TOPICAL REVIEW — Quantum information

Quantum cloning machines and their implementation in physical systems*

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(Received 12 September 2013; revised manuscript received 8 October 2013)

We review the basic theory of approximate quantum cloning for discrete variables and some schemes for implementing quantum cloning machines. Several types of approximate quantum clones and their expansive quantum clones are introduced. As for the implementation of quantum cloning machines, we review some design methods and recent experimental results.

Keywords: quantum cloning machine, discrete-variable, expansive quantum clone, implementation

PACS: 03.67.–a, 03.65.–w **DOI:** 10.1088/1674-1056/22/11/110311

1. Introduction

In recent years, quantum information processing has been attracting much interest, not only because it offers new methods of computation and communication, but also because quantum information differs from classical information in several fundamental ways. If information is encoded in classical forms, it can be replicated perfectly, i.e., a process called cloning. However, when information is encoded in quantum states, perfect cloning is impossible. Either the copies are imperfect or perfect, sometimes the copying process simply gives no outcome. The no-cloning theorem,[1] one of the most fundamental differences between classical and quantum information theory, guarantees absolute security of quantum cryptography.[2] Due to the no-cloning theorem, quantum information provides a means to perform some tasks that would be impossible using only ordinary information, such as detecting any eavesdropper on a communication channel. This tells us that it is impossible to accurately copy an arbitrary quantum state. Nevertheless, it is still an open question how well one can copy quantum states, namely, when ideal copies are not available, how close can the copy state be to the original state? The concepts and tools of cloning have proved to be useful for the foundations of quantum mechanics such as state estimation, cryptography, etc. Now one can obtain approximate copies in the deterministic cloning process[3–6] or perfect ones in the probabilistic cloning process.[7,8] In 1996, Bužek and Hillery[2] first investigated an optimal $1 \to 2$ universal quantum cloning machine (UQCM) in two dimensions. Since then, quantum cloning has attracted a vast amount of research and has become a particularly interesting field within quantum information science,[9] and the UQCM idea has been further studied.[10–14] Other quantum cloners have been conceived, such as phase-covariant cloning machine (PCCM),[15–20] economical phase-covariant cloning (EPCC),[21–28] the real state cloning machine (RSCM),[29] and the state-dependent quantum cloning machine,[6] entanglement state cloning,[30,31] and orthogonal qubit cloning.[32–34] Furthermore, according to the characteristics of the quantum clones, the approximate quantum cloning machines can be divided into two types: symmetric and asymmetric. As is well known, scalability is very important for quantum computation and quantum communications. Likewise, scalability is also a key characteristic of quantum cloning methods. The number of quantum copies and the dimension of Hilbert space are the two aspects of the scalability of quantum cloning. Some kinds of quantum cloning machines for discrete variables have been implemented in experiment.[35–42] Continuous-variable cloning has been investigated both theoretically[43–48] and experimentally.[49–52]

In the present paper, we review the basic theory of approximate quantum cloning for discrete variables and some schemes for implementing quantum cloning machines. It is organized as follows. In Section 2, we review the basic theory of the quantum cloning. We discuss different types of quantum cloning and their corresponding scalable form. In Section 3, we compare and discuss different design methods for implementing quantum cloning machines. In Section 4, we discuss the experimental progress. We give a wrap-up in Section 5.

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11074002, 61275119, and 11247009) and the Doctoral Foundation of the Ministry of Education of China (Grant No. 20103401110003).

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2. Basic theory of quantum cloning

The no-cloning theorem prohibits a perfect copy of an arbitrary quantum state. Thus, imperfect quantum cloning was proposed by researchers. One type of quantum cloning machine, called "approximate quantum cloning" or "deterministic quantum cloning", can provide imperfect quantum clones with a determinate probability of 1. The other type, called probabilistic cloning machines, can provide perfect quantum clones with a certain probability, and sometimes it cannot produce any quantum clones.

In this paper, we consider that the quantum information is encoded in the input quantum state $|\psi\rangle_1$. For performing quantum cloning, a blank state is needed and an ancillary system is also needed in some cases. A unitary transformation is applied on the whole system. The output systemic state is denoted as $|\Psi\rangle$. Take the partial trace over the state $|\Psi\rangle$ to obtain the reduced density operator of the No. j clone, denoted as $\rho_j = \mathrm{Tr}|\Psi\rangle\langle\Psi|$. Then, the fidelity of the No. j clone can be calculated by $F_j = {}_1\langle\psi|\rho_j|\psi\rangle_1$, which specifies the quality of the approximate quantum cloning machine.

