Nonlinear Landau damping of high frequency waves in non-Maxwellian plasmas*

M. N. S. Qureshi[a)b)†], Sumbul Sehar[a)], J. K. Shi[b)], and H. A. Shah[a)]

[a)] *Department of Physics, GC University, Lahore*

[b)] *Center for Space Science and Applied Research, Chinese Academy of Sciences, Beijing* 100190, *China*

(Received 11 March 2013; revised manuscript received 28 May 2013)

Space plasmas often possess non-Maxwellian distribution functions which have a significant effect on the plasma waves. When a laser or electron beam passes through a dense plasma, hot low density electron populations can be generated to alter the wave damping/growth rate. In this paper, we present theoretical analysis of the nonlinear Landau damping for Langmuir waves in a plasma where two electron populations are found. The results show a marked difference between the Maxwellian and non-Maxwellian instantaneous damping rates when we employ a non-Maxwellian distribution function called the generalized (r, q) distribution function, which is the generalized form of the kappa and Maxwellian distribution functions. In the limiting case of $r = 0$ and $q \to \infty$, it reduces to the classical Maxwellian distribution function, and when $r = 0$ and $q \to \kappa + 1$, it reduces to the kappa distribution function.

Keywords: nonlinear Landau damping, non-Maxwellian distribution function, nonlinear Langmuir waves, wave–particle interaction

PACS: 52.25.Dg, 52.27.Cm, 52.35.Fp, 52.35.Mw **DOI:** 10.1088/1674-1056/22/11/115201

1. Introduction

Landau damping is a fundamental process in plasma physics and is responsible for the transfer of wave energy to particles. Landau, in his elegant paper,[1] studied the collisionless damping caused by the resonant interactions of the particles moving with velocities comparable to the wave phase velocity. In his perturbation theory, the electric field of the wave was assumed to be small and the collisionless-linearized Boltzmann equation was considered. In the early 1960s, Landau's result was generalized, and it was proved that the solutions to the linearized Vlassov–Poisson's system Landau damp whenever the equilibrium distribution is single humped, i.e., when it has only one maximum, as in the case of Maxwellian distribution.[2–4] Experimental evidence for Landau damping were presented later.[5,6] However, when the wave amplitude is large, the linear theory cannot be applied and it is difficult to obtain strict solutions; only the particular solutions can be obtained. For sinusoidal waves, this problem was first discussed by O'Neil,[7] who considered electron plasma waves of constant and uniform amplitude that grew infinitely quickly in an initially Maxwellian plasma. The validity of most of the linear results is limited to the short time scales on which the linearization is accurate. On the large time scales, nonlinear effects can prevent the complete damping of the sinusoidal wave and lead to a fixed amplitude time asymptotic travelling wave mode sustained by the oscillations of the particles trapped in the wave's potential well.

The appearance of a strongly nonlinear behavior is associated with the tuning points in the trapped-electron trajectories. Hence, the time scale at which the nonlinear effects become important can be estimated to be the same as the bounce time τ_B. If $\tau_L \ll \tau_B$, where τ_L is the time at which linear Landau damping takes place, the wave will damp before the nonlinear effects become important and the linear theory is reasonably accurate.[7] If $\tau_L \geqslant \tau_B$, the initial decay of the wave amplitude will soon turn into nonlinear oscillations around a value somewhat lower than the initial value. After a few oscillations, the amplitude will approach a steady value and the wave becomes the so called Bernstein–Greene and Kruskal (BGK) mode.[8] However, Brodin[9] concluded that the amplitude never settles to a steady value although the nonlinearities obstruct the linear decay. Instead, the energy of the resonant particles diffuses slowly into higher harmonics. In the literature, nonlinear Landau damping has been studied theoretically by numerous authors,[10–14] however, all the studies were restricted to Maxwellian plasmas. Verification of the nonlinear Landau damping of an electron plasma wave has also been reported both by observation in space as well through experiments.[15,16] To our best knowledge, no study has been done on the nonlinear Landau damping with a non-Maxwellian distribution comprising of two electron populations.

Xiao *et al.*[17] studied nonlinear Landau damping of ion-acoustic waves by investigating the threshold of nonlinearity with the drifting electrons in Maxwellian plasmas. In this pa-

*Project supported by the Pakistan Science Foundation Project No. PSF/Res/P-GCU/Phys. (143), the National Natural Science Foundation of China (Grant Nos. 41074114 and 41274146), and the Specialized Research Fund for State Key Laboratories of China.

