Symmetrical dynamical characteristic of peak and valley current-mode controlled single-phase H-bridge inverter*

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Complex dynamical phenomenon was studied in the single phase H-bridge inverter which was controlled by either a peak current or a valley current. The state functions and the discrete iterative map equations were established to analyze the dynamical phenomenon in the single phase H-bridge inverter. The dynamical characteristics of the single phase H-bridge inverter, such as time domain waveform diagram, bifurcation diagram, and folding map, were obtained by using the numerical calculation when the circuit parameters varied in specific range. Moreover, the simulation results were obtained by using the OrCAD-PSpice software to validate the numerical calculation. Both the numerical calculation and the circuit simulation show that the symmetrical dynamical phenomenon occurs in the single phase H-bridge inverter controlled by the peak current or the valley current.

Keywords: single-phase H-bridge inverter, symmetrical dynamical phenomenon, folding map, bifurcation diagram

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1. Introduction

Research on nonlinear phenomenon in a power electronic circuit first began in DC–DC converters which are typical strong nonlinear circuits.[1–10] The period bifurcation and the chaos phenomenon in buck converter were studied by Hamill in 1988[1] and the DC–DC buck converter exhibits a wide range of bifurcation and chaos phenomena even if small changes occur in the parameter of the device. The nonlinear phenomenon in other types of DC–DC converters was further studied, such as the second-order boost converter,[7] the higher order sepic converter,[8] and cuk converter.[11,12] Recently, the nonlinear research in power electronic circuits was extended to the single phase H-bridge inverter,[13–16] and its chaos phenomenon under the proportion control method was obtained by theoretical analysis, numerical simulation, and experimental validation.

At first, the state space average model is applied to the research on nonlinear phenomenon in switching converters. However, this method cannot be used to analyze the influence of the parameters on the converter working state when only one parameter changes. On this condition, it can only approximately obtain the stable region and the unstable region. Later, scholars mainly used "S" switching mapping, "A" switching mapping, and stroboscopic mapping to analyze the nonlinearity of the converter. In fact, from the view of dynamic analysis, these methods were discretization of a continuous system, which was called Poincare mapping. At present, this Poincare mapping principle is used to analyze the influence of the parameters in a switching converter on bifurcation and chaos.

The current mode controlled switch converter was applied widely in engineering, and its nonlinear phenomenon was also studied in depth. The control technology based on current mode mainly includes peak current control,[3,6,17,18] average current control,[19,20] and valley current control.[21–23] Up to now, only the control method based on peak current has been used to the single phase H-bridge inverter, the nonlinear phenomenon in the valley current control was not studied. In this paper, the single phase H-bridge inverter controlled by the peak current or the valley current is comparatively studied, and its symmetrical dynamical phenomenon is analyzed by bifurcation diagram, folding map, and time domain waveform diagram.

2. Peak/Valley current-mode control in single phase H-bridge inverter

2.1. Principle of single phase H-bridge inverter

A block diagram of the single phase H-bridge inverter controlled by the peak current mode control is shown in Fig. 1(a). Figure 1(b) shows the waveform diagram under steady operation (as every switch cycle is very short, the reference sinusoidal wave within a switch cycle is approximately a constant). This is a first-order circuit which consists of four

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switches (from VT1 to VT4), an inductor L, and a resistor R. The switches (from VT1 to VT4) are controlled by a feedback path comprising an RS trigger and a comparator. The inductor current is compared with a reference current $i_{\rm ref}$ to generate the driving signal for the switches from VT1 to VT4). There are two circuit configurations according to whether the VT1 and the VT3 are closed or open. At the beginning of each cycle, the inductor current rises linearly until $i = i_{\rm ref}$; during this period, any clock pulse arriving is ignored. When $i = i_{\rm ref}$, the VT1 and the VT3 are closed until the next clock pulse arrives. Then, they open again. The waveform of the current through the inductor is shown in Fig. 1(b). The single phase H-bridge inverter will be in chaotic state if this current is lower than $i_{\rm ref}$ until this period is over.

