Quantum interferometer with two-mode squeezed vacuum: $\hat{S}_z^2$ measurement*

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Quantum interferometric strategy with input two-mode squeezed vacuum [*Phys. Rev. Lett.* **104** 103602] is reexamined for both parity and $\hat{S}_z^2$ measurements. Unlike the previous scheme, we find that phase sensitivity obtained with the $\hat{S}_z^2$ measurement is minimized at phase origin, which may be useful to estimate a small phase shift at high precision. For the phase deviated from zero, the sensitivity increases more slowly than that of the parity detection.

Keywords: Mach–Zehnder interferometer, phase sensitivity

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1. Introduction

Quantum phase estimation with its precision beyond shot-noise limit (SNL) is of significant importance for quantum interferometry and quantum metrology. For standard Mach–Zehnder (or equivalently, Ramsey) interferometer, the achievable sensitivity depends on the input state[1–10] and the observable being measured at output ports.[11–16] With a coherent state as the input, the achievable phase uncertainty is limited by the SNL, i.e., $\Delta\varphi \geq 1/\sqrt{\bar{n}}$, where $\bar{n}$ is the average number of photons. The ultimate Heisenberg limit (HL) with $\Delta\varphi \propto 1/\bar{n}$ is attainable by using highly entangled states, such as the so-called twin-Fock states,[3] or the NOON states.[4,6–8]

Formally, the quantum interferometer can be described by multiple rotations to the input state:[2,17–21] $|\psi_{\rm out}\rangle = {\rm e}^{-{\rm i}(\pi/2)\hat{S}_x}{\rm e}^{{\rm i}\varphi\hat{S}_z}{\rm e}^{{\rm i}(\pi/2)\hat{S}_x}|\psi_{\rm in}\rangle = {\rm e}^{-{\rm i}\varphi\hat{S}_y}|\psi_{\rm in}\rangle$, where the three unitary transformations (from left to right) represent output 50:50 beam-splitter, linear phase shifter, and input beam-splitter, respectively. The angular momentum operators are defined as $\hat{S}_x = (\hat{a}^\dagger\hat{b} + \hat{b}^\dagger\hat{a})/2$, $\hat{S}_y = (\hat{a}^\dagger\hat{b} - \hat{b}^\dagger\hat{a})/(2{\rm i})$, and $\hat{S}_z = (\hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b})/2$, where $\hat{a}$ and $\hat{b}$ denote annihilation operators of two bosonic modes. Common eigenstates of $\hat{\boldsymbol{S}}^2$ and $\hat{S}_z$ are given by $|s,m\rangle \equiv |s+m, s-m\rangle_{a,b}$ with $m = -s, -s+1, \ldots, s$, representing $s+m$ and $s-m$ photons in the a and the b modes. If the interferometer is felt with the NOON or the twin-Fock state, a detection of the light-intensity difference ($\propto \hat{S}_z$) gives no information about the phase shift, due to the output signal $\langle\hat{S}_z\rangle_{\rm out} = 0$. Therefore, a phase-dependent observable other than $\hat{S}_z$ has been proposed, such as the parity $(-1)^{\hat{a}^\dagger\hat{a}}$[11,12,14,15] and the fluctuation of the population imbalance, i.e., the $\hat{S}_z^2$ measurement.[18,22–24]

Recently, Anisimov *et al.* proposed a lossless optical interferometer with two-mode squeezed vacuum (TMSV) and found it is possible to overpass the HL through the parity measurement.[15] However, this sub-Heisenberg limit of phase sensitivity occurs at $\varphi \sim \pi/2$. In this paper, we focus on the measurement scheme of $\hat{S}_z^2$ and show that the best sensitivity is achievable for phase shift $\varphi \sim 0$. Our work is mainly motivated by the following considerations. Firstly, the goal of interferometric quantum metrology is to estimate a small phase shift induced by certain weak interactions.[1] Secondly, the detection of $\hat{S}_z^2$ is now experimentally available.[24]

Our paper is organized as follows. In Section 2, we reexamine the lossless optical interferometer felt with the TMSV. For the parity measurement, we obtain the output signal and the phase sensitivity in a simple way other than that of Anisimov *et al.*[15] Then, in Section 3, we focus on the S_z^2 measurement and present analytical results of the best sensitivity, as well as the lowest bound of the phase sensitivity defined by quantum Fisher information. We finish by discussing the resultant sensitivity for the case that the input state deviates from the TMSV.