†Corresponding author. E-mail: nouman_sarwar@yahoo.com

© 2013 Chinese Physical Society and IOP Publishing Ltd

per, we study nonlinear Landau damping of Langmuir waves in non-Maxwellian plasmas by employing the generalized (r, q) distribution function[18,19] and discuss how the nonlinear effects become important on short time scales. We then present instantaneous nonlinear damping rates and compare our results with the Maxwellian instantaneous damping rates.

2. Mathematical formulism

We consider a plasma which is initially uniform and unmagnetized, containing ions, cold bulk electrons, and a small population of hot electrons. In this case, we solve the 1-dimensional (1D) Vlasov–Poisson system of equations

$$\frac{\partial f}{\partial t}+v\frac{\partial f}{\partial x}-\frac{e}{m}E\frac{\partial f}{\partial v}=0, \tag{1}$$

$$\frac{\partial E}{\partial x}=\int_{-\infty}^{\infty}f\,\mathrm{d}v-1, \tag{2}$$

where $f(x,v,t)$ is the two-temperature electron distribution function, and the electrostatic wave is monochromatic and has the form

$$E\left(x,t\right)=E_0\left(t\right)\sin\left(kx-\omega t\right). \tag{3}$$

We assume that the amplitude $E_0(t)$ evolves with a slow time scale compared to the frequency ω, i.e., $|(\mathrm{d}E_0/\mathrm{d}t)/E_0|\ll\omega$. Oscillations are excited by initializing the fundamental mode $k=2\pi/\lambda$, where k is the wave vector and λ is the wavelength. Then

$$f\left(x,v,0\right)=f_{rq}\left(v\right)\left[1+\varepsilon\cos\left(kx\right)\right], \tag{4}$$

where $\varepsilon=\partial n/n$ is the density perturbation amplitude, and the electron distribution function satisfies the 1D Vlasov equation.

Linear Landau damping occurs when those (resonant) electrons whose phase velocities match the velocity of the wave take energy from the wave while the bulk distribution (having velocities that are mismatched with the wave phase velocity) does not exchange energy with the wave. As an increase in the kinetic energy of the electrons is equal to a decrease in the wave energy, therefore, we are interested in the rate of change of kinetic energy of the resonant electrons in the field $E\left(x,t\right)$. The rate of change of kinetic energy is then given by

$$\frac{\mathrm{d}K}{\mathrm{d}t}=\frac{m}{2}\int\mathrm{d}v v^2\left\langle\frac{\partial f}{\partial t}\right\rangle_x, \tag{5}$$

where the bracket $\langle\ \rangle_x$ denotes the spatial average. We start by considering a perturbative approach, and divide the distribution into an unperturbed background distribution and a perturbed distribution evolved due to the field $E\left(x,t\right)$. We then put this total distribution consisting of an unperturbed part and a perturbed part into the Vlasov Eq. (1) and expressing Eq. (5) in the lowest order as

$$\begin{aligned}\frac{\mathrm{d}K}{\mathrm{d}t}&=-\pi\frac{\varepsilon_0\omega_\mathrm{p}^2}{2k}E_0^2\left(t\right)v\left.\frac{\partial f_0/N}{\partial v}\right|_{v=\omega/k}\\&\quad-\frac{\varepsilon_0}{4}\frac{\mathrm{d}E_0^2(t)}{\mathrm{d}t}\frac{\partial}{\partial\omega}\left\{\omega\frac{\omega_\mathrm{p}^2}{k^2}\mathrm{P}\int\mathrm{d}v\frac{N\partial f_0/\partial v}{v-\omega/k}\right\},\end{aligned} \tag{6}$$

where $\omega_\mathrm{p}^2=ne^2/(m\varepsilon_0)$ is the square of the plasma frequency, and P stands for the principle part. The first term on the right-hand side of Eq. (6) gives the contribution from the resonant particles, while the second term corresponds to the contribution from the bulk of the distribution. For a fixed amplitude sinusoidal wave, the bulk of the distribution simply oscillates back and forth in the electrostatic field, and therefore it experiences no secular gain or loss in energy. However, the resonant particles see a nearly constant field, and thus can be efficiently accelerated or decelerated.