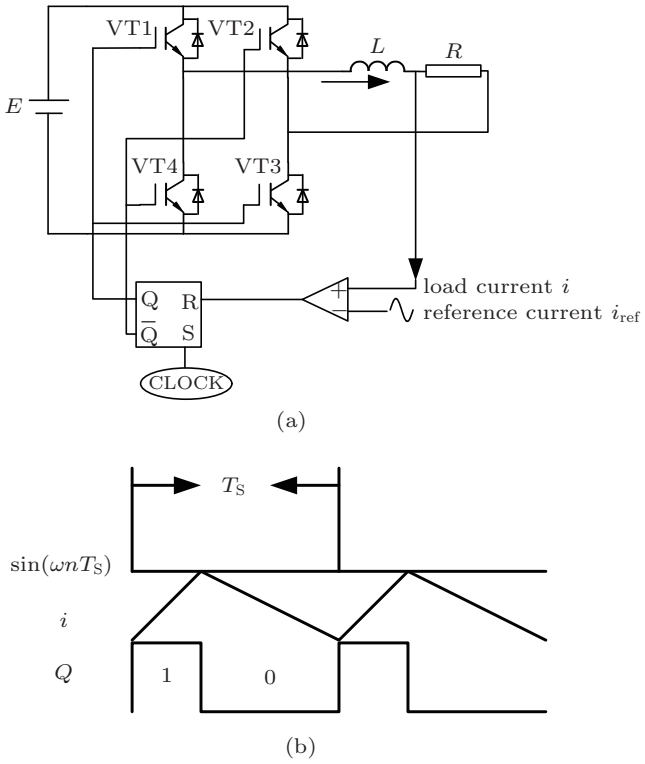


Fig. 1. The single phase H-bridge inverter controlled by the peak current control. (a) Block diagram and (b) waveform diagram under steady operation.

The block diagram of the single phase H-bridge inverter controlled by the valley current mode control is shown in Fig. 2(a), and the waveform diagram under steady operation is shown in Fig. 2(b). The circuit structure is approximately the same as that of Fig. 1(a), except that the inductor current descends linearly, as shown in Fig. 2 at the beginning of each cycle. The inductor current descends linearly until $i = i_{\rm ref}$. When $i = i_{\rm ref}$, the VT1 and the VT3 are opened until the next clock pulse arrives. Then, they are closed again. If the inductor current is higher than $i_{\rm ref}$ until this period is over, the single phase H-bridge inverter will also be in chaotic state.

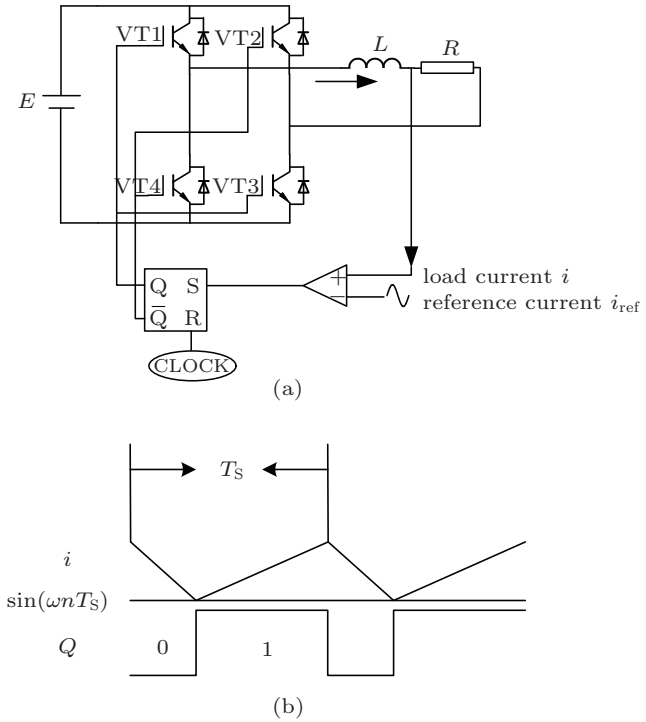


Fig. 2. The single phase H-bridge inverter controlled by the valley current control. (a) Block diagram and (b) waveform diagram under steady operation.