2. Optical interferometer with two-mode squeezed vacuum

Following Anisimov *et al.*,[15] we consider the Mach–Zehnder interferometer (MZI) felt with the two-mode squeezed vacuum

$$|\psi\rangle_{\rm in} = {\rm e}^{r(\hat{a}^\dagger\hat{b}^\dagger - \hat{a}\hat{b})}|0,0\rangle_{a,b} = \sum_{s=0}^{\infty}\sqrt{p_s(\bar{n})}\,|s,0\rangle\,, \qquad (1)$$

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which is indeed a superposition of the twin-Fock states $\{|s,0\rangle\}$, with the probability

$$p_s(\bar{n}) = \frac{2(\bar{n})^s}{(2+\bar{n})^{s+1}}, \tag{2}$$

and the average number of photons $\bar{n} = 2\sinh^2 r$. Via a detection of an observable $\hat{\mathcal{O}}$ at the output ports, the interferometric signal is given by $\langle\hat{\mathcal{O}}\rangle_{\rm out} = \langle\psi|_{\rm in} e^{i\varphi\hat{S}_y}\hat{\mathcal{O}}e^{-i\varphi\hat{S}_y}|\psi\rangle_{\rm in}$. The accumulated phase φ manifests itself as oscillations of $\langle\hat{\mathcal{O}}\rangle_{\rm out}$, with its sensitivity quantified by error-propagation formula

$$\Delta\varphi = \frac{(\Delta\hat{\mathcal{O}})_{\rm out}}{\left|{\rm d}\langle\hat{\mathcal{O}}\rangle_{\rm out}/{\rm d}\varphi\right|}, \tag{3}$$

which depends on the variance $(\Delta\hat{\mathcal{O}})_{\rm out} = [\langle\hat{\mathcal{O}}^2\rangle_{\rm out} - \langle\hat{\mathcal{O}}\rangle^2_{\rm out}]^{1/2}$ and the slope of the signal ${\rm d}\langle\hat{\mathcal{O}}\rangle_{\rm out}/{\rm d}\varphi$. Since the detection of $\hat{S}_z$ gives no information about the phase shift, one has to perform other measurement schemes in the MZI.

To obtain the phase-dependent signal, a detection of the parity on one of the two output modes has been proposed and extensively studied in the literature. In 1996, Bollinger *et al.* suggested that the parity measurement to the MZI injected with the NOON state could produce the Heisenberg-limited sensitivity.[11] Gerry *et al.* found a way to generate the NOON state by using a nonlinear beam-splitter.[12] With the parity measurement, they found it is also possible to achieve the HL. Recently, a higher precision of phase estimation has been obtained with this detection strategy for different input states, such as the two-mode squeezed vacuum[15] and the entangled coherent state.[25]

The well-known parity operator on the mode a can be defined as:[11,12,14,15]

$$\hat{\Pi} = (-1)^{\hat{a}^\dagger\hat{a}} = e^{i\pi\hat{S}_z}e^{i\pi\hat{N}/2}, \tag{4}$$

with photon number operator $\hat{N} = \hat{a}^\dagger\hat{a} + \hat{b}^\dagger\hat{b}$. Following Ref. [15], we now calculate the output signal with the TMSV. Using Eq. (1), we obtain

$$\langle\hat{\Pi}\rangle_{\rm out} = \sum_{s=0}^{\infty}\sum_{s'=0}^{\infty} e^{i\pi s}\sqrt{p_s(\bar{n})p_{s'}(\bar{n})}\,\langle s',0|\,e^{i\varphi\hat{S}_y}e^{i\pi\hat{S}_z}e^{-i\varphi\hat{S}_y}\,|s,0\rangle,$$

where we used $[\hat{N},\hat{S}_y] = 0$ and $\hat{N}|s,0\rangle = 2s|s,0\rangle$. In the above equation, the matrix element can be simplified as

$$\begin{aligned}
&\langle s',0|e^{i\varphi\hat{S}_y}e^{i\pi\hat{S}_z}e^{-i\varphi\hat{S}_y}|s,0\rangle \\
&= \langle s',0|e^{i\pi\hat{S}_z}(e^{-i\pi\hat{S}_z}e^{i\varphi\hat{S}_y}e^{i\pi\hat{S}_z})e^{-i\varphi\hat{S}_y}|s,0\rangle \\
&= \langle s',0|\,e^{-2i\varphi\hat{S}_y}|s,0\rangle = d^s_{0,0}(2\varphi)\delta_{s',s},
\end{aligned}$$

