Effects of bi-kappa distributed electrons on dust-ion-acoustic shock waves in dusty superthermal plasmas

M. S. Alam[a)†], M. M. Masud[b)], and A. A. Mamun[a)]

[a)] *Department of Physics, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh*
[b)] *Department of Physics, Bangladesh University of Engineering and Technology (BUET), Bangladesh*

(Received 24 February 2013; revised manuscript received 7 April 2013)

The basic properties of dust-ion-acoustic (DIA) shock waves in an unmagnetized dusty plasma (containing inertial ions, kappa distributed electrons with two distinct temperatures, and negatively charged immobile dust grains) are investigated both numerically and analytically. The hydrodynamic equation for inertial ions has been used to derive the Burgers equation. The effects of superthermal bi-kappa electrons and ion kinematic viscosity, which are found to modify the basic features of DIA shock waves significantly, are briefly discussed.

Keywords: bi-kappa electrons, shock waves, dust-ion-acoustic waves, dusty plasma

PACS: 52.27.–h, 52.27.Lw, 52.30.–q **DOI:** 10.1088/1674-1056/22/11/115202

1. Introduction

Nowadays, the study and analysis of the properties of dusty plasmas have become a popular and important research topic. A particular field of study which has received a great deal of attention is that of dust-ion-acoustic (DIA) waves (e.g., DIA solitary and shock waves) and has been extensively studied both theoretically and experimentally. About 21 years ago, Shukla and Silin[1] theoretically showed that because of the equilibrium charge neutrality condition $n_{i0} = n_{e0} + Z_d n_{d0}$, and the strong inequality $n_{e0} \ll n_{i0}$ (where n_{e0}, n_{d0}, and n_{i0} are electron, dust, and ion number densities at equilibrium respectively, and Z_d is the number of electrons residing onto the dust grain surface) a dusty plasma (with negatively charged immobile dust) supports the low-frequency DIA wave. In this situation, the ion mass provides the inertia and the restoring force comes from the electron thermal pressure. The presence of DIA waves has been verified by the laboratory experiments.[2,3] The linear properties of the DIA waves in an unmagnetized bounded[4,5] plasma, as well as in magnetized[6–9] and inhomogeneous[10] dusty plasmas have also been studied. A number of authors have studied the DIA solitary waves (SWs)[11–15] by using the dusty plasma model consisting of negatively charged static dust and single-temperature electrons as well as ions. These SWs[11–15] are formed due to the delicate balance between nonlinearity and dispersion.

However, a plasma medium with significant dissipative properties, assists the formation of shock structures.[16–19] The Landau damping, kinematic viscosity among the plasma species, and the collision between ion-neutral, dust-neutral, etc. are the major causes of the dissipation that is mainly responsible for the formation of shock structures in a plasma medium. The shock structures in dusty plasmas have been observed in a number of major research laboratories throughout the world. There are also plans to carry out such practical experiments during the mission of the International Space Station. For this reason, studies of the propagation of DIA shock waves in dusty plasmas have great significance. A critical review of shock wave phenomena in dusty plasmas can be obtained in some significant review papers,[20] referenced papers, and books.[21] The DIA shock waves were observed by Nakamura *et al.*[22,23] in a dusty plasma associated with a single type of thermal electrons. The dust-ion interaction produces a kinematic viscosity which is responsible for the formation of DIA shock waves. The results provide that both monotonic and oscillatory shock structures exist and the dust density has significant effects on shock structures as well as the phase velocity of the wave. The detailed theoretical models for the DIA shock waves were given by Shukla[24] for a weakly coupled dusty plasma. The study of the DIA shock waves in a strongly coupled dusty plasma was also presented by Shukla and Mamun.[25] Recently, Sayeed and Mamun[26] generalized the work of Mamun and Shukla[27] to include the roles of Maxwellian electrons and ions. In 2012, Asaduzzaman *et al.*[28] considered a dusty plasma medium where two-temperature Maxwellian ions play a significant role in the wave dynamics. Tasnim *et al.*[29] also considered two-temperature nonthermal ions and discussed the properties of shock waves. In the same year, Masud *et al.*[30–32] considered two-temperature electrons following Maxwellian distributions in a dusty plasma environment and analyzed both the SWs and shock waves in respective articles. However, most of the theoretical works on the DIA shock waves[20,22,24,33–35] are based on single-temperature electrons.