2.2. Discrete iterative map equation

The inductor current is sampled by applying the stroboscopic mapping principle, so a discrete iterative mathematical model of the single phase H-bridge inverter is obtained. The system can be described as the following state equations:

(i) State 1

$$\frac{{\rm d}i}{{\rm d}t} = -\frac{R}{L}i + \frac{E}{L}, \quad U = E. \tag{1}$$

(ii) State 2

$$\frac{{\rm d}i}{{\rm d}t} = -\frac{R}{L}i - \frac{E}{L}, \quad U = -E, \tag{2}$$

where i is the inductor current and U is the output voltage.

When the single phase H-bridge inverter works in the peak current control mode, the inductor current linearly rises at the beginning of each period as described by Eq. (1); when the inductor current is higher than the reference current during the switch cycle, the trigger is reset, and the inductor current linearly declines as described by Eq. (2). Therefore, the discrete iterative model of the single phase inverter controlled by the peak current is

$$i_{n+1} = (i_{\rm ref} + \alpha)\exp[-\beta \times (1 - D)] - \alpha, \tag{3}$$

where $\alpha = E/R$, $\beta = (RT_{\rm S})/L$, $D = -\{\log[(\alpha - i_{\rm ref})/(\alpha - i_n)]\}/\beta$, i_n is the inductor current in initial state, i_{n+1} is the inductor current in final state during the switching period T_s.

When the single phase H-bridge inverter works in the valley current control mode, the inductor current linearly declines as described by Eq. (2); when the inductor current is smaller than the reference current in switching period, the inductor

current linearly rises as described by Eq. (1). Therefore, the discrete iterative model of the single phase inverter controlled by the valley current is

$$i_{n+1} = (i_{\text{ref}} - \alpha)\exp[-\beta \times (1-D)] + \alpha, \tag{4}$$

where $\alpha = E/R$, $\beta = (RT_{\text{S}})/L$, $D = -\{\log[(\alpha + i_{\text{ref}})/(\alpha + i_n)]\}/\beta$.

3. Analysis of symmetrical dynamical phenomenon

Based on the iterative equation proposed in Section 2, the nonlinear characteristic in the single phase H-bridge inverter can be analyzed by using the numerical method. Methods that can be utilized to visualize the nonlinear phenomenon are time domain waveforms of state variable, folding maps, and bifurcation diagrams.

3.1. Time domain waveform of inductor current

Parameters of the single phase H-bridge inverter are fixed at the following values: $E = 100$ V, $R = 10\ \Omega$, $L = 10$ mH, $T_{\text{S}} = 100\ \mu$s, $i_{\text{ref}} = k\sin(40\pi t)$. The inductor currents of the single phase H-bridge inverter controlled by the peak current and the valley current are shown in Figs. 3 and 4.

By comparing Fig. 3 with Fig. 4, the similarity of the peak current control and the valley current control is that when the proportion coefficient k increases, the inductor current changes from the stable state towards the nonlinear state gradually. The difference between the peak current control and the valley current control is that when proportion coefficients k are the same, the position where the inductor current in chaotic state is different, the inductor current of the peak current control appears chaotic in the positive half period, and the inductor current of the valley current control appears chaotic in the negative half period. Both of them show the symmetrical dynamical characteristic.

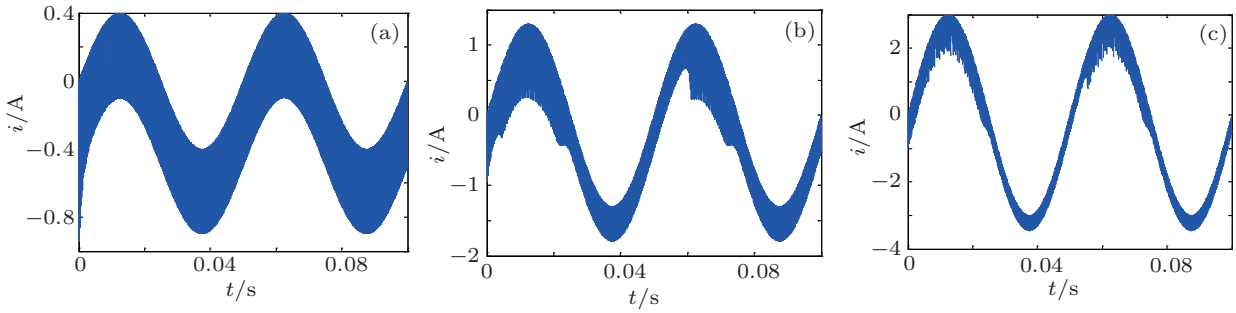


Fig. 3. (color online) Inductor currents of the single phase inverter controlled by the peak current with different amplitude coefficients: (a) $k = 0.4$; (b) $k = 1.3$; (c) $k = 3$.