The space averaged potential energy of the wave can be written as

$$U=\left\langle\frac{\varepsilon_0}{2}E^2(x,t)\right\rangle_x=\frac{\varepsilon_0}{4}E_0^2\left(t\right). \tag{7}$$

Recalling the law of conservation of energy, we can write Eq. (6) as

$$\begin{aligned}&\frac{\varepsilon_0}{4}\frac{\mathrm{d}E_0^2\left(t\right)}{\mathrm{d}t}\frac{\partial}{\partial\omega}\left[\omega\left(1-\frac{\omega_\mathrm{p}^2}{k^2}\mathrm{P}\int\mathrm{d}v\frac{N\partial f_0/\partial v}{v-\omega/k}\right)\right]\\&=\pi\frac{\varepsilon_0}{2}\frac{\omega_\mathrm{p}^2}{k}E_0^2\left(t\right)v\left.\frac{\partial f_0/N}{\partial v}\right|_{v=\omega/k}.\end{aligned} \tag{8}$$

The right-hand side of the above equation gives the variation of the kinetic energy of the resonant particles, while the left-hand side shows the time variation of the wave energy, which is the sum of the kinetic energy of the bulk population and the potential energy of the wave. Therefore, the rate of change of wave energy is equal to the rate of change of kinetic energy of the resonant particles and can be written as

$$\frac{\mathrm{d}E_\mathrm{wave}}{\mathrm{d}t}=-\frac{\mathrm{d}K}{\mathrm{d}t}, \tag{9}$$

where

$$E_\mathrm{wave}=\frac{\varepsilon_0}{4}E_0^2\left(t\right)\frac{\partial}{\partial\omega}\left(\omega\varepsilon_\mathrm{R}(\omega)\right), \tag{10}$$

and

$$\varepsilon_\mathrm{R}\left(k,\omega\right)=1-\frac{\omega_\mathrm{p}^2}{k^2}\mathrm{P}\int\mathrm{d}v\frac{N\partial f_0\partial v}{v-\omega/k} \tag{11}$$

is the real part of the dielectric function, i.e., the non-resonant part. By assuming an exponential decay of the wave and taking the long-wavelength limit, i.e., $k\lambda_\mathrm{D}\ll1$, where $\lambda_\mathrm{D}=\left(T_\mathrm{c}/4\pi Ne^2\right)$, the linear damping rate in the resonant limit can be written as

$$\Omega_\mathrm{i}=-\pi\frac{\omega_\mathrm{P}^2}{k}v\frac{\partial f_0/N}{\partial v}. \tag{12}$$

Equation (12) gives the general linear damping rate of electrostatic waves satisfied by any distribution function. The basic procedure for handling the nonlinear evolution of the wave's amplitude is essentially the same as that for the linear regime addressed above. The nonlinear calculation differs however from the linear case in the way the distribution of the resonant particles is computed, which cannot be derived by applying a perturbative approach but must be calculated from the exact Vlasov equation.

Linear Landau damping remains valid until $\partial f_1/\partial v < \partial f_0/\partial v$. However, when $\partial f_1/\partial v$ becomes of the order of $\partial f_0/\partial v$, the linear damping breaks down and nonlinear effects have to be taken into account. Away from resonance, terms grow linearly with time t, however, at resonance, terms grow as t^2 and become dominant quickly, and eventually the linear theory breaks down. If $t=\tau$, τ is the time at which the breakdown occurs, one can find at resonance $\partial f_1/\partial v \sim \partial f_0/\partial v$ or $t > \tau_b = (m/eE_0k)^{1/2} = 1/\omega_b$, where ω_b is the bounce frequency of the electrons trapped deeply in the potential well of the wave. Away from the resonance, i.e., in the bulk, we can have $t > m\omega/eE_0k = \omega/\omega_b^2$. Therefore, the linear results in the bulk of the distribution remain valid for a much longer time than the resonant region assuming $\omega \gg \omega_b$. Thus, the wave damps out before the linear approximation breaks down, i.e., $\Omega_i \gg \omega_b$.

If we consider the case where $\omega_b^2/\omega \ll \Omega_i \ll \omega_b \ll \omega$, then the trapped electrons have time to bounce back and forth many times in the potential well of the electrostatic field before the amplitude of the field is damped significantly. In this case, for the resonant particles, we can consider the fully nonlinear trajectories of the electrons in the sinusoidal field; however, the linear response of the bulk distribution remains valid over the characteristic time scales of damping. The effect of the resonant particles includes significant contributions from both the trapped electrons and the passing electrons.