†Corresponding author. E-mail: soyonplasma@gmail.com

© 2013 Chinese Physical Society and IOP Publishing Ltd

Also, most of the studies have been centered on Maxwellian plasmas. The authors have also studied non-Maxwellian plasmas with only one electron component, for instance, in the form of the Cairns distribution,[36] kappa distribution[37,38] or Tsallis distribution.[39] It is due to these features that kappa distributions are convenient in analyzing observational data in space plasmas that show a Maxwellian core at low energies and a power law-like tail at higher energies. For example, kappa distributions have been used to interpret spacecraft data in the Earth's magnetospheric plasma sheet[40] and the solar wind,[41] Jupiter and Saturn;[42] to explain the velocity filtration effect in the solar corona[41,43] and to analyze the field-aligned conductance values in the auroral region by using the Freja satellite data.[44] Schippers *et al.*[45] analysed the CAPS/ELS and MIMI/ LEMMS data from the Cassini spacecraft orbiting Saturn over a range of 5.420 R_S, where $R_S \approx 60300$ km is the radius of Saturn. They found a best fit for the electron velocity distribution using a combination of a hot and a cool electron components, with both species being kappa distributions with individual low values of κ. In particular, the cool electrons typically have $\kappa_c \simeq 1.8$–3,[45] rising to 8 to 10 in the inner magnetosphere. On the other hand, for the hot electrons, κ_h typically lies in the range 3 to 7.[45] The plasmas composed of two kappa-distributed electron components[45–50] are very relevant to the Saturnian magnetosphere.[47]. That is, this model is also the basis of a recent kinetic theory study of the waves in Saturn's magnetosphere.[47]

The family of κ distributions, a power law in particle speed, was introduced on phenomenological grounds to model superthermal structures.[51–54] Vasyliūnas (1968) was the first to propose an empirical functional form for describing the distribution of energy over the whole spectrum of the high-energy power-law tail, and this is now widely known as the kappa distribution. In order to provide the missing link between the Tsallis nonextensive q-statistics to the family of the phenomenologically introduced κ distributions favored in space and astrophysical plasma modeling, we stress that fundamental and generalized physics is provided within the framework of an entropy modification consistent with q nonextensive statistics, and we perform the transformation $\kappa = 1/(q-1)$[55] or $q = 1 + 1/k$.[56] $\kappa \to \infty$ corresponds to $q = 1$ and recovers the extensive limit-Maxwell–Boltzmann distribution. The family of kappa distributions are obtained from the positive definite part[46,57] $3/2 < \kappa < \infty$ corresponding to $-1 \leq q \leq 1$ of the general statistical formalism where in analogy the spectral index kappa is a measure of the degree of nonextensivity. The nonextensive distribution function provides access to nonlocal effects, where κ measures the degree of nonextensivity within the system. Nonextensive q distribution is successfully applied to demonstrate the solar neutrino problem,[58] peculiar velocity distributions of galaxies,[59] fractal like space-times, etc. On the other hand, kappa-distributions are favored in any kind of space plasma modeling[60] among others, where a reasonable physical background was not apparent.

The isotropic three-dimensional (3D) kappa velocity distribution of particles of mass m is of the form[61–65]

$$F_\kappa(v) = \frac{\Gamma(\kappa+1)}{(\pi\kappa\theta^2)^{3/2}\Gamma(\kappa-1/2)}\left(1+\frac{v^2}{\kappa\theta^2}\right)^{-(\kappa+1)}, \quad (1)$$

where θ is the most probable speed (effective thermal speed), related to the usual thermal velocity $V_t = (K_B T/m)^{1/2}$ by $\theta = [(2\kappa-3)/\kappa]V_t$, T being the characteristic kinetic temperature, i.e., the temperature of the equivalent Maxwellian with the same average kinetic energy,[66] and K_B is the Boltzmann constant. The most probable speed, and hence the κ distribution, is defined for $\kappa > 3/2$. The parameter κ is the spectral index, which is a measure of the slope of the energy spectrum of the superthermal particles forming the tail of the velocity distribution function and, thus, allows for a family of power law-like distributions. Low values of κ represent a hard spectrum with a strong non-Maxwellian (power law-like) tail, an enhanced velocity distribution at low speeds, and a depressed distribution at intermediate speeds.[61]. For $\kappa \to \infty$, the Maxwellian distribution is recovered.[67] Inclusion of multiple-temperature electrons is of potential importance and represents more realistic phenomena since such plasma environments are obtained to be common both in space[68,69] and laboratory[70,71] environments. Kundu *et al.*[72] considered a plasma model consisting of kappa distributed electrons of distinct temperature and discussed the properties of dust acoustic waves. In 2010, Pakzad[73] derived the K-dV Burgers equation and properties of ion-acoustic shock waves in the plasma model were analyzed. The effects of ion kinematic viscosity and the superthermal parameter on the ion acoustic waves were observed there, but the presence of dust particles was not considered in that investigation. In the next year, Pakzad[74] also considered a plasma model where the ions are superthermally distributed, and the characteristic of shock waves were investigated by deriving the K-dV Burgers equation. Recently, the behavior of dust acoustic waves was also analyzed by Pakzad[75] in a plasma environment where the ions follow a kappa distribution. In that paper, it was found that the coefficients of the K-dV Burgers equation are significantly modified by the κ-parameter, where κ is the spectral index for superthermal distribution. In addition, until now, no theoretical or experimental investigation on DIA shock waves has been performed considering a dusty plasma model where two populations of thermal electrons following kappa distribution with distinct temperatures and the presence of immobile dust grains.

In this paper, we present a generalized work to investigate the DIA shock waves in an unmagnetized quasi-neutral

dusty plasma. For this purpose, we consider ion continuity and momentum equations, supplemented by the kappa electron density distribution with distinctive temperatures (bi-kappa distribution), to derive the Burgers equation (BE). We have analyzed that the relative number densities of electrons-ions and the temperature ratio of two distinctive electrons are the sources of dissipation and are responsible for the formation of DIA shock waves.