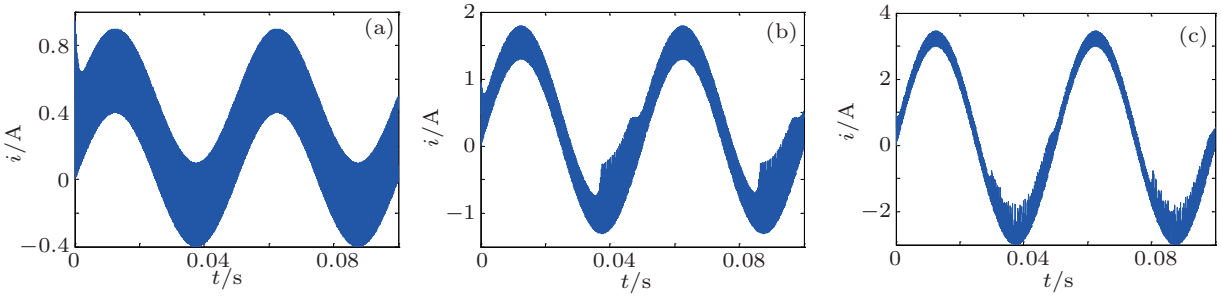


Fig. 4. (color online) Inductor currents of the single phase inverter controlled by the valley current with different amplitude coefficients: (a) $k = 0.4$; (b) $k = 1.3$; (c) $k = 3$.

3.2. Folding map analysis

According to the folding map, the phenomena of bifurcation and chaos can be clearly judged. The theory of the folding map is as follows:

Step 1: substitute arbitrary initial value into Eqs. (3) and (4);

Step 2: after fifty times of the iterative process, the first twenty that are unstable are omitted, then, the remaining ones are aligned by the time in which the samples are taken;

Step 3: the folding map Figs. 5 and 6 are obtained.

Comparing Fig. 5 with Fig. 6, we can see that the chaotic region is located respectively in the positive half cycle and the negative half cycle of the sinusoidal wave, which is in consonance with the symmetry that the time domain diagram has illustrated.

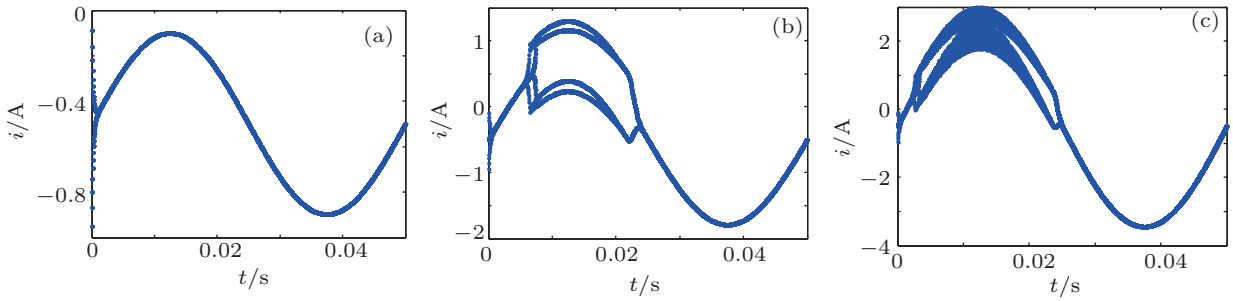


Fig. 5. (color online) Folding maps of peak current control with different reference current amplitudes: (a) $k = 0.4$; (b) $k = 1.3$; (c) $k = 3$.