2.1. Universal quantum cloning machine

The input quantum state $|\psi\rangle_1$ is often expressed as

$$|\psi\rangle_1 = \cos\frac{\theta}{2}|0\rangle_1 + \sin\frac{\theta}{2}\mathrm{e}^{\mathrm{i}\phi}|1\rangle_1, \tag{1}$$

where the angle $\theta \in [0,\pi]$ denotes the amplitude parameter, and the angle $\varphi \in [0,2\pi)$ denotes the phase parameter. If the input quantum state carrying the quantum information is completely unknown, in other words, the angles θ and ϕ are any unknown values, this kind of quantum cloning can be grouped into several categories according to different coding modes and different purposes. Next, we discuss it in detail.

2.1.1. The optimal $1 \to 2$ symmetric (asymmetric) UQCM in 2-dimensional Hilbert space

The basic type of the optimal $1 \to 2$ symmetric UQCM (SUQCM) was proposed by Bužek and Hillery. The explicit unitary transformation of the optimal $1 \to 2$ SUQCM in two dimensions is defined as[2]

$$\begin{aligned}
|0\rangle_1|0\rangle_2|0\rangle_a &\to \sqrt{2/3}|0\rangle_1|0\rangle_2|0\rangle_a \\
&\quad + \sqrt{1/6}(|01\rangle+|10\rangle)_{12}|1\rangle_a, \\
|1\rangle_1|0\rangle_2|0\rangle_a &\to \sqrt{2/3}|1\rangle_1|1\rangle_2|1\rangle_a \\
&\quad + \sqrt{1/6}(|10\rangle+|01\rangle)_{12}|0\rangle_a,
\end{aligned} \tag{2}$$

where the state $|0\rangle_2$ is the blank state, much like the blank paper in classical information theory, and $|0\rangle_a$ is the ancillary system. For all transformations acting on the initial input state $|\psi\rangle_1|0\rangle_2|0\rangle_a$, the first qubit and the second qubit of Eq. (2) are the two quantum clones. The quality of the quantum clone is described using fidelity, which is defined as

$$F_{1\to2,j}^{\mathrm{SUQC},2} = {}_1\langle\psi|\rho_j|\psi\rangle_1, \tag{3}$$

where SUQC denotes the type of the quantum cloning machine, the number 2 in the superscript denotes the number of dimensions, $1 \to 2$ denotes that one input state is copied into two quantum states, j denotes the No. j quantum clone, and ρ_j denotes the reduced density operator of the No. j clone. Here, ρ_j is given by

$$\begin{aligned}
\rho_j = &\left(\frac{4}{6}\cos^2\frac{\theta}{2}+\frac{1}{6}\right)|0\rangle_j\langle 0| + \left(\frac{1}{6}+\frac{4}{6}\sin^2\frac{\theta}{2}\right)|1\rangle_j\langle 1| \\
&+\frac{1}{3}\sin\theta(\mathrm{e}^{\mathrm{i}\phi}|1\rangle_j\langle 0| + \mathrm{e}^{-\mathrm{i}\phi}|0\rangle_j\langle 1|).
\end{aligned} \tag{4}$$

The fidelities of the two clones are $F_1 = F_2 = 5/6 \approx 0.833$. The fidelities of two clones are equal, so the quantum cloning machine is symmetric. It has been proven that the fidelities of the two clones are maximal and independent of the input state $|\psi\rangle_1$,[3,5] so the UQCM is optimal.

People's needs for copy fidelity vary, so the optimal $1 \to 2$ asymmetric universal quantum cloning machine (AUQCM) is suitable for various situations. Therefore, the unitary transformation of the optimal $1 \to 2$ AUQCM in two dimensions can be expressed as[19]

$$\begin{aligned}
|0\rangle_1|00\rangle_{2,x} &\to (|000\rangle + p|011\rangle + q|101\rangle)_{1,2,x}/N, \\
|1\rangle_1|00\rangle_{2,x} &\to (|111\rangle + p|100\rangle + q|010\rangle)_{1,2,x}/N.
\end{aligned} \tag{5}$$

Here, the cloning parameters $p,q \geq 0$ satisfy the condition $p+q=1$, and the normalization factor is $N^2 = 1+p^2+q^2$. After the application of the unitary transformation, the fidelities $F_{1\to2,1}^{\mathrm{AUQC},2} = (1+p^2)/N^2$ and $F_{1\to2,1}^{\mathrm{AUQC},2} = (1+q^2)/N^2$ are not equal. Thus, it is asymmetric.