with the Wigner–D function defined as usual $d^s_{0,0}(2\varphi) = \mathscr{P}_s(\cos 2\varphi) = \langle s,0|e^{-i2\varphi\hat{S}_y}|s,0\rangle$, and $\mathscr{P}_s(\cos 2\varphi)$ denoting Legendre polynomial. Finally, we obtain the explicit form of the output signal

$$\langle\hat{\Pi}\rangle_{\rm out} = \frac{1}{\sqrt{1+\bar{n}(\bar{n}+2)\cos^2(\varphi)}}, \tag{5}$$

where we have used the identity

$$\sum_{s=0}^{\infty} a^s\mathscr{P}_s(x) = \frac{1}{\sqrt{1+a^2-2ax}}.$$

Due to $\hat{\Pi}^2 = 1$, the variance of the parity $(\Delta\hat{\Pi})_{\rm out} = \sqrt{1-\langle\hat{\Pi}\rangle^2_{\rm out}}$. With the parity measurement, one obtains the phase sensitivity

$$\Delta\varphi = \frac{1+\bar{n}(2+\bar{n})\cos^2\varphi}{|\sin\varphi|\sqrt{\bar{n}(2+\bar{n})}}. \tag{6}$$

Note that Anisimov *et al.* have obtained a similar result to Eq. (6) but they made a shift $\varphi\to\varphi-\pi/2$.[15] In Fig. 1, we depict the sensitivity $\Delta\varphi$ as a function of the accumulated phase shift φ with $\bar{n} = 5$ and $\bar{n} = 25$. One can easily see that the best sensitivity occurs at $\varphi = \pi/2$, with $\Delta\varphi_{\rm min} = 1/\sqrt{\bar{n}(2+\bar{n})}$. It is worthwhile to mention that the sensitivity diverges at $\varphi = 0$, which implies that the parity measurement fails to estimate a small phase shift induced by certain weak interactions. To overcome this problem, we turn to another measurement scheme—fluctuation of the population imbalance $\hat{S}_z^2$.[18,22–24]

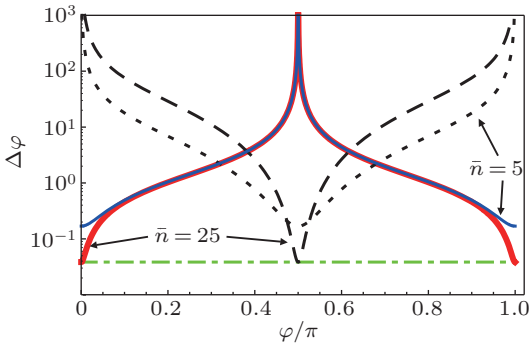


Fig. 1. (color online) Phase sensitivity as a function of phase shift for detecting the parity (dotted and dashed curves) and the $\hat{S}_z^2$ (solid curves) with average photon number $\bar{n} = 5$ and $\bar{n} = 25$. The green dash-dotted line indicates the best sensitivity $\Delta\varphi_{\rm min} = 1/\sqrt{\bar{n}(\bar{n}+2)}$ for $\bar{n} = 25$.

3. Phase sensitivity with S_z^2 measurement

The $\hat{S}_z^2$ measurement to the MZI with the twin-Fock state $|n/2,n/2\rangle_{a,b}$ was originally proposed by Kim *et al.*[18] They found that the sensitivity is minimum at $\varphi = 0$, with

$$\Delta\varphi_{\rm min} = \frac{\sqrt{2}}{\sqrt{n(n+2)}} \sim \frac{\sqrt{2}}{n}, \tag{7}$$

where n is the total photon number. Rodríguez *et al.*[22,23] pointed out that the above interferometric strategy can be realized with two-component Bose–Einstein condensate in an optical lattice. Recently, Lücke *et al.* prepared a superposition of the twin-Fock states by using spin dynamics of spin-1 Bose–Einstein condensate.[24] As an input of atomic interferometer, they found that it is capable of obtaining a better sensitivity beyond the SNL via detecting $\hat{S}_z^2$.