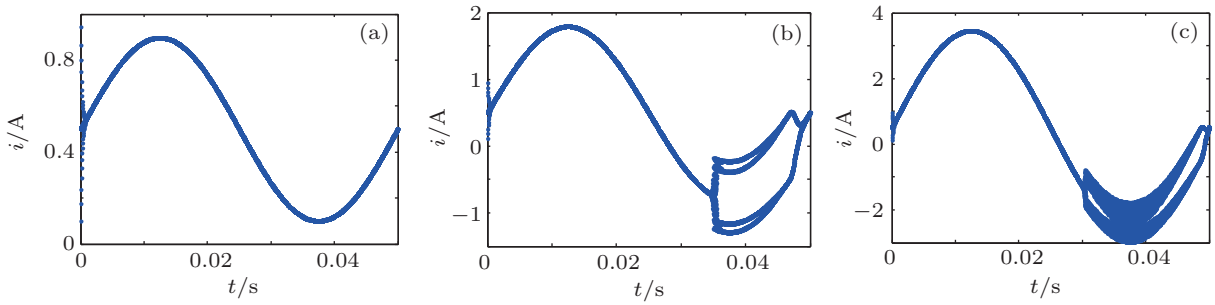


Fig. 6. (color online) Folding maps of valley current control with different reference current amplitudes (a) $k = 0.4$; (b) $k = 1.3$; (c) $k = 3$.

3.3. Bifurcation diagram analysis

Among characteristic quantity methods, the bifurcation diagram is the most powerful tool to investigate the nonlinear phenomenon. In the bifurcation diagram, a periodic steady state of the system is represented as a signal point or several points that equal to the periodicity of the system for a fixed parameter, the chaotic state of the system is represented as numerous points because chaos means period infinity and does not fall at the same position. Therefore, in such a bifurcation diagram, the change of behavior of a system is clearly shown when a parameter is varied.

From the theoretical point of view, any circuit parameters can work as the bifurcation parameters. In this section, discrete iterative map Eqs. (3) and (4) of the single phase H-bridge inverter controlled by the peak current or the valley current are used. The accurate analysis is performed by simulation in Matlab for symmetrical dynamical phenomena of the single phase H-bridge inverter. We select below the resistor R, the power supply E, the inductance L, and the reference current amplitude coefficient k as the bifurcation parameters. The bifurcation diagrams of the single phase H-bridge inverter are shown in Figs. 7–10.

In the single phase H-bridge inverter, parameter A (A represents k, R, E, and T_S respectively) is taken as the bifurcation parameter. The parameter A is varied from smaller values to bigger values at a step of 0.01, while other parameters are kept the same as that illustrated in Section 3. Among them, the coefficient k changes from 0.01 to 3.5, the power supply changes from 5 V to 150 V, the resistor R changes from 0.01 Ω to 100 Ω and the period T_S changes from 1 ms to 10 ms.

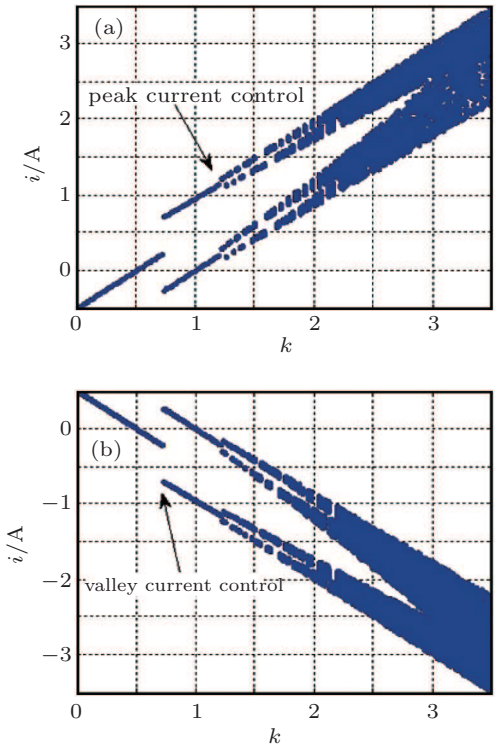


Fig. 7. (color online) Bifurcation diagrams using k as the bifurcation parameter for (a) peak current control and (b) valley current control.