This paper is organized as follows. The basic equations are presented in Section 2. The Burgers equation is derived in Section 3, and finally a brief discussion and summary are provided in Sections 4 and 5.

2. Governing equations

We consider the nonlinear propagation of the DIA waves in an unmagnetized dusty plasma system consisting of negatively charged static dust, inertial ions, and kappa distributed electrons with two distinctive temperatures. Hence, at equilibrium, $n_{i0} = n_{e10} + n_{e20} + Z_d n_{d0}$, where n_{i0} is the unperturbed ion number density, n_{e10} (n_{e20}) is the density of an unperturbed lower (higher) temperature electron, n_{d0} is the unperturbed dust number density, and Z_d is the number of electrons residing on the dust grain surface. It is to be noted that the possible ranges[5] of Z_d can vary in different plasma environments, viz. $Z_d = 10^3$–10^4, 10^5–10^6, 10^3–10^4 in Q-machine, dc, and rf discharges, respectively. The nonlinear dynamics[29,32] of the DIA waves, whose phase speed is much smaller (larger) than the electron (ion) thermal speed in a planar geometry is governed by

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0, \tag{2}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \phi}{\partial x} + \eta \frac{\partial^2 u_i}{\partial x^2}, \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \mu + \mu_{e1}\left(1 - \frac{\sigma_1 \phi}{\kappa_{e1} - 3/2}\right)^{-\kappa_{e1}+1/2} + \mu_{e2}\left(1 - \frac{\sigma_2 \phi}{\kappa_{e2} - 3/2}\right)^{-\kappa_{e2}+1/2} - n_i, \tag{4}$$

where n_i is the ion particle number density normalized by its equilibrium value n_{i0}, u_i is the ion fluid speed normalized by $C_i = (k_B T_{ef}/m_i)^{1/2}$, η is the co-efficient of viscosity normalized by $m_i n_{i0} \omega_{pi} \lambda_{Dm}^2$, ϕ is the electrostatic wave potential normalized by $k_B T_{ef}/e$, $\sigma_1 = T_{ef}/T_{e1}$, $\sigma_2 = T_{ef}/T_{e2}$, $\mu_{e1} = n_{e10}/n_{i0}$, $\mu_{e2} = n_{e20}/n_{i0}$, $\mu = Z_d n_{d0}/n_{i0} = \mu_{e1} + \mu_{e2} - 1$ where $T_{ef} = n_{e0} T_{e1} T_{e2}/(n_{e10} T_{e2} + n_{e20} T_{e1})$. Here n_{e0} is the total electron number density[76] at equilibrium. It should be noted that T_{e1} (T_{e2}) is the lower (higher) electron temperature, T_{ef} is the effective temperature[76] of two electrons, T_i is the ion temperature, k_B is the Boltzmann constant, and e is the magnitude of the electron charge. The time variable t is normalized by $\omega_{pi}^{-1} = (m_i/4\pi n_{i0} e^2)^{1/2}$ and the space variable x is normalized by the effective electron Debye length[5,30–32,77,78] $\lambda_{Dm} = (k_B T_{ef}/4\pi n_{i0} e^2)^{1/2}$.

It is important to mention here that we are interested in studying the properties of shock waves associated with DIA waves[1,32] in which inertia comes from the ion mass and restoring force comes from the electron thermal pressure. We have considered bi-kappa distributed massless electrons (in comparison with ion mass)[46,47] of two distinct temperatures[46,47] and inertial ions. Thus, we have assumed only the ion kinematic viscosity[32] and neglected the electrons kinematic viscosity. This theory is therefore applicable when $V_{Ti} \ll \omega/k \ll V_{Te1}, V_{Te2}$.

We note that we have considered negatively charged stationary (immobile) dust, and thus the contribution of negatively charged dust appeared only in Eq. (4). We further add here that we have assumed constant dust charge, i.e., neglected the dust charge fluctuation. However, the dust charge fluctuation is important only when the dust charging time period (T_{ch}) is comparable to the DIA wave time period (T_w). Thus, our present theory is no longer valid when T_{ch} is comparable to T_w.

It is important to note here that our plasma model containing superthermal electrons of distinct temperatures is applicable when the wave time scale is much smaller than the time scale of the establishment of the thermal equilibrium, and when the two group of electrons occupy different regions of the phase space.[79–81]

3. Derivation of the Burgers equation

We first derive the BE. The DIA BE (with two-electron temperature) has been introduced by the following stretched[29,32] coordinates:

$$\zeta = \epsilon(x - V_p t), \tag{5}$$

$$\tau = \epsilon^2 t, \tag{6}$$

where V_p is the phase speed of the DIA waves and ϵ is a smallness parameter measuring the weakness of the dispersion ($0 < \epsilon < 1$). We then expand n_i, u_i, and ϕ in power series of ϵ:

$$n_i = 1 + \epsilon n_i^{(1)} + \epsilon^2 n_i^{(2)} + \epsilon^3 n_i^{(3)} + \cdots, \tag{7}$$

$$u_i = 0 + \epsilon u_i^{(1)} + \epsilon^2 u_i^{(2)} + \epsilon^3 u_i^{(3)} + \cdots, \tag{8}$$