We now consider the TMSV as the input state of the MZI with the $\hat{S}_z^2$ measurement. Due to $\hat{S}_z|\psi\rangle_{\rm in}=0$, it is easy to obtain the output signal

$$\langle\hat{S}_z^2\rangle_{\rm out}=\langle\hat{S}_x^2\rangle_{\rm in}\sin^2\varphi, \tag{8}$$

where $\langle\hat{A}\rangle_{\rm in}=\langle\psi|_{\rm in}\hat{A}|\psi\rangle_{\rm in}$, denoting the expectation value of any observable $\hat{A}$ with respect to the input TMSV. In Eq. (8), we have used the relation $e^{i\varphi\hat{S}_y}\hat{S}_z^2e^{-i\varphi\hat{S}_y}=(\hat{S}_z\cos\varphi-\hat{S}_x\sin\varphi)^2$. In order to get the variance of $\hat{S}_z^2$, we further calculate the fourth moment of the population imbalance

$$\langle\hat{S}_z^4\rangle_{\rm out}=\langle\hat{S}_x^4\rangle_{\rm in}\sin^4\varphi+\langle\hat{S}_x\hat{S}_z^2\hat{S}_x\rangle_{\rm in}\sin^2\varphi\cos^2\varphi, \tag{9}$$

where we have used $\langle f(\hat{\boldsymbol{S}})\hat{S}_z\rangle_{\rm in}=\langle\hat{S}_z f(\hat{\boldsymbol{S}})\rangle_{\rm in}=0$, with $f(\hat{\boldsymbol{S}})$ being arbitrary function of the spin $\hat{\boldsymbol{S}}=(\hat{S}_x,\hat{S}_y,\hat{S}_z)$. In addition, we can prove $\langle\hat{S}_x\hat{S}_z\hat{S}_x^2\rangle_{\rm in}=0$ for the TMSV. Using Eqs. (8) and (9), we obtain the phase sensitivity with the $\hat{S}_z^2$ measurement

$$\Delta\varphi^2=\frac{\langle\hat{S}_x\hat{S}_z^2\hat{S}_x\rangle_{\rm in}+(\Delta\hat{S}_x^2)_{\rm in}^2\tan^2\varphi}{4\langle\hat{S}_x^2\rangle_{\rm in}^2}, \tag{10}$$

which keeps hold for the input twin-Fock state.[22,23] To get the scaling rule of $\Delta\varphi$, we have to calculate the expectation values with respect to the TMSV. For instance, we obtain

$$\begin{aligned}\langle\hat{S}_x^2\rangle_{\rm in}&=\sum_{s=0}^{\infty}\sum_{s'=0}^{\infty}\sqrt{p_s(\bar{n})p_{s'}(\bar{n})}\langle s',0|\hat{S}_x^2|s,0\rangle\\&=\sum_{s=0}^{\infty}p_s(\bar{n})\frac{s(s+1)}{2},\end{aligned}$$

where $p_s(\bar{n})$ is given by Eq. (2). Using the relations

$$\sum_{s=0}^{\infty}p_s(\bar{n})s=\frac{\bar{n}}{2},\quad\sum_{s=0}^{\infty}p_s(\bar{n})s^2=\frac{\bar{n}(\bar{n}+1)}{2}, \tag{11}$$

we obtain $\langle\hat{S}_x^2\rangle_{\rm in}=(2\bar{n}+\bar{n}^2)/4$. Since $\hat{S}_x\hat{S}_z^2\hat{S}_x=\hat{S}_x^2+{\rm i}2\hat{S}_x\hat{S}_y\hat{S}_z+\hat{S}_x\hat{S}_z^2$, we can prove

$$\langle\hat{S}_x\hat{S}_z^2\hat{S}_x\rangle_{\rm in}=\langle\hat{S}_x^2\rangle_{\rm in}=\frac{\bar{n}}{4}(\bar{n}+2). \tag{12}$$