The nonlinear damping rate in the limit $\omega_b \gg \Omega_i$ can be written as[7]

$$\gamma(t) = \Omega_i \sum_{n=0}^{\infty} \frac{64}{\pi} \int_0^1 \mathrm{d}\, j \left[\frac{2n\pi^2 \sin\left(\dfrac{n\pi\omega_b t}{j\mathcal{F}}\right)}{j^5\mathcal{F}^2\left(1+q^{2n}\right)\left(1+q^{-2n}\right)} + \frac{(2n+1)\,\pi^2 j \sin\left(\dfrac{(2n+1)\pi\omega_b t}{2\mathcal{F}}\right)}{\mathcal{F}^2\left(1+q^{2n+1}\right)\left(1+q^{-2n-1}\right)} \right], \tag{13}$$

where $q = \exp\left(\pi\mathcal{F}'/\mathcal{F}\right), \mathcal{F} = \mathcal{F}\left(j^2\right)$ is the complete elliptic integral of the first kind and $\mathcal{F}' = \mathcal{F}'\left(j^2\right) = \mathcal{F}(\sqrt{1-j^2})$. Equation (13) includes the integral $\int \mathrm{d}\, j$ over the energy variable j, characterizing the different orbits of the resonant particles in phase space. The first term in the integrand corresponds to the contribution from the resonant passing particles, while the second term is related to the trapped particles. In the limit $t \to 0$, Eq. (13) reduces to the linear damping rate Ω_i.

Assuming E_0 to vary as $E_0 \exp\left(-\int_0^t \gamma(t')\mathrm{d}t'\right)$, we can invoke the law of conservation of energy and obtain

$$\int_0^t \mathrm{d}t\gamma(t) = \frac{\Omega_i}{\omega_b} \sum_{n=0}^{\infty} \frac{64}{\pi} \int_0^1 \mathrm{d}\, j \left\{ \frac{2\pi\left[1-\cos\left(\dfrac{n\pi t}{j\mathcal{F}\tau}\right)\right]}{j^4\mathcal{F}\left(1+q^{2n}\right)\left(1+q^{-2n}\right)} + \frac{2\pi j\left[1-\cos\left(\dfrac{(2n+1)\pi t}{2\mathcal{F}\tau}\right)\right]}{\mathcal{F}\left(1+q^{2n+1}\right)\left(1+q^{-2n-1}\right)} \right\}. \tag{14}$$

3. Generalized (r, q) distribution function and numerical results

The generalized (r, q) distribution function, which consists of a cold dense electron population and a fractional (F) hot population, is of the form[19]

$$f_{rq}(v) = \frac{(1-F)C}{\pi}\left(\frac{a}{DT_c}\right)^{3/2}\left[1+\frac{1}{q-1}\left(\frac{a}{D^2}\frac{v^2}{T_c}\right)^{r+1}\right]^{-q} + \frac{FC}{\pi}\left(\frac{a}{DT_h}\right)^{3/2} \times \left[1+\frac{1}{q-1}\left(\frac{a}{D^2}\frac{v^2}{T_h}\right)^{r+1}\right]^{-q}, \tag{15}$$

where

$$C = \frac{3(q-1)^{-3/2(1+r)}\Gamma(q)}{4\Gamma\left(q-\dfrac{3}{2(1+r)}\right)\Gamma\left(1+\dfrac{3}{2(1+r)}\right)}, \tag{16}$$

$$D = \frac{3(q-1)^{-1/(1+r)}\Gamma\left(q-\dfrac{3}{2(1+r)}\right)\Gamma\left(\dfrac{3}{2(1+r)}\right)}{\Gamma\left(q-\dfrac{5}{2(1+r)}\right)\Gamma\left(d\dfrac{5}{2(1+r)}\right)}. \tag{17}$$

Here, Γ is the Gamma function; T_c and T_h are the cold and hot electron temperatures, respectively, and $a = m/2k_B$, where m is the electron mass and k_B is the Boltzmann constant. The total electron density is the sum of the hot and cold electron densities. In the limit $r=0$ and $q=\kappa+1$, Eq. (15) reduces to the kappa distribution function; and in the limiting case $r=0$ and $q \to \infty$, it reduces to the well-known Maxwellian distribution function. We note that $q>1$ and $q(1+r)>5/2$ are the conditions which arise from the normalization and the definition of temperature for the distribution function.[20]