2.1.2. Generalize the UQCM to multiple copies

To provide more quantum clones, the SUQCM can be generalized into the multiple copies. Gisin and Massar have discovered a unitary transformation $U_{1,M}$[3] that can provide optimal multiple quantum clones in two dimensions, and the fidelity of all the quantum clones is equal. In the optimal $1 \to M$ SUQCM in two dimensions, the corresponding unitary transformation $U_{1\to M}$ can be described by

$$\begin{aligned}
&U_{1,M}|0\rangle\otimes R = \sum_{j=0}^{M-1}\alpha_j|(M-j)\,0, j1\rangle\otimes R_j, \\
&U_{1,M}|1\rangle\otimes R \\
&= \sum_{j=0}^{M-1}\alpha_{M-1-j}|(M-1-j)\,0, (j+1)\,1\rangle\otimes R_j.
\end{aligned} \tag{6}$$

Here, R is the initial state of the copy machine, and the $M-1$ blank copies R_j are orthogonal normalized internal states of the SUQCM. When the input state is an arbitrary state $|\psi\rangle$,

R_j can be denoted as $\left|(M-1-j)\,\psi, j\psi^{\perp}\right\rangle$. In Eq. (6), the cloning coefficient α_j is given by

$$\alpha_j = \sqrt{\frac{2\,(M-j)}{M\,(M+1)}}. \tag{7}$$

Note that $|(M-j)\,0, j1\rangle$ denotes an orthogonal normalized state. For example, for $M=3$ and $j=1$, the orthogonal normalized state is given by

$$|21,10\rangle = (|011\rangle + |101\rangle + |110\rangle)/\sqrt{3}. \tag{8}$$

The fidelity of each clone is

$$F_{1\to M}^{\mathrm{SUQC},2} = \frac{2M+1}{3M}. \tag{9}$$

Obviously, if $M=2$, the unitary transformation $U_{1,M}$ reduces to the unitary transformation of Eq. (2).

Consider the more general UQCM that takes N identical input qubits into M identical copies. Here, the arbitrary input state of each qubit is denoted as $|\psi\rangle$, so the N identical input qubits are $|N\psi\rangle$, the unitary transformation $U_{N\to M}$ of the optimal $N\to M$ SUQCM in two dimensions is given by[3]

$$U_{N,M}\,|N\psi\rangle \otimes R = \sum_{j=0}^{M-N} \alpha_j \left|(M-j)\,\psi, j\psi^{\perp}\right\rangle \otimes R_j\,(\psi)\,. \tag{10}$$

Similarly, R is the initial state of the $N\to M$ SUQCM system in two dimensions, including the $M-N$ blank copies. The R_j is the auxiliary state, which is orthogonally normalized and can be expressed as $R_j\,(\psi) = \left|(M-1-j)\,\psi, j\psi^{\perp}\right\rangle$, and the cloning coefficient α_j is given by

$$\alpha_j = \sqrt{\frac{N+1}{M+1}}\sqrt{\frac{(M-N)!\,(M-j)!}{(M-N-j)!M!}}. \tag{11}$$

The corresponding fidelity of each clone is

$$F_{N\to M}^{\mathrm{SUQC},2} = \frac{M\,(N+1)+N}{M\,(N+2)}. \tag{12}$$

2.1.3. Extend the UQCM into multi-dimensional case

Consider the quantum state to be encoded in a d-dimensional quantum system: the input state is $|\psi\rangle_{\mathrm{in}} = \sum_{i=0}^{d-1} a_i \mathrm{e}^{\mathrm{i}\phi_i}\,|i\rangle$, where $a_i \in [0,1]$ satisfying $\sum_{i=0}^{d-1} a_i^2 = 1$ are the amplitude parameters of the state of the system, $\varphi_i \in [0, 2\pi)$ are the phase parameters, and a_i and φ_i are completely unknown in the UQCM. The unitary transformation of an optimal $1\to 2$ SUQCM in d dimensions is given by[10]

$$|i\rangle_1\,|0\rangle_2\,|0\rangle_{\mathrm{a}} \to \alpha\,|i\rangle_1\,|i\rangle_2\,|i\rangle_{\mathrm{a}} + \beta \sum_{\substack{j=0\\ i\neq j}}^{d-1} (|ij\rangle + |ji\rangle)_{12}\,|j\rangle_{\mathrm{a}}, \tag{13}$$

where $i,j = 0,1,\ldots,d-1$, $\alpha^2 = 2/(d+1)$ and $\beta^2 = 1/2(d+1)$ are the cloning coefficients. The fidelity of each clone is