Similarly, we calculate the fourth moment of $\hat{S}_x$

$$\begin{aligned}\langle\hat{S}_x^4\rangle_{\rm in}&=\sum_{s=0}^{\infty}p_s(\bar{n})\frac{s(s+1)(3s^2+3s-2)}{8}\\&=\frac{\bar{n}(\bar{n}+2)}{16}(9\bar{n}^2+18\bar{n}+4),\end{aligned} \tag{13}$$

where we have used Eq. (11), as well as

$$\begin{aligned}\sum_{s=0}^{\infty}p_s(\bar{n})s^3&=\frac{3\bar{n}(\bar{n}+1)^2-\bar{n}}{4},\\\sum_{s=0}^{\infty}p_s(\bar{n})s^4&=\frac{\bar{n}(\bar{n}+1)[3(\bar{n}+1)^2-2]}{2}.\end{aligned}$$

Finally, equation (10) can be simplified as

$$\Delta\varphi=\frac{1}{\sqrt{\bar{n}(\bar{n}+2)}}\sqrt{1+[2\bar{n}(\bar{n}+2)+1]\tan^2\varphi}, \tag{14}$$

which is minimized at the phase shift $\varphi=0$, with the best sensitivity

$$\Delta\varphi_{\rm min}=\frac{1}{\sqrt{\bar{n}(\bar{n}+2)}}. \tag{15}$$

Equation (14) is the main result of this work, which enables us to dig out advantages of the $\hat{S}_z^2$ measurement. From Fig. 1, one can find that $\Delta\varphi$ is minimized in the vicinity of the phase origin. It means that the $\hat{S}_z^2$ measurement is capable of estimating a small phase shift with a precision better than the HL. One can also note that the value of $\Delta\varphi$ increases rapidly as the phase shift φ deviates from zero. However, we will show that the increase rate is smaller than that of the parity measurement (see the inset of Fig. 2).

It is interesting to note that both the parity and the $\hat{S}_z^2$ detections lead to the same sensitivity, given by Eq. (15). Actually, independent of the measurement schemes, there exists a lower bound of phase uncertainty for the interferometer with the input TMSV, i.e., the quantum Cramer–Rao bound[26–29]

$$(\Delta\varphi)_{\rm QCR}=\frac{1}{\sqrt{F_Q[\hat{\rho}_{\rm in},\hat{S}_y]}}, \tag{16}$$

where $F_Q[\hat{\rho}_{\rm in},\hat{S}_y]$ denotes quantum Fisher information,[28,29] depending solely on the input state $\hat{\rho}_{\rm in}$. For a pure state, such as the TMSV, $F_Q[\hat{\rho}_{\rm in},\hat{S}_y]=4(\Delta\hat{S}_y)_{\rm in}^2=4\langle\hat{S}_y^2\rangle_{\rm in}$, where we have used $\langle\hat{S}_y\rangle_{\rm in}=0$. For the TMSV, we obtain $\langle\hat{S}_y^2\rangle_{\rm in}=(2\bar{n}+\bar{n}^2)/4$ and thereby the QCR $(\Delta\varphi)_{\rm QCR}=1/\sqrt{\bar{n}(\bar{n}+2)}$,[15] as shown by the empty circles in Fig. 2.

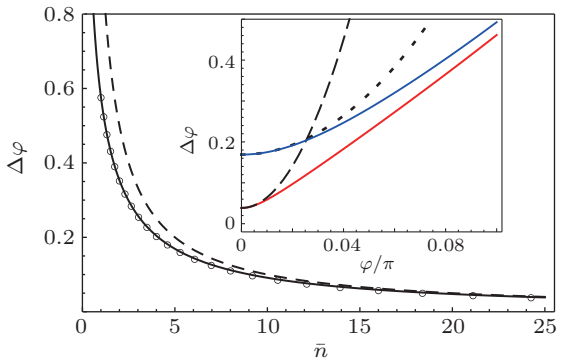


Fig. 2. (color online) The best sensitivity obtained by the $\hat{S}_z^2$ measurement against average photon number $\bar{n}$ (solid line). Empty circles are given by the Quantum Cramer–Rao bound. The dashed line is the Heisenberg limit, $1/\bar{n}$. Inset: Phase sensitivity as a function of the accumulated phase shift for measuring the parity (dotted and dashed lines) and the $\hat{S}_z^2$ (solid red and blue lines). The dotted and the solid blue lines correspond to $\bar{n}=5$; the dashed and the solid red lines are given for $\bar{n}=25$.