First we consider the case of high frequency electrostatic waves where ion dynamics is not important. This procedure yields the propagation and damping characteristics of electron plasma waves in the limiting case of $k\lambda_D \ll 1$, where

$$\lambda_{\rm D} = \left(\frac{T_{\rm c}}{4\pi N e^2}\right) = \left(\frac{(q-1)^{1/(1+r)}\Gamma\left(q-\frac{5}{2(1+r)}\right)\Gamma\left(\frac{5}{2(1+r)}\right)}{3\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(\frac{3}{2(1+r)}\right)}\right)^{1/2}\left(\frac{\psi_{\rm thc}}{\omega_{\rm p}}\right) \tag{18}$$

is the modified electron Debye length,

$$\psi_{\rm thc} = \left\{(T_{\rm c}/m)\left(\frac{3(q-1)^{-1/(1+r)}\Gamma\left[q-\frac{3}{2(1+r)}\right]\Gamma\left[\frac{3}{2(1+r)}\right]}{\Gamma\left[q-\frac{5}{2(1+r)}\right]\Gamma\left[\frac{5}{2(1+r)}\right]}\right)\right\}^{1/2}$$

is the generalized electron thermal velocity for the (r, q) distributed plasma, and N is the total number density. Following the general procedure for the electrostatic waves modeled with the generalized (r,q) distribution function, we can obtain the damping rate for the Langmuir wave as[19]

$$\begin{aligned}\Omega_{\rm i} = &-\frac{\pi C}{D^{3/2}}\frac{1}{k^3\lambda_{\rm D}^3}\\ &\times\left[(1-F)\left(1+\frac{1}{q-1}\left(\frac{1}{Dk^2\lambda_{\rm D}^2}\right.\right.\right.\\ &\left.\left.+2K(1-F+F(T_{\rm h}/T_{\rm c}))\right)^{r+1}\right)^{-q}\\ &+F\left(\frac{T_{\rm c}}{T_{\rm h}}\right)^{3/2}\left(1+\frac{1}{q-1}\left(\frac{1}{Dk^2\lambda_{\rm D}^2}\right.\right.\\ &\left.\left.\left.+2K(1-F+F(T_{\rm h}/T_{\rm c}))\left(\frac{T_{\rm c}}{T_{\rm h}}\right)\right)^{r+1}\right)^{-q}\right],\end{aligned} \tag{19}$$

$$K = \frac{3(q-1)^{\frac{-1}{1+r}}\Gamma\left(q-\frac{5}{2(1+r)}\right)\Gamma\left(1+\frac{5}{2(1+r)}\right)}{10\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(1+\frac{3}{2(1+r)}\right)}. \tag{20}$$

The instantaneous rate of change of nonlinear Landau damping as a function of time is plotted in Figs. 1 and 2 by using Eq. (13) for different values of r and q when $k\lambda_{\rm D} = 0.25$. Here time t is normalized by $\tau_{\rm b} = 2\pi/\omega_{\rm b}$ and the rate $\gamma(t)$ corresponds to the change in kinetic energy of the resonant particles. Figure 1 depicts the instantaneous rate of change of nonlinear Landau damping for different values of q when r is fixed. The positive damping reflects that majority of the electrons absorb energy from the wave and are accelerated due to the increase in kinetic energy, and as a result the wave damps out. The negative damping reflects that a majority of the electrons give energy to the wave and are decelerated due to the decrease in kinetic energy, and as a result the wave grows. From Fig. 2, we can see that more electrons are in resonance with the wave for the case of generalized (r, q) distribution function as compared to the Maxwellian plasma. As q increases for a fixed value of r, we can see that the nonlinear damping rate decreases and approaches to Maxwellian. In Fig. 1, the dotted line indicates the nonlinear damping rate when the Maxwellian distribution contains only the cold bulk electron population.