$$F_{1\to 2}^{\mathrm{SUQC},d} = \frac{d+3}{2\,(d+1)}. \tag{14}$$

Consider the more general case of UQCM, that is, the optimal $N\to M$ SUQCM in d dimensions. The input state is

$$|\psi\rangle^{\otimes N} = \sum_{n=0}^{N} \sqrt{\frac{N!}{n_1!\cdots n_d!}}\, x_o^{n_1}\cdots x_{d-1}^{n_d}\,|n\rangle\,,$$

where $|n\rangle = |n_1\cdots n_d\rangle$ is a symmetric state with n_i states in state $|i\rangle$. The unitary transformation of an optimal $N\to M$ SUQCM in d dimensions is given by[53]

$$|n\rangle \otimes R \to \sum_{j=0}^{M-N} \beta_{nj}\,|n+j\rangle \otimes R_j, \tag{15}$$

where R is the initial state of the $N\to M$ SUQCM system in d dimensions, including $M-N$ blank copies, and R_j is the state of the auxiliary system, which is orthogonally normalized. The coefficient is $\beta_{nj} = \sqrt{\prod_{i=1}^{d} C_{n_i+j_i}^{j_i}/C_{M+d-1}^{M-N}}$.

Werner[4] proved that the fidelity is

$$F_{N\to M}^{\mathrm{SUQC},d} = \frac{M-N+N\,(M+d)}{M\,(N+d)}. \tag{16}$$

In a d-dimensional quantum system, for the asymmetric case, the unitary transformation of the optimal $1\to 2$ AUQCM in d dimensions is[11,12]

$$|i\rangle_1\,|0\rangle_2\,|0\rangle_{\mathrm{a}} \to \alpha\,|i\rangle_1\,\left|\Phi^+\right\rangle_{2a} + \beta\,|i\rangle_2\,\left|\Phi^+\right\rangle_{1\mathrm{a}}, \tag{17}$$

where the cloning parameters α and β satisfy the condition $\alpha^2+\beta^2+2\alpha\beta/d = 1$, and $|\Phi^+\rangle = \sqrt{d}\sum_{i=0}^{d-1}|ii\rangle$ is the Bell state of the d-dimensional quantum system. The two fidelities are $F_{1\to 2,1}^{\mathrm{AUQC},d} = 1-\frac{d-1}{d}\beta^2$ and $F_{1\to 2,2}^{\mathrm{AUQC},d} = 1-\frac{d-1}{d}\alpha^2$. Obviously, if $d=2$, the unitary transformation of Eq. (17) will be reduced to Eq. (5).

So far, researchers have only found the unitary transformation of the optimal $1\to 3$ AUQCM in d dimensions, which is[54]

$$\begin{aligned} &|i\rangle_1\,|00\rangle_{2345} \\ \to\ & P[\alpha\,|i\rangle_1\,(\left|\Phi^+\right\rangle_{24}\otimes\left|\Phi^+\right\rangle_{35} + \left|\Phi^+\right\rangle_{25}\otimes\left|\Phi^+\right\rangle_{34}) \\ &+\beta\,|i\rangle_2\,(\left|\Phi^+\right\rangle_{14}\otimes\left|\Phi^+\right\rangle_{35} + \left|\Phi^+\right\rangle_{15}\otimes\left|\Phi^+\right\rangle_{34}) \\ &+\gamma|i\rangle_3\,(\left|\Phi^+\right\rangle_{14}\otimes\left|\Phi^+\right\rangle_{25} + \left|\Phi^+\right\rangle_{15}\otimes\left|\Phi^+\right\rangle_{24})], \end{aligned} \tag{18}$$

where the cloning parameters $\alpha, \beta, \gamma \geq 0$ satisfy

$$\alpha^2+\beta^2+\gamma^2+\frac{2(\alpha\beta+\beta\gamma+\gamma\alpha)}{d} = 1,$$

and $P = \sqrt{d/2(d-1)}$ is the normalization constant. The three fidelities are

$$\begin{aligned} F_{1\to 3,1}^{\mathrm{AUQC},d} &= 1-\frac{d-1}{d}\left(\beta^2+\gamma^2+\frac{2\beta\gamma}{d+1}\right), \\ F_{1\to 3,2}^{\mathrm{AUQC},d} &= 1-\frac{d-1}{d}\left(\alpha^2+\gamma^2+\frac{2\alpha\gamma}{d+1}\right), \\ F_{1\to 3,3}^{\mathrm{AUQC},d} &= 1-\frac{d-1}{d}\left(\alpha^2+\beta^2+\frac{2\alpha\beta}{d+1}\right). \end{aligned} \tag{19}$$

2.2. Optimal phase-covariant quantum cloning machine

If the partial information of the input state is known, one can design a type of quantum cloning with better quality than the UQCM. For example, if the input state is $|\psi\rangle_1 = \cos\theta/2\,|0\rangle + \mathrm{e}^{\mathrm{i}\varphi}\sin\theta/2\,|1\rangle$, here $\theta \in [0,\pi)$ is certain and known, but $\varphi \in [0,2\pi)$ is unknown. In other words, the amplitude information of the input state is known, but the phase information is completely unknown. This type of restrictive quantum cloning is referred to as phase-covariant quantum cloning machine (PCCM).