$$\phi = 0 + \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \epsilon^3 \phi^{(3)} + \cdots, \tag{9}$$

and develop equations in various powers of ϵ. To the lowest order in ϵ, Eqs. (2)–(9) give

$$u_i^{(1)} = \frac{1}{V_p}\psi, \tag{10}$$

$$n_i^{(1)} = \frac{1}{V_p^2}\psi, \tag{11}$$

$$V_{\rm p}=\frac{1}{\sqrt{\dfrac{\mu_{\rm e1}(\kappa_{\rm e1}-1/2)\sigma_1}{\kappa_{\rm e1}-3/2}+\dfrac{\mu_{\rm e2}(\kappa_{\rm e2}-1/2)\sigma_2}{\kappa_{\rm e2}-3/2}}}, \quad (12)$$

where $\psi=\phi^{(1)}$. Equation (12) describes the linear dispersion relation for the DIA waves.

To the next higher order of ϵ, we obtain a set of equations,[29,32] which, after using Eqs. (10)–(12), can be simplified as

$$\frac{\partial n_{\rm i}^{(1)}}{\partial\tau}-V_{\rm p}\frac{\partial n_{\rm i}^{(2)}}{\partial\zeta}+\frac{\partial u_{\rm i}^{(2)}}{\partial\zeta}+\frac{\partial}{\partial\zeta}\left[n_{\rm i}^{(1)}u_{\rm i}^{(1)}\right]=0, \quad (13)$$

$$\frac{\partial u_{\rm i}^{(1)}}{\partial\tau}-V_{\rm p}\frac{\partial u_{\rm i}^{(2)}}{\partial\zeta}+u_{\rm i}^{(1)}\frac{\partial u_{\rm i}^{(1)}}{\partial\zeta}+\frac{\partial\phi^{(2)}}{\partial\zeta}-\eta\frac{\partial^2 u_{\rm i}^{(1)}}{\partial\zeta^2}=0, \quad (14)$$

$$\frac{\mu_{\rm e1}P_1P_2\sigma_1^2}{P_3^2}\psi\frac{\partial\psi}{\partial\zeta}+\frac{\partial n_{\rm i}^{(2)}}{\partial\zeta}+\frac{\mu_{\rm e2}P_4P_5\sigma_2^2}{P_6^2}\psi\frac{\partial\psi}{\partial\zeta}-\frac{\mu_{\rm e1}P_1\sigma_1}{P_3}\frac{\partial\phi^{(2)}}{\partial\zeta}+\frac{\mu_{\rm e2}P_4\sigma_2}{P_6}\frac{\partial\phi^{(2)}}{\partial\zeta}=0, \quad (15)$$

where $P_1=-1/2-\kappa_{\rm e1}$, $P_2=1/2-\kappa_{\rm e1}$, $P_3=-3/2+\kappa_{\rm e1}$, $P_4=-1/2-\kappa_{\rm e2}$, $P_5=1/2-\kappa_{\rm e2}$, and $P_6=-3/2+\kappa_{\rm e2}$.

Next, combining Eqs. (13)–(15), we obtain an equation[29,32]

$$\frac{\partial\psi}{\partial\tau}+A\psi\frac{\partial\psi}{\partial\zeta}-C\frac{\partial^2\psi}{\partial\zeta^2}=0, \quad (16)$$

where

$$A=\frac{V_{\rm p}^3}{2}\left[\frac{3}{V_{\rm p}^4}-\frac{\mu_{\rm e1}P_1P_2\sigma_1^2}{P_3^2}-\frac{\mu_{\rm e2}P_4P_5\sigma_2^2}{P_6^2}\right], \quad (17)$$

$$C=\frac{\eta}{2}. \quad (18)$$

Equation (16) is known as the Burgers equation.[29,32,82] The stationary localized solution of the BE is given by

$$\psi=\psi_m[1-\tanh(\zeta/\delta)], \quad (19)$$

where $\delta=2C/U_0$ is the width[29,32] and $\psi_m=U_0/A$ is the amplitude of the shock wave. It is obvious from Eq. (17), for $A>(<)0$, the dusty plasma supports compressive (rarefactive) DIA shock waves which are associated with a positive (negative) potential, and no shock waves exist at $A=0$. It is obvious that A is a function of $\mu_{\rm e1}$, $\mu_{\rm e2}$, $\kappa_{\rm e1}$, $\kappa_{\rm e2}$, σ_1, and σ_2. Therefore, $A(\mu_{\rm e1}=\mu_{\rm c})=0$ and $\mu_{\rm c}$ can be expressed as

$$\begin{aligned}\mu_{\rm e1} &= \mu_{\rm c}\\ &= \frac{1}{6R_1R_2\sigma_1^2}[Q_1Q_2\sigma_1^2-6Q_3Q_4\mu_{\rm e2}\sigma_1\sigma_2\\ &\quad+\sqrt{R_1R_2R_3\sigma_1^2}],\end{aligned} \quad (20)$$

where $Q_1=-1+4\kappa_{\rm e1}^2$, $Q_2=(3-2\kappa_{\rm e2})^2$, $Q_3=3+4(-2+\kappa_{\rm e1})\kappa_{\rm e1}$, $Q_4=3+4(-2+\kappa_{\rm e2})\kappa_{\rm e2}$, $R_1=(1-2\kappa_{\rm e1})^2$, $R_2=(3-2\kappa_{\rm e2})^2$, $R_3=(1+2\kappa_{\rm e1})^2R_2\sigma_1^2-12(-3+2\kappa_{\rm e1})(1+2\kappa_{\rm e1})Q_4\mu_{\rm e2}\sigma_1\sigma_2+12(3-2\kappa_{\rm e1})^2(-1+4\kappa_{\rm e2}^2)\mu_{\rm e2}\sigma_2^2$.