Figure 2 depicts the instantaneous rate of change of nonlinear Landau damping for different values of r when q is fixed. From Fig. 2, we can see that more electrons are in resonance with the wave for the case of generalized (r, q) distribution function as compared to the Maxwellian plasma. As r increases for a fixed value of q, the nonlinear damping rate decreases and approaches to Maxwellian. In Fig. 2, the dotted line indicates the nonlinear damping rate when the Maxwellian distribution contains only the cold bulk electron population. A comparison of Figs. 1 and 2 shows that the nonlinear damping rate reduces to the Maxwellian damping rate more quickly when shoulders or the flat part of the distribution becomes more flattened.

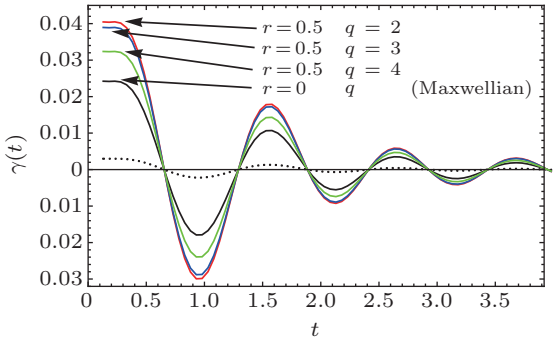


Fig. 1. (color online) Instantaneous nonlinear Landau damping rates for different q when r is fixed.

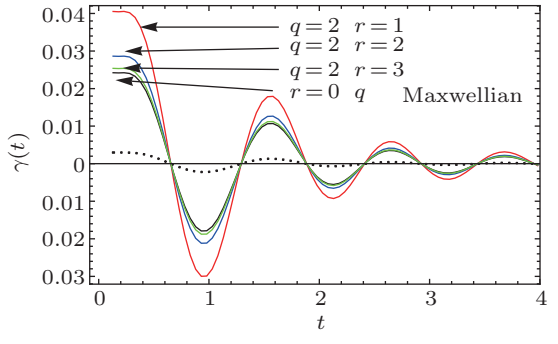


Fig. 2. (color online) Instantaneous nonlinear Landau damping rates for different r when q is fixed.

The instantaneous wave amplitude $E(t)$ can be defined as

$$\frac{E(t)}{E_0} = \exp\left[-\int_0^t \mathrm{d}t'\gamma(t')\right], \tag{21}$$

which can be solved numerically for a given value of $\Omega_\mathrm{i}/\omega_\mathrm{b}$ using Eq. (21). Figure 3 depicts the normalized wave amplitude plotted against the time t normalized by ω_b for different values of $\Omega_\mathrm{i}/\omega_\mathrm{b}$. We can see that the amplitude of oscillation decreases as the ratio $\Omega_\mathrm{i}/\omega_\mathrm{b}$ decreases. The phase mixing of the electron orbits and the evolution to a constant amplitude state as $t \to \infty$ is also prominent in Fig. 3.

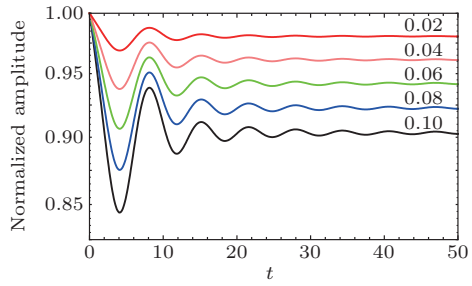


Fig. 3. (color online) Instantaneous evolution of wave amplitude.

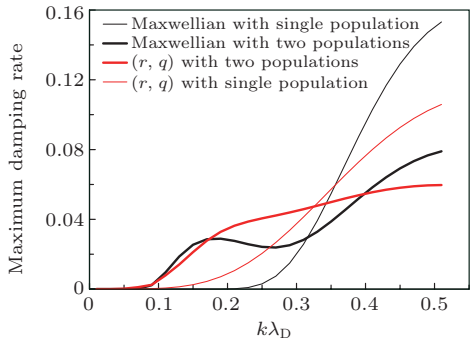


Fig. 4. (color online) Comparison of the maximum amplitude of the nonlinear damping rate against the normalized wave number for the (r, q) distribution, here $r = 1$ and $q = 2$.