2.2.1. The optimal $1 \to 2$ symmetric (asymmetric) PCCM in 2-dimensional Hilbert space

Karimipour discovered the unitary transformation of the optimal $1 \to 2$ symmetric PCCM (SPCCM) in two dimensions, which can be expressed as[55]

$$\begin{aligned}
&|0\rangle_1 \otimes |0\rangle_2 |0\rangle_{\mathrm{a}} \\
\to\ & A(\theta)|0\rangle_1|0\rangle_2|0\rangle_{\mathrm{a}} + B(\theta)(|0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2)|1\rangle_{\mathrm{a}}] \\
&|1\rangle_1 \otimes |0\rangle_2 |0\rangle_{\mathrm{a}} \\
\to\ & A(\theta)|1\rangle_1|1\rangle_2|1\rangle_{\mathrm{a}} + B(\theta)(|1\rangle_1|0\rangle_2 + |0\rangle_1|1\rangle_2)|0\rangle_{\mathrm{a}}], \quad (20)
\end{aligned}$$

where the state $|0\rangle_2$ is the blank state, $|0\rangle_{\mathrm{a}}$ is the ancillary system,

$$A(\theta) = \frac{1}{\sqrt{2}}\left(1 + \frac{1}{\sqrt{1+2\tan^4\theta}}\right)^{1/2},$$

and

$$B(\theta) = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1+2\tan^4\theta}}\right)^{1/2}.$$

Choose the state of qubits 1 and 2 to be output state. The corresponding fidelities of two clones can be expressed as

$$F_{1\to2}^{\mathrm{SPCC},2} = \left(2 + \cos^2\theta + \sqrt{\cos^4\theta + 2\sin^4\theta}\right)/4. \quad (21)$$

Because the equator states $|\psi\rangle_1 = \left(|0\rangle + \mathrm{e}^{\mathrm{i}\phi}|1\rangle\right)_1/\sqrt{2}$, $\phi = 0,\ \pi/2,\ 3\pi/2,\ \pi$ are used as the encoded state in the BB84 cryptographic scheme, copying the equator states has been given much attention. If the input state is defined as the equator state $|\psi\rangle_1 = \left(|0\rangle + \mathrm{e}^{\mathrm{i}\varphi}|1\rangle\right)_1/\sqrt{2}$ with $\varphi \in [0,2\pi)$ being unknown, the unitary transformation of Eq. (20) becomes

$$\begin{aligned}
|0\rangle_1|0\rangle_2|0\rangle_{\mathrm{a}} \to\ & 1/\sqrt{2}\,|0\rangle_1|0\rangle_2|0\rangle_{\mathrm{a}} \\
& + 1/2\,(|01\rangle + |10\rangle)_{12}|1\rangle_{\mathrm{a}}, \\
|1\rangle_1|0\rangle_2|0\rangle_{\mathrm{a}} \to\ & 1/\sqrt{2}\,|1\rangle_1|1\rangle_2|1\rangle_{\mathrm{a}} \\
& + 1/2\,(|10\rangle + |01\rangle)_{12}|0\rangle_{\mathrm{a}}. \quad (22)
\end{aligned}$$

Equation (22) is also often called the optimal $1 \to 2$ SPCCM in two dimensions, which is given by Refs. [15] and [16]. The fidelities of the two clones reduce to $F_{1\to2}^{\mathrm{SPCC},2} = \left(1 + 1/\sqrt{2}\right)/2 \approx 0.853$.