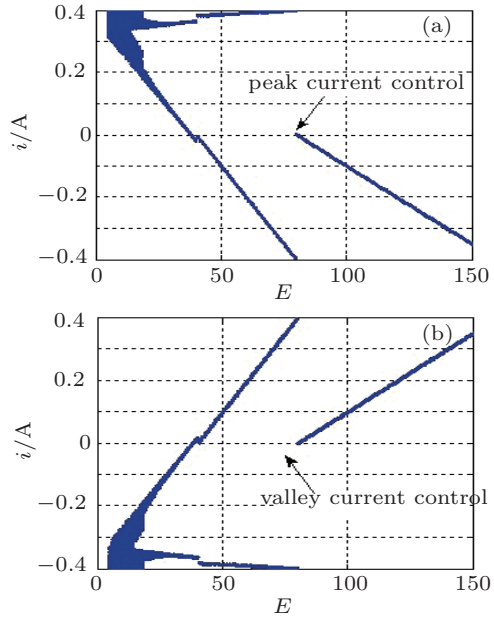


Fig. 8. (color online) Bifurcation diagrams using E as the bifurcation parameter for (a) peak current control and (b) valley current control.

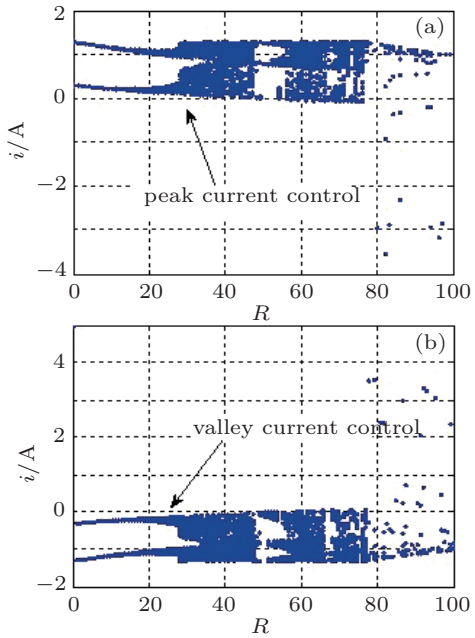


Fig. 9. (color online) Bifurcation diagrams using R as the bifurcation parameter for (a) peak current control and (b) valley current control.

As shown in Figs. 7–10, the single phase H-bridge inverter goes through period-1, period-2 and eventually exhibits chaos as the parameters are varied. When the k, E, T_S are bifurcation parameters, the inductor current shows border collision bifurcation phenomenon. Among the above figures, it can be interestingly observed that the inductor current shows the symmetrical dynamical characteristic on the horizontal axis.

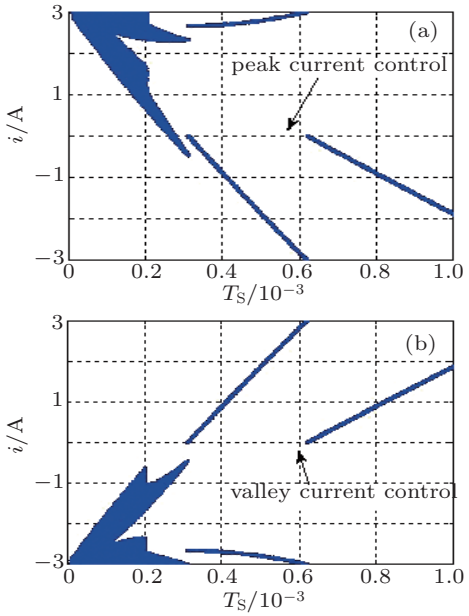


Fig. 10. (color online) Bifurcation diagrams using switch period T_S as the bifurcation parameter for (a) peak current control and (b) valley current control.

3.4. PSPICE circuit simulation verification of the system

The circuit simulation experiment of the single phase inverter is performed by using OrCAD-PSpice software. The circuit is accomplished by using MOSFET IRF150, comparators, resistances, inductance, and RS flip flop. The circuit simulation experiment of the system suggests that the result of the circuit simulation experiment is the same as the numerical simulation. Carefully considering the linear working range of these devices, the result of the circuit simulation experiment is the same as the real circuit. Therefore, this circuit simulation experiment can reflect the real circuit experiment of the single phase H-bridge inverter. The results are shown in Figs. 11 and 12.