Equation (20) represents the critical value of $\mu_{\rm e1}$ above (below) which the shock waves with a positive (negative) potential exists, which can be clearly understood from Figs. 1 and 2.

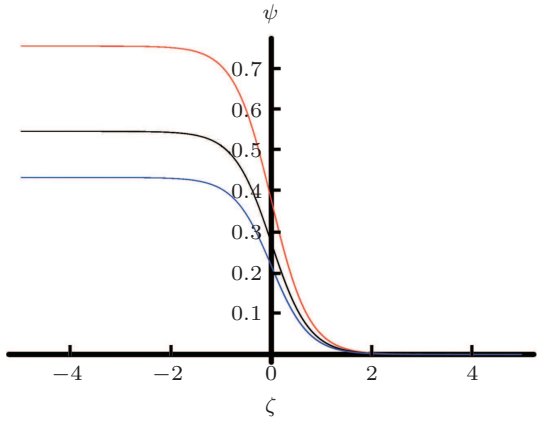


Fig. 1. (color online) Positive potential shock structures for $\mu_{\rm e2}=0.4$, $\sigma_1=2.5$, $\sigma_2=0.1$, $\eta=0.3$, $\kappa_{\rm e1}=1.6$, $\kappa_{\rm e2}=7$, $\mu_{\rm e1}=0.75$ (upper curve), $\mu_{\rm e1}=0.8$ (middle curve), and $\mu_{\rm e1}=0.85$ (lower curve).

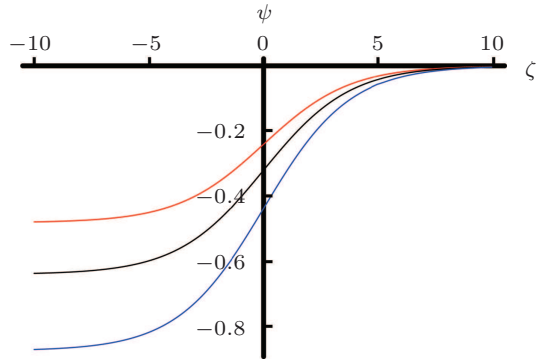


Fig. 2. (color online) Negative potential shock structures for $\mu_{\rm e1}=0.6$, $\mu_{\rm e2}=0.4$, $\sigma_1=2.5$, $\sigma_2=0.1$, $\eta=0.3$, $\kappa_{\rm e2}=7$, $\kappa_{\rm e1}=1.6$ (upper curve), $\kappa_{\rm e1}=1.62$ (middle curve), and $\kappa_{\rm e1}=1.64$ (lower curve).

We have found numerically the critical value of $\mu_{\rm e1}$ ($\mu_{\rm c}=\mu_{\rm e1}=0.63$) for typical dusty plasma parameters, viz. $\mu_{\rm e2}=0.4$, $\sigma_1=2.5$, $\sigma_2=0.1$, $\kappa_{\rm e1}=1.6$, and $\kappa_{\rm e2}=7$. It is clear that $\psi_{\rm m}=\infty$ at $\mu_{\rm e1}=\mu_{\rm c}$ and the BE that we have derived is no longer valid at this condition, so the shock waves are found for $\mu_{\rm e1}\neq\mu_{\rm c}$. The small amplitude shock waves with a positive (negative) potential exist for a set of dusty plasma parameters.[83–86] Figure 1 (2) shows the positive (negative) potential shock waves for different values of $\mu_{\rm e1}$ ($\kappa_{\rm e1}$). Figure 3 (4) shows the variation of the amplitudes of positive (negative) potential shock waves for different values of σ_2 (σ_1). Figure 5 represents the variation of widths of the shock waves with η for different values of U_0.

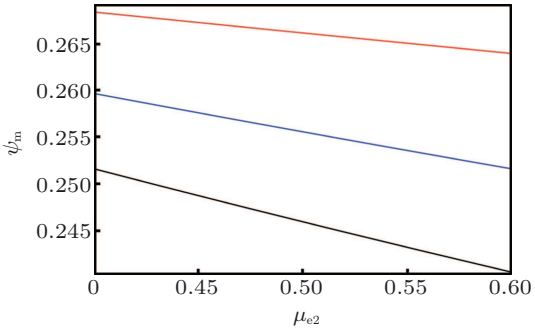


Fig. 3. (color online) Effects of variation of μ_{e2} and σ_2 on amplitudes of positive shock profiles. Here, $\sigma_1 = 2.5$, $\eta = 0.3$, $\kappa_{e1} = 1.6$, $\kappa_{e2} = 7$, $\sigma_2 = 0.2$ (upper curve), $\sigma_2 = 0.4$ (middle curve), and $\sigma_2 = 0.6$ (lower curve).