Next, we discuss the PCCM in the asymmetric case. The $1 \to 2$ asymmetric PCCM (APCCM) was considered in Ref. [19]. The transformation of $1 \to 2$ APCCM in two dimensions can be expressed as

$$\begin{aligned}
&|0\rangle_1|0\rangle_2|0\rangle_{\mathrm{a}} \\
\to\ & 1/\sqrt{2}\,[|0\rangle_1|0\rangle_2|0\rangle_{\mathrm{a}} + (\cos\phi\,|01\rangle + \sin\phi\,|10\rangle)_{12}|1\rangle_{\mathrm{a}}], \\
&|1\rangle_1|0\rangle_2|0\rangle_{\mathrm{a}} \\
\to\ & 1/\sqrt{2}\,[|1\rangle_1|1\rangle_2|1\rangle_{\mathrm{a}} + (\cos\phi\,|10\rangle + \sin\phi\,|01\rangle)_{12}|0\rangle_{\mathrm{a}}], \quad (23)
\end{aligned}$$

where the cloning parameter $\phi \in [0,\pi/2]$. The corresponding fidelities are

$$F_{1\to2,1}^{\mathrm{APCC},2} = (1+\cos\phi)/2, \quad F_{1\to2,2}^{\mathrm{APCC},2} = (1+\sin\phi)/2. \quad (24)$$

2.2.2. Generalize the PCCM to multiple copies

If we expect to provide more quantum clones, the SPCCM can be generalized to multiple copies.[22,23] Fan *et al.* have proved the transformation $U_{1\to M}^{\mathrm{PCC}}$ of the optimal $1 \to M$ SPCCM in two dimensions, which can be described by[22]

$$\begin{aligned}
& U_{1\to M}^{\mathrm{PCC}}|0\rangle \otimes R = \sum_{j=0}^{M-1} \alpha_j |(M-j)0, j1\rangle \otimes R_j, \\
& U_{1\to M}^{\mathrm{PCC}}|1\rangle \otimes R \\
&= \sum_{j=0}^{M-1} \alpha_{M-1-j}|(M-1-j)0, (j+1)1\rangle \otimes R_j. \quad (25)
\end{aligned}$$

Here, R is the initial state of the PCCM including the $M-1$ blank copies, and R_j are the auxiliary states, which are orthogonally normalized. When the input state is described as the equator state, R_j can be denoted as $\left|(M-1-j)\psi, j\psi^{\perp}\right\rangle$. α_j is the cloning coefficient which satisfies these conditions: if M is even, $\alpha_j = \sqrt{1/2}$ with $j = M/2-1$ and $j = M/2$, otherwise $\alpha_j = 0$; if M is odd, $\alpha_j = 1$ with $j = (M-1)/2$, otherwise $\alpha_j = 0$.

The corresponding fidelity of each clone is

$$F_{1\to M}^{\mathrm{SPCC},2} = \begin{cases} \dfrac{1}{2} + \dfrac{\sqrt{M(M+2)}}{4M}, & M \text{ is even}, \\ \dfrac{1}{2} + \dfrac{M+1}{4M}, & M \text{ is odd}. \end{cases} \quad (26)$$

Note that D'Ariano and Macchiavello have also verified the corresponding fidelity independently.[23]

Consider the more general PCCM that takes N identical input qubits into M identical copies. Here, the input state of each qubit is denoted as $|\psi\rangle_1 = \left(|0\rangle + \mathrm{e}^{\mathrm{i}\phi}|1\rangle\right)_1/\sqrt{2}$, so the N identical input qubits are

$$|\psi\rangle^{\otimes N} = \frac{1}{2^{N/2}}\sum_{j=0}^{N} \mathrm{e}^{\mathrm{i}j\phi}\sqrt{C_N^j}\,|(N-j)0, j1\rangle. \quad (27)$$

In Ref. [22], the authors conjecture the unitary transformation $U_{N\to M}^{\mathrm{SPCCM}}$ of the optimal $N \to M$ SPCCM in two dimensions is given by

$$U_{N\to M}^{\mathrm{SPCCM}}|(N-j)0, j1\rangle \otimes R$$

$$= \left|\left(\frac{M+N}{2}-j\right)0,\left(\frac{M-N}{2}+j\right)1\right\rangle R_{(M-N)/2}, \quad (28)$$

where $M+N$ is even. The corresponding fidelity distribution of each clone is

$$F_{N\to M}^{\mathrm{SPCC},2} = \frac{1}{2}+\frac{1}{M2^N}\sum_{j=0}^{N-1}\sqrt{C_N^j C_N^{j+1}} \times\sqrt{\left(\frac{M-N}{2}+j+1\right)\left(\frac{M+N}{2}+j\right)}. \quad (29)$$

