-c_2}/h_1)$. Equation (31) admits that $1/D > a_2 > 0$. With known parameters C and D, a_2 can be obtained via Eq. (30), so the other coefficients can be obtained from Eqs. (22)–(29). It is obvious from Eqs. (32) and (33) that there is a small phase difference β between the n and ϕ fields.

4. Analysis and discussion

It is revealed in Eqs. (19), (20), (32), and (33) that the two groups of solutions are not only exponential with $a_1 x + a_2 y$ but also periodical in the x and y directions, and instability and a limitation exist. From Eqs. (12) and (22), with $a_1 < 0$ for $x > 0$ and $a_1 > 0$ for $x < 0$, the solutions are stable in the x direction; otherwise they are unstable. From Eqs. (11) and (31), $1/D > a_2 > CD^2/(D^2 - 2CD + 4) > 0$, so the solutions are stable for $y < 0$ in the y direction, however they are unstable for $y > 0$ and strong fluctuations occur at the edge. The a_2 is determined by C and D, therefore the adiabatic parameter and the diffusion coefficient determine the distributions of n and ϕ and the change of the fluctuant strength.

According to the solutions without phase shift, we first discuss the effect of C and D on the fluctuant density n. Substituting Eq. (11) into Eqs. (12)–(18), we can obtain all coefficients including C and D. Then from Eq. (19), we can obtain the relationship among n, C, and D, which is complex and influenced also by other parameters. With a group of parameters, Fig. 1 displays that the fluctuation is between positive and negative n. It first decreases and then increases with C increasing, changes periodically with D, and decreases with D increasing. So the fluctuant density may be controllable by controlling the adiabaticity and diffusion.

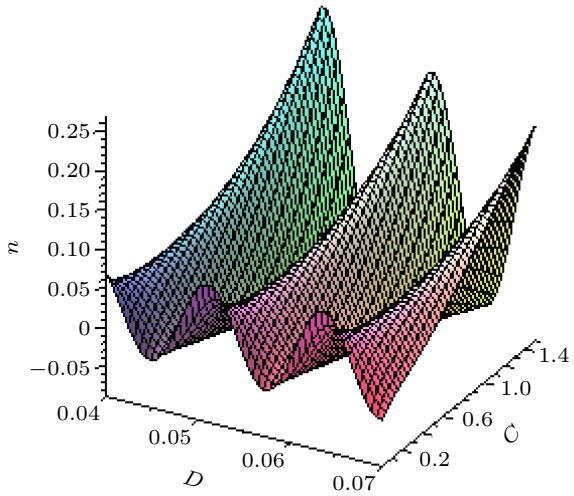


Fig. 1. (color online) Variation of the fluctuant density n given by Eq. (19) with the adiabatic parameter C and diffusion coefficient D, where $c_1 = k_1 = h_1 = 0.5$, $\alpha = 0$, $t = 1000$, $x = y = 1$, and $a_1 > 0$.

Because the solutions with phase shift are more realistic, we analyze the fluctuant solutions (32) and (33) in detail by choosing the adiabatic parameter C and the diffusion coefficient D in the quasi-adiabatic regime.[15] When $C = 0.49$ and $D = 0.018$, we can obtain $a_2 = 0.18$ from Eqs. (30) and (31), $a_1 = \pm 3.14$ from Eq. (22), and the other parameters from Eqs. (23)–(29) by selecting the nonzero constants $c_1 = 10^{-3}$, $h_4 = 8.2 \times 10^{-5}$, and $k_1 = 0.1$. The contour plots of the fluctuant density given by Eq. (32) are shown in Fig. 2. It is observed

that when the positive x increases with $a_1 = 3.14$, or the negative x decreases with $a_1 = -3.14$, the contour curves become dense, and the density gradient becomes larger. In Figs. 2(a) and 2(c), the contours change from lines to curves, and the spacing of contours indicates the periodic distribution in the x and y directions. In Figs. 2(b) and 2(d), the contours become more curved and localized closed structures appear toward the edge, which shows that the boundary is strongly perturbed.