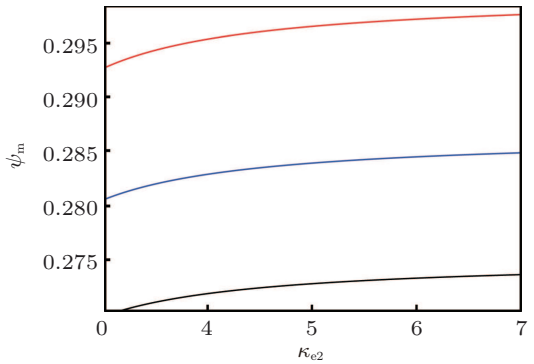


Fig. 4. (color online) Effects of variation of σ_1 and κ_{e2} on amplitudes of negative shock profiles. Here, $\eta = 0.3$, $\mu_{e1} = 0.8$, $\mu_{e2} = 0.4$, $\kappa_{e1} = 1.6$, $\sigma_1 = 2$ (upper curve), $\sigma_1 = 2.2$ (middle curve), and $\sigma_1 = 2.4$ (lower curve).

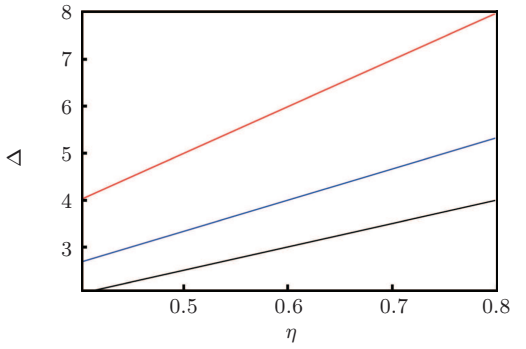


Fig. 5. (color online) Variation of widths of shock structures with η. Here, $\sigma_1 = 2.5$, $\sigma_2 = 0.1$, $\kappa_{e1} = 1.6$, $\kappa_{e2} = 7$. $U_0 = 0.1$ (upper curve), $U_0 = 0.15$ (middle curve), and $U_0 = 0.2$ (lower curve).

4. Discussion

We have considered an unmagnetized dusty plasma system consisting of inertial ions, negatively charged immobile dust, and kappa distributed electrons of two distinct temperatures. The well known BE has been derived by using the reductive perturbation method. We have also analyzed and found that the dusty plasma system under consideration supports finite amplitude shock waves, whose basic features (polarity, amplitude, width, etc.) depend on the relative temperature-ratio of electrons (i.e., σ_1 and σ_2) as well as relative dust number density μ which is a function of relative electron number densities (i.e., μ_{e1} and μ_{e2}).

The results which have been found from this investigation can be pinpointed as follows.

1) In our present investigation, shock waves are formed for above or below the critical value (i.e., when $\mu_{e1} > \mu_c$ or $\mu_{e1} < \mu_c$).

2) We have observed that at $\mu_{e1} > 0.63$, positive (compressive) shock waves exist, whereas at $\mu_{e1} < 0.63$, negative (rarefactive) shock waves exist (shown in Figs. 1 and 2).

3) We have observed that when $\mu_{e1} > 0.63$ (i.e., $\mu > 0.03$, where μ is a function μ_{e1} and μ_{e2} as already described after governing equations) positive potential shock structures are formed (as shown in Fig. 1), and when $\mu_{e1} < 0.63$ (i.e., $\mu < 0.03$), negative potential shock structures are formed (as shown in Fig. 2). This means that the presence of dust may change the polarity of the DIA shock structures. It is also observed from Fig. 3 that the presence of dust grains significantly modifies the basic properties (amplitude and width) of the DIA shock structures.

4) The magnitude of the amplitude of positive shock waves decreases (increases) with the increase (decrease) of relative-electron-number-density μ_{e1} (shown in Fig. 1).

5) It is found that the magnitude of the amplitude of negative shock waves increases (decreases) with the increase (decrease) of the relative-index-parameter κ_{e1} (shown in Fig. 2).

6) The magnitude of the amplitude of shock waves increases (decreases) with the increase (decrease) of κ_{e2} and the magnitude of the amplitude of shock waves decreases (increases) with the increase (decrease) of σ_1 (shown in Fig. 4).

7) The effect of relative-temperature-ratio σ_2 on the shock profiles is found as with the increase of σ_2, the height of the positive potential shock structure gradually decreases (shown in Fig. 3).

8) From our investigation, it is obtained that the width of the shock waves increases with the increase of η. In addition, it can also be said that with the increase of dissipation, the shock waves become smoother and weaker (shown in Fig. 5).

9) The effect of speed on the propagation of DIA shock waves that we have observed is that the amplitude (width) of shock waves decreases (increases) with the increase (decrease) of U_0 (shown in Fig. 5).