Similarly, the authors conjecture that the unitary transformation $U_{N\to M}^{\mathrm{SPCCM}}$ of the optimal $N\to M$ SPCCM in two dimensions is given by

$$U_{N\to M}^{\mathrm{SPCCM}}|(N-j)0,j1\rangle\otimes R = \frac{1}{\sqrt{2}}\left|\left(\frac{M+N+1}{2}-j\right)0,\left(\frac{M-N-1}{2}+j\right)1\right\rangle R_{(M-N-1)/2} + \frac{1}{\sqrt{2}}\left|\left(\frac{M+N-1}{2}-j\right)0,\left(\frac{M-N+1}{2}+j\right)1\right\rangle R_{(M-N+1)/2}, \quad (30)$$

with $M+N$ being odd. The corresponding fidelity distribution of each clone is

$$F_{N\to M}^{\mathrm{SPCC},2} = \frac{1}{2}+\frac{1}{M2^{N+1}}\sum_{j=0}^{N-1}\sqrt{C_N^j C_N^{j+1}} \times\left[\sqrt{\left(\frac{M-N+1}{2}+j\right)\left(\frac{M+N+1}{2}-j\right)} +\sqrt{\left(\frac{M-N+3}{2}+j\right)\left(\frac{M+N-1}{2}-j\right)}\right]. \quad (31)$$

2.2.3. Extend the PCCM to multidimensional case

On the other hand, we can generalize the cloner to d dimensions, if the quantum information is encoded in the input state $|\psi\rangle_1=\frac{1}{\sqrt{d}}\sum_{i=0}^{d-1}\mathrm{e}^{\mathrm{i}\phi_i}|i\rangle$, where $\varphi\in[0,2\pi)$ is unknown, and $i=0,1,\ldots,d-1$. The unitary transformation of optimal $1\to2$ SPCCM in d dimensions is defined as[16]

$$|i\rangle_1|0\rangle_2|0\rangle_a\to\alpha|i\rangle_1|i\rangle_2|i\rangle_a+\beta\sum_{\substack{j=0\\ i\neq j}}^{d-1}(|ij\rangle+|ji\rangle)_{12}|j\rangle_a. \quad (32)$$

The cloning coefficients are

$$\alpha^2=\frac{1}{2}\left(1-\frac{d-2}{\sqrt{d^2+4d-4}}\right), \quad \beta^2=\frac{1}{4(d-1)}\left(1+\frac{d-2}{\sqrt{d^2+4d-4}}\right). \quad (33)$$

The fidelity of each clone is

$$F_{1\to2}^{\mathrm{SPCC},d}=\frac{1}{d}+\frac{1}{4d}\left(d-2+\sqrt{d^2+4d-4}\right). \quad (34)$$

In the asymmetric case, the unitary transformation in d dimensions can be expressed as

$$|i\rangle_1|0\rangle_2|0\rangle_a \to p\left[|i\rangle_1|i\rangle_2|i\rangle_a+q\sum_{\substack{j=0\\ i\neq j}}^{d-1}(\cos\phi|ij\rangle+\sin\phi|ji\rangle)_{12}|j\rangle_a\right], \quad (35)$$

where $i=0,1,\ldots,d-1$, and $\phi\in[0,\pi/2]$. The cloning coefficients are

$$p^2=\frac{\sqrt{d^2+4d-4}-d+2}{2\sqrt{d^2+4d-4}}, \quad q^2=\frac{\sqrt{d^2+4d-4}+d-2}{(d-1)\left(2-d+\sqrt{d^2+4d-4}\right)}. \quad (36)$$

The optimal fidelities are

$$F_{1\to2,1}^{\mathrm{APCC},d}=\frac{p^2}{d}\{(d-1)q[(d-1)q\cos^2\phi+2\cos\phi+q\sin^2\phi]+1\}, \quad F_{1\to2,2}^{\mathrm{APCC},d}=\frac{p^2}{d}\{(d-1)q[(d-1)q\sin^2\phi+2\sin\phi+q\cos^2\phi]+1\}. \quad (37)$$

2.3. Optimal economic phase-covariant cloning

In this subsection, we discuss economic phase-covariant quantum cloning. This type of quantum cloning, which can perform phase-covariant quantum cloning without requiring any auxiliary qubits, was proposed by Durt and Du.[21] Suppose the input state is $|\psi\rangle_1=\cos\theta/2|0\rangle+\mathrm{e}^{\mathrm{i}\varphi}\sin\theta/2|1\rangle$, and the angle $\theta\in[0,\pi]$ is specified and fixed. This means that the amplitude of the input state is known, but the phase $\varphi\in[0,2\pi)$ is unknown.