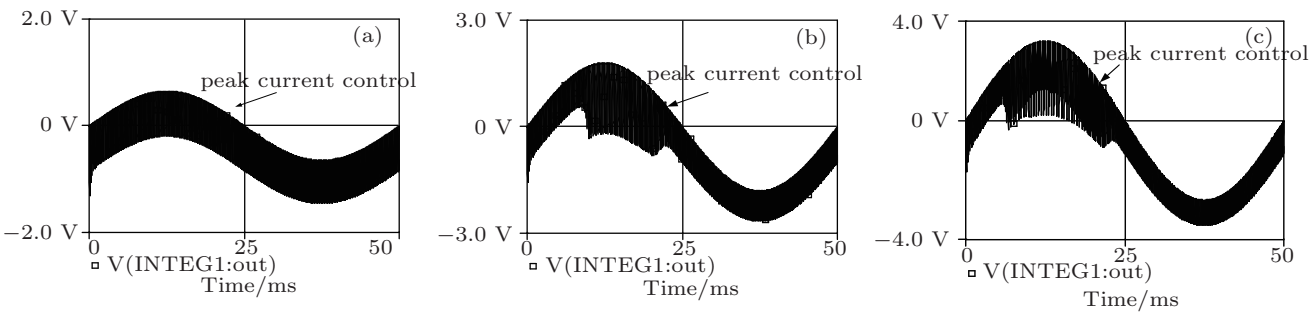


Fig. 11. Inductor current simulation experiment waveform of single phase inverter controlled by peak current with different amplitude coefficient k: (a) $k = 0.4$; (b) $k = 1.3$; (c) $k = 3$.

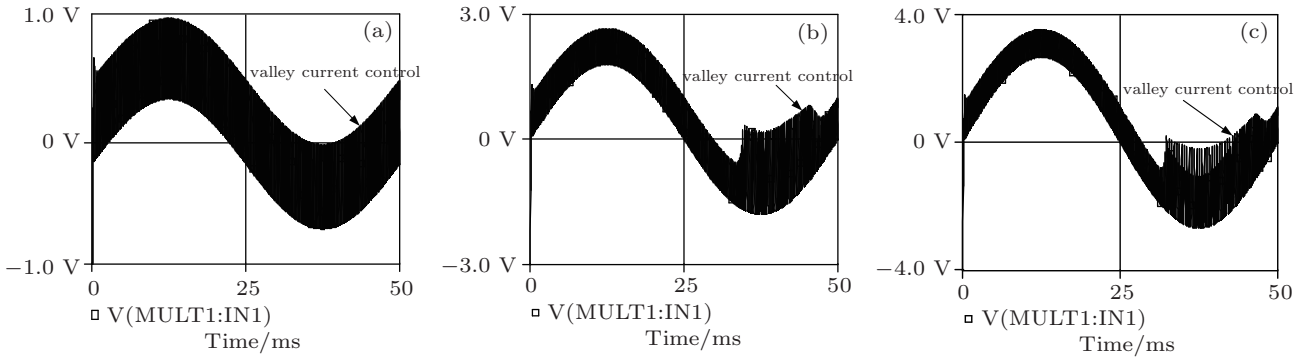


Fig. 12. Inductor current simulation experiment waveform of single phase inverter controlled by valley current with different amplitude coefficient k: (a) $k = 0.4$; (b) $k = 1.3$; (c) $k = 3$.

4. Conclusion

In this paper, the peak current control and the valley current control are studied comparatively in the single phase H-bridge inverter. The nonlinear phenomena such as bifurcation and chaos have been illustrated. The results show that they have the symmetrical dynamical characteristic on the horizontal axis (x axis). To study these phenomena, state equations of the single phase H-bridge inverter controlled by the peak current or the valley current are established, iterative map equation is deduced by using the stroboscopic map principle. The corresponding simulation results verified the symmetry. Time domain simulation results based on Matlab/Simulink and OrCAD-Pspice verify numerical simulation results, and it also reveals that the single phase H-bridge inverter controlled by either the peak current or the valley current shows symmetrical dynamical phenomena. It is of significance for designing a single phase H-bridge inverter.

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