5. Summary

The Burgers equation for DIA waves in an unmagnetized dusty plasma system consisting of two-electron temperatures following kappa distribution is numerically investigated. The ranges ($\sigma_1 = 1.8$–5, $\sigma_2 = 0.2$–0.8, $\mu_{e1} = 0.7$–0.9 and $\mu_{e2} = 0.4$–0.6)[30,46,47] of the dusty plasma parameters used in this numerical analysis are very wide, and correspond to space and laboratory dusty plasma situations. The results of our present investigation should be useful in understanding the nonlinear features of electrostatic disturbances in

space dusty plasmas, viz. saturn's magnetosphere,[46] pulsar magnetosphere[72] etc., in which negatively charged dust fluid, ions, and electrons of two different temperatures (hot and cold) can be the major plasma species.

References

[1] Shukla P K and Silin V P 1992 *Phys. Scr.* **45** 508
[2] Barkan A, D'Angelo N and Merlino R L 1996 *Planet Space Sci.* **44** 239
[3] Merlino R L and Goree J A 2004 *Phys. Today* **57** 32
[4] Shukla P K and Rosenberg M 1999 *Phys. Plasmas* **6** 1038
[5] Shukla P K and Mamun A A 2002 *Introduction to Dusty Plasma Physics* (Bristol: IOP Publishing Ltd.)
[6] Shukla P K 2001 *Phys. Plasmas* **8** 1791
[7] Masud M M and Mamun A A 2012 *JETP Lett.* **96** 765
[8] Kundu N R, Masud M M, Ashrafi K S and Mamun A A 2013 *Astrophys. Space Sci.* **343** 279
[9] Masud M M, Kundu N R and Mamun A A 2013 *Can. J. Phys.* **91** 530
[10] Shukla P K 1991 *J. Geophys. Res.* **96** 21343
[11] Rahman O and Mamun A A 2011 *Phys. Plasmas* **18** 083703
[12] Mamun A A 2007 *Phys. Lett. A* **372** 1490
[13] Mamun A A, Jahan N and Shukla P K 2009 *J. Plasma Phys.* **75** 413
[14] Alinejad H 2010 *Astrophys. Space Sci.* **327** 131
[15] Alinejad H 2011 *Astrophys. Space Sci.* **334** 331
[16] Xie B S, He K F and Chen Y P 2000 *Chin. Phys.* **9** 922
[17] Feng S D and Michihisa T 2001 *Acta Phys. Sin.* **50** 1006 (in Chinese)
[18] Huang X G, Wu J, Wang R R, Ma M X, He J H, Ye J J, Gu Y and Fu S Z 2003 *Acta Phys. Sin.* **52** 1877 (in Chinese)
[19] Bian B M, Yang L, Zhang P, Ji Y J, Li Z H and Ni X W 2006 *Acta Phys. Sin.* **55** 4181 (in Chinese)
[20] Shukla P K 2003 *Phys. Plasmas* **10** 1619
[21] Verheest F 2000 *Waves in Dusty Space Plasmas* (Dordrecht: Kluwer)
[22] Nakamura Y, Bailung H and Shukla P K 1999 *Phys. Rev. Lett.* **83** 1602
[23] Nakamura Y 2002 *Phys. Plasmas* **9** 440
[24] Shukla P K 2000 *Phys. Plasmas* **7** 1044
[25] Shukla P K and Mamun A A 2001 *IEEE Trans. Plasma Sci.* **29** 221
[26] Sayeed F and Amun A A 2007 *Phys. Plasmas* **14** 014501
[27] Mamun A A and Shukla P K 2002 *Geophys. Res. Lett.* **29** 1874
[28] Asaduzzaman M and Mamun A A 2012 *J. Plasma Phys.* **78** 125
[29] Tasnim I, Masud M M and Mamun A A 2013 *Astrophys. Space Sci.* **343** 647
[30] Masud M M, Asaduzzaman M and Mamun A A 2012 *Phys. Plasmas* **19** 103706
[31] Masud M M, Asaduzzaman M and Mamun A A 2012 *Astrophys. Space Sci.* **343** 221
[32] Masud M M, Asaduzzaman M and Mamun A A 2013 *J. Plasma Phys.* **79** 215
[33] Luo Q Z, D'Angelo N and Merlino R L 1999 *Phys. Plasmas* **7** 3457
[34] Popel S I, Gisko A A, Golub A P, Losseva T V, Bingham R and Shukla P K 2000 *Phys. Plasmas* **7** 2410
[35] Eliasson B and Shukla P K 2005 *Phys. Plasmas* **12** 024502
[36] Cairns R A, Mamum A A, Bingham R, Bostrom R, Dendy R O, Nairn C M C and Shukla P K 1995 *Geophys. Res. Lett.* **22** 2709
[37] Abbasi H and Pajouh H H 2007 *Phys. Plasmas* **14** 012307
[38] Saini N S, Kourakis I and Hellberg M A 2009 *Phys. Plasmas* **16** 062903
[39] Dubinova I D and Dubinov A E 2006 *Tech. Phys. Lett.* **32** 575