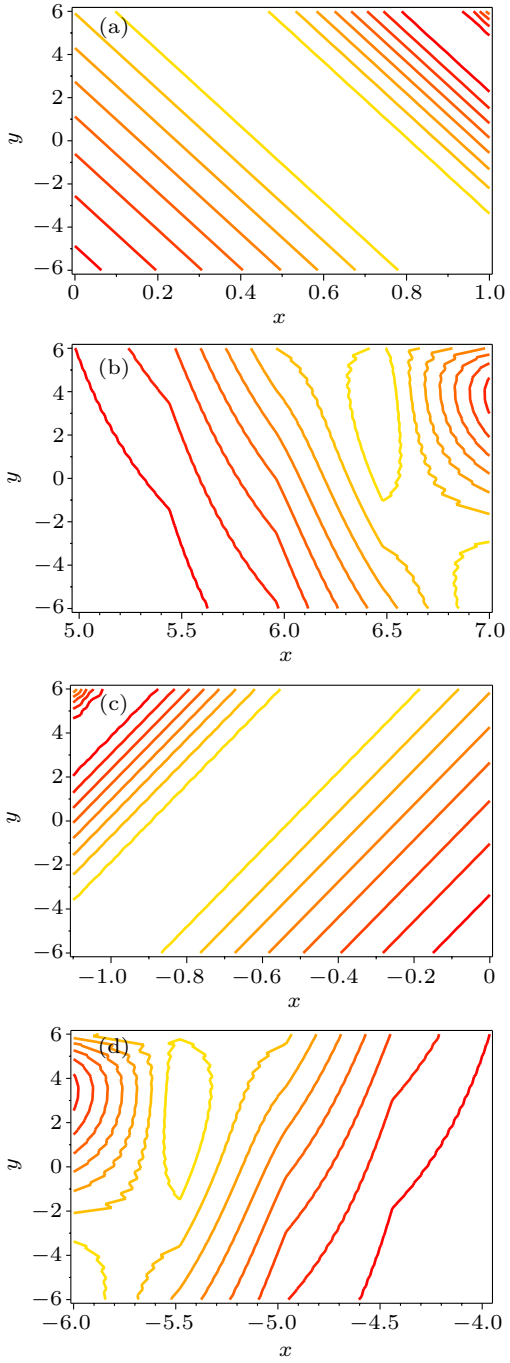


Fig. 2. (color online) Contour plots of the fluctuant density n given by Eq. (32) in the x–y plane with $C = 0.49$, $D = 0.018$, $c_1 = 10^{-3}$, $h_4 = 8.2 \times 10^{-5}$, $k_1 = 0.1$, and (a) $\alpha = 18$, $t = 70$, $a_1 = 3.14$; (b) $\alpha = 4$, $t = 50$, $a_1 = 3.14$; (c) $\alpha = 58$, $t = 20$, $a_1 = -3.14$; (d) $\alpha = 1$, $t = 50$, $a_1 = -3.14$.

The fluctuant potential presents similar spatio-temporal changes to the fluctuant density. We can see from Eqs. (32) and (33) that the fluctuant density and the electrostatic potential have the same periodicity but different phases under this nonviscous condition. When $C = 0.8$, $D = 0.068$, the phase difference is $\beta \simeq \arctan(0.26k_1/\sqrt{k_1^2}) \simeq \pm 0.25$, which has no relationship with the nonzero constants c_1, h_4, k_1 and thus represents the nature of the system.

According to Ref. [12], the edge turbulence gives $|n_1/n_0| \simeq |e\phi/T_e|$ in the coupling HW system. For the Caltech Research Tokamak, the edge turbulence has $|n_1/n_0| \simeq 0.22$.[1] It is remarkable that our results can illustrate the experimental phenomena of density fluctuation and the nonuniform distribution in the plasma edge.

5. Summary

For a tokamak plasma, considering the nonviscosity case and ignoring the high-order terms, we obtain a two-dimensional system of Hasegawa–Wakatani equations to describe the resistive drift-wave fluctuation. Using a special transformation method, we obtain two groups of analytic solutions. It is noted that the fluctuant density and the potential have the same periodicity and similar spatio–temporal distribution characters. From the solutions without phase shift between the fluctuant potential and the density, it is found that the change of the fluctuant density with the adiabatic and diffusion coefficients is complex, and the fluctuation may be controlled through the adiabaticity and diffusion. The other solutions with phase shift are discussed in detail using the typical parameters. It is noticed that the phase difference has no relationship with the nonzero constants and represents the nature of the system. The density gradient becomes larger toward the plasma edge in the x–y plane, and the density distribution shows an irregular structure, which reveals the nonuniform distribution in the tokamak edge. These results may provide beneficial theoretical references for resistive drift-wave fluctuation in an ideal tokamak plasma where the viscosity is very small and can be neglected. More realistic cases including the viscosity and the high-order terms will be studied later.

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