The best sensitivity is illustrated as a function of the average photon number in Fig. 2. Clearly, one can see that $\Delta\varphi_{\rm min}$ for the $\hat{S}_z^2$ measurement (the solid line) saturates the lower bound defined by the QCR (the empty circles). Indeed, both the parity and the $\hat{S}_z^2$ measurements give the same scaling

of $\Delta\varphi_{\min}$, which is better than the Heisenberg limit $1/\bar{n}$ (the dashed line). In the inset of Fig. 2, we compare the sensitivity obtained by the two measurements for $\bar{n}=5$ and $\bar{n}=25$. The solid lines are given by Eq. (14), corresponding to the case of the $\hat{S}_z^2$ measurement; while the dashed and the dotted lines are obtained from Eq. (6), i.e., for the parity measurement. To compare them, we have shifted φ in Eq. (6) by $\varphi-\pi/2$, as done in Ref. [15]. From the inset, one can easily find out the advantage of the $\hat{S}_z^2$ measurement. Specifically, the increase rate of $\Delta\varphi$ as $\varphi>0$ is smaller than that of the parity detection.

4. Discussion

So far, we have only considered the pure state as a probe of the Mach–Zender interferometer. We now discuss a mixed state of the twin-Fock state and the TMSV as follows:

$$\hat{\rho}=\sin^2(\theta)\,|\psi\rangle_{\rm in}\langle\psi|_{\rm in}+\cos^2(\theta)\left|\frac{n}{2},\frac{n}{2}\right\rangle_{\rm a,b}\left\langle\frac{n}{2},\frac{n}{2}\right|_{\rm a,b}, \quad (17)$$

where $|\psi\rangle_{\rm in}$ is the TMSV, given by Eq. (1). The average photon number now becomes $\langle\hat{N}\rangle=\bar{n}\sin^2(\theta)+n\cos^2(\theta)$. For brevity, we take $n=\bar{n}$ so $\langle\hat{N}\rangle=\bar{n}$. Based upon the $\hat{S}_z^2$ measurement, the output signal is given by

$$\langle\hat{S}_z^2\rangle_{\rm out}=\frac{1}{8}\bar{n}(\bar{n}+2)(1+\sin^2\theta)\sin^2\varphi. \quad (18)$$

We can also obtain the fourth moment of $\hat{S}_z$ but it is too cumbersome and will not be shown here. In Fig. 3, we plot the best sensitivity as a function of the mixed angle θ. When $\theta=\pi/2$, it scales as $\Delta\varphi_{\min}=1/\sqrt{\bar{n}(\bar{n}+2)}$, as predicted by Eq. (15). If θ deviates $\pi/2$, the resultant phase sensitivity becomes slightly worse. For the case $\theta=0$ or π, the twin-Fock state is the probe of the MZI, which gives $\Delta\varphi_{\min}\sim\sqrt{2}/\bar{n}$ [see Eq. (7)]. From the curves of Fig. 3 (and Fig. 2), one can find that the above two scalings are almost the same for large enough $\bar{n}$ ($\bar{n}>25$).

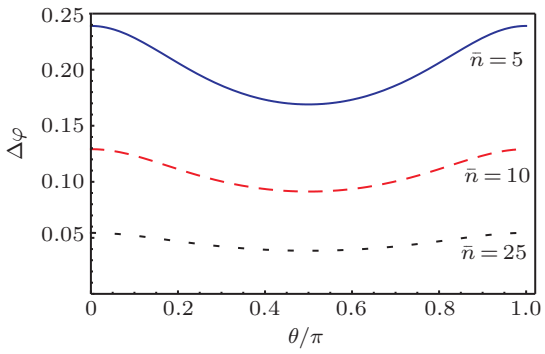


Fig. 3. (color online) The best sensitivity against θ for the mixed input state, Eq. (17), and the $\hat{S}_z^2$ measurement. From top to bottom, the three curves correspond to different average photon numbers, as indicated. For $\theta=\pi/2$, scaling of $\Delta\varphi_{\min}$ obeys Eq. (15); while for $\theta=0$ or π, it is given by Eq. (7).

5. Conclusion

In summary, we have presented analytical results of the phase sensitivity for a lossless Mach-Zehnder interferometer felt with the two-mode squeezed vacuum. In contrast to the previous parity measurement,[15] we show that the $\hat{S}_z^2$ measurement can estimate a small phase shift at high precision. More specifically, it is found that the sensitivity is minimized at $\varphi=0$. For the phase deviated from zero, the phase uncertainty obtained by the latter scheme increases more slowly in comparison with the previous one.

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