[40] Christon S P, Mitchell D G, Williams D J, Frank L A, Huang C Y and Eastman T E 1988 *J. Geophys. Res.* **93** 2562
[41] Pierrard V and Lemaire J 1996 *J. Geophys. Res.* **101** 7923
[42] Krimigis S M, Carbary J F, Keath E P, Armstrong T P, Lanzerotti L J and Gloeckler G 1983 *J. Geophys. Res.* **88** 8871
[43] Scudder J D 1992 *Astrophys. J.* **398** 299
[44] Olsson A and Janhunen P 1998 *Ann. Geophys.* **16** 1567
[45] Schippers P, Blanc M, Andre N, Dandouras I, Lewis G R, Gilbert L K, Persoon A M, Krupp N, Gurnett D A, Coates A J, Krimigis S M, Young D T and Dougherty M K 2008 *J. Geophys. Res.* **113** A07208
[46] Baluku T K and Hellberg M A 2012 *Phys. Plasmas* **19** 012106
[47] Baluku T K, Hellberg M A and Mace R L 2011 *J. Geophys. Res.* **116** A04227
[48] Alam M S, Masud M M and Mamun A A 2013 *Astro. Phys. Space. Sci.*
[49] Alam M S, Masud M M and Mamun A A 2013 *Plasma Phys. Rep.*
[50] Masud M M, Sultana S and Mamun A A 2013 *Astro. Phys. Space. Sci.*
[51] Ghosh U N, Chatterjee P and Kundu S K 2012 *Astrophys. Space Sci.* **339** 255
[52] Bora M P, Choudhury B and Das G C 2012 *Astrophys. Space Sci.* **341** 515
[53] Alinejad H 2012 *Astrophys. Space Sci.* **339** 249
[54] Eslami P, Mottaghizadeh M and Pakzad H R 2011 *Astrophys. Space Sci.* **333** 263
[55] Beck C 2000 *Physica A* **277** 249
[56] Leubner M P 2002 *Astrophys. Space Sci.* **282** 573
[57] Ashraf S, Yasmin S, Asaduzzaman M and Mamun A A 2013 *Astrophys. Space Sci.* **344** 145
[58] Kaniadakis G, Lavagno A and Quarati P 1996 *Phys. Lett. B* **369** 308
[59] Lavagno A, Kaniadakis G, Rego-Monteiro M, Quarati P and Tsallis C 1998 *Astrophys. Lett. Commun.* **35** 449
[60] Mendis D A and Rosenberg M 1994 *Annu. Rev. Astron. Astrophys.* **32** 419
[61] Summers D and Thorne R M 1991 *Phys. Fluids B* **3** 1835
[62] Lu Q M, Zhou L H and Wang S 2010 *J. Geophys. Res.* **115** A02213
[63] Liu S Q and Li J 2011 *Phys. Scr.* **84** 035504
[64] Hussain S 2012 *Chin. Phys. Lett.* **29** 065202
[65] Chen H and Liu S Q 2011 *Phys. Scr.* **84** 055502
[66] Hellberg M A, Mace R L, Baluku T K, Kourakis I and Saini N S 2009 *Phys. Plasmas* **16** 094701
[67] Shahmansouri M 2012 *Chin. Phys. Lett.* **29** 105201
[68] Dubouloz N, Treumann R A, Pottelette R and Malingre M 1993 *Geophys. Res. Lett.* **98** 17415
[69] Pickett J S, Chen L J, Mutel R L, Christopher I W, Santolk O, Lakhina G S, Singh S V, Reddy R V, Gurnett D A, Tsurutani B T, Lucek E and Lavraud B 2008 *Adv. Space Res.* **41** 1666
[70] Derfler H and Simonen T C 1969 *Phys. Fluids* **12** 269
[71] Henry D and Treguier J P 1972 *J. Plasma Phys.* **8** 311
[72] Kundu S K, Ghosh D K, Chatterjee P and Das B 2011 *Bulg. J. Phys.* **38** 409
[73] Pakzad H R 2010 *Astrophys. Space Sci.* **331** 169
[74] Pakzad H R 2011 *World Acad. Sci., Engin. Technol.* **49** 741
[75] Pakzad H R 2012 *Indian J. Phys.* **86** 743
[76] Jones W D, Lee A, Gleman S M and Doucet H J 1975 *Phys. Rev. Lett.* **35** 1349
[77] Das G C, Surchandra S S and Ibohanbi S K 1996 *Chaos Soliton. Fract.* **7** 309
[78] Kakad A P, Singh S V, Reddy R V, Lakhina G S and Tagare S G 2009 *Adv. Space Res.* **43** 1945
[79] Buti B 1980 *Phys. Lett. A* **76A** 251
[80] Nishihara K and Tajiri M 1981 *J. Phys. Soc. Jpn.* **50** 4047
[81] Yu M Y and Luo H Q 2008 *Phys. Plasmas* **15** 024504
[82] Ling H 2006 *Acta Phys. Sin.* **55** 3864 (in Chinese)
[83] Kamal K G, Buddhadeb P, Chandra D and Sailendra N P 2008 *J. Phys. A: Math. Theor.* **41** 335501
[84] Shalaby M, El-Labany S K, El-Shamy E F and El-Sherif L S 2009 *Chaos Soliton. Fract.* **41** 1208
[85] Moslem W M and El-Taibany W F 2005 *Phys. Plasmas* **12** 122309
[86] Chatterjee P and Roychoudhury R 1997 *Can. J. Phys.* **75** 337