A comparison between the maximum starting normalized nonlinear amplitudes of the damping rates for (r,q) distribution and Maxwellian distribution functions comprised of one and two electron populations are presented in Fig. 4. The bold lines represent the damping rates based on the two population distributions whereas the thin lines depict the damping rates derived for one electron population. The nonlinear damping rate for the (r,q) distribution function (bold red line) remains higher than the Maxwellian damping rate (bold black line) in the range $0.15 < k\lambda_\mathrm{D} < 0.4$, however, the Maxwellian damping rate remains higher in the ranges $0.1 < k\lambda_\mathrm{D} < 0.15$ and $k\lambda_\mathrm{D} > 0.4$. The normalized amplitudes of the nonlinear damping rates derived for distributions containing one electron population are given as thin lines. The amplitudes of damping rates for both $(r,\ q)$ and Maxwellian distributions become higher than the corresponding amplitudes of both distributions when they contain two electron populations at about $k\lambda_\mathrm{D} > 0.32$.

4. Conclusion

The nonlinear damping increases as compared to the Maxwellian damping when we use the generalized (r,q) distribution function in the range $0.15 < k\lambda_\mathrm{D} < 0.4$, as more electrons are in resonance with the wave when either of the spectral indices r and q decreases for the generalized (r,q) distribution function. In the high velocity region, the generalized (r,q) distribution function contains a power law tail whose slope (in magnitude) increases with decreasing q or r, and more particles take part in the energy exchange process, which explains why smaller values of q and r enhance the nonlinear damping as compared to the Maxwellian one. Moreover, the normalized amplitude of the nonlinear damping rate for the generalized (r,q) distribution function as well as that for the Maxwellian distribution remains high when the distributions contain two electron populations as compared to that with one electron component for $k\lambda_\mathrm{D} \approx 0.32$. Moreover, the maximum instantaneous amplitude for the generalized (r,q) distribution function remains higher than the Maxwellian amplitude for any values of r and q when $k\lambda_\mathrm{D} < 0.35$ except for a small range $0.12 < k\lambda_\mathrm{D} < 0.175$. We also study how the normalized nonlinear wave amplitude varies with the time t normalized by ω_b. Numerical results show that the wave amplitude decreases as the ratio $\Omega_\mathrm{i}/\omega_\mathrm{b}$ decreases. Significant initial damping occurs before the start of the oscillation for the ratio $\Omega_\mathrm{i}/\omega_\mathrm{b} > 0.02$ and after a few oscillations, the amplitude settles down to a constant value as $t \to \infty$ due to the phase mixing of the electron orbits.

References

[1] Landau L D 1946 *J. Phys.* (*Moscow*) **10** 25
[2] Penrose O 1960 *Phys. Fluids* **3** 258
[3] Jackson A J 1960 *J. Nucl. Energy C* **1** 171
[4] Backus G 1960 *J. Math. Phys.* **1** 178
[5] Malberg J H and Warton C B 1966 *Phys. Rev. Lett.* **17** 175
[6] Defler H and Simonen T 1967 *J. Appl. Phys.* **38** 5014
[7] O'Neil T M 1965 *Phys. Fluids* **8** 2255
[8] Bernstein I B, Greene J M and Kruskal M D 1957 *Phys. Rev.* **108** 546
[9] Brodin G 1997 *Phys. Rev. Lett.* **78** 1263
[10] Matthieussent G 1976 *J. Plasma Phys.* **15** 371
[11] Brunetti M, Califano F and Pegoraro F 2000 *Phys. Rev. E* **62** 4109
[12] Belakhovsky V B and Maltsev Y P 2005 *Physics of Auroral Phenomenoa, Proc. XXVIII Annual Seminar* **89**
[13] Benisti D, Strozzi D J, Gremillet L and Morice O 2009 *Phys. Rev. Lett.* **103** 155002
[14] Yampolsky N A and Fisch N J 2009 *Phys. Plasma* **16** 072104
[15] Tsunehiro M, Ryuji H, Yoshiaki K, Shohei U, Masao S and Reiji S 2001 *J. Fusion Res.* **4** 322
[16] Soucek J, Krasnoselskikh V, Wit T de, Pickett J and Kletzing C 2005 *J. Geophys. Res.* **110** A08102
[17] Xiao Z, Wang W Q and Hao Y Q 2004 *Chin. Phys. Lett.* **21** 2461
[18] Qureshi M N S, Shi J K and Ma S Z 2005 *Phys. Plasmas* **12** 122902
[19] Qureshi M N S, Sehar S, Shah H A and Cao J B 2013 *Chin. Phys. B* **22** 035201
[20] Qureshi M N S, Shah H A, Murtaza G, Schwartz S J and Mahmood F 2004 **11** 3819