2.3.1. The optimal $1\to2$ symmetric (asymmetric) economic PCCM in 2-dimensional Hilbert space

Depending on the value of θ, the optimal cloning transformations have two different forms. In both cases, the optimal cloning transformation is a unitary operation on two qubits: the input qubit and the blank qubit. If the input state is on the northern hemisphere of the Bloch sphere ($\theta\in[0,\pi/2]$), the unitary transformation of the optimal $1\to2$ symmetric economic PCCM (SEPCCM) in two dimensions is[26]

$$|0\rangle_1|0\rangle_2\to|00\rangle_{12}, \quad |1\rangle_1|0\rangle_2\to\frac{1}{\sqrt{2}}(|10\rangle+|01\rangle)_{12}. \quad (38)$$

The fidelities of two copies are

$$F_{1\to2}^{\mathrm{SEPCC},2}=\frac{1}{2}\sin^2\frac{\theta}{2}+\cos^4\frac{\theta}{2}+\frac{\sqrt{2}}{4}\sin^2\theta. \tag{39}$$

On the contrary, if the input state is on the southern hemisphere ($\theta\in[\pi/2,\pi]$), the transformation is

$$|0\rangle_1|0\rangle_2\to\frac{1}{\sqrt{2}}(|10\rangle+|01\rangle)_{12},\quad |1\rangle_1|0\rangle_2\to|11\rangle_{12}. \tag{40}$$

The fidelities of two copies can be expressed as Eq. (39).

In accord with the above discussion, the equator states are used as the encoded state in the BB84 cryptographic scheme,[24] so copying the equator state has also been given much attention. The fidelities of the optimal $1\to2$ SEPCCM in two dimensions for copying the equator state are $F_{1\to2}^{\mathrm{SEPCC},2}=1/2+\sqrt{2}/4$, which are equal to $F_{1\to2}^{\mathrm{SPCC},2}$.

In the corresponding asymmetric case, the main research objective is to copy the equator state $|\psi\rangle=\left(|0\rangle+\mathrm{e}^{\mathrm{i}\varphi}|1\rangle\right)/\sqrt{2}$. The transformation of the optimal $1\to2$ asymmetric EPCCM (AEPCCM) is[21]

$$\begin{aligned}&|0\rangle_1|0\rangle_2\to|00\rangle_{12},\\&|1\rangle_1|0\rangle_2\to\cos\phi|10\rangle_{12}+\sin\phi|01\rangle_{12},\end{aligned} \tag{41}$$

where $\phi\in[0,\pi/2]$. The fidelities are $F_{1\to2,1}^{\mathrm{AEPCC},2}=(1+\cos\phi)/2$ and $F_{1\to2,2}^{\mathrm{AEPCC},2}=(1+\sin\phi)/2$, which are also equal to $F_{1\to2,1}^{\mathrm{APCC},2}$ and $F_{1\to2,2}^{\mathrm{APCC},2}$, respectively.

2.3.2. Generalize the EPCCM to multiple copies

In the case of multiple copies, the optimal $1\to M$ SEPCCM is also the focus of the attention.[23,28] As demonstrated by D'Ariano *et al.* and Delgado *et al.*, the unitary transformation of the $1\to M$ SEPCCM in two dimensions can be expressed as

$$\begin{aligned}&U_{1\to M}^{\mathrm{EPCC}}|0\rangle|(M-1)0\rangle=\sum_{j=0}^{M-1}\alpha_j|(M-j)0,j1\rangle,\\&U_{1\to M}^{\mathrm{EPCC}}|1\rangle|(M-1)0\rangle\\&=\sum_{j=0}^{M-1}\alpha_{M-1-j}|(M-1-j)0,(j+1)1\rangle.\end{aligned} \tag{42}$$

Here, $|(M-1)0\rangle$ is the initial state of the SEPCCM, that is, the $M-1$ blank copies. α_j is the cloning coefficient which satisfies these conditions: if M is even, $\alpha_j=1$ with $j=M/2-1$, otherwise $\alpha_j=0$; if M is odd, $\alpha_j=1$ with $j=(M-1)/2$, otherwise $\alpha_j=0$.

The corresponding fidelity of each clone is

$$F_{1\to M}^{\mathrm{SEPCC},2}=\begin{cases}\dfrac{1}{2}+\dfrac{\sqrt{M(M+2)}}{4M}, & M\ \text{is even},\\[2mm]\dfrac{1}{2}+\dfrac{M+1}{4M}, & M\ \text{is odd}.\end{cases} \tag{43}$$

2.3.3. Extend the PCCM to multi-dimensional case

In this subsection, the asymmetric case is discussed first. In order to generalize to d dimensions for cloning the state $|\psi\rangle_1=\frac{1}{\sqrt{d}}\sum_{i=0}^{d-1}\mathrm{e}^{\mathrm{i}\phi_i}|i\rangle$ with $\varphi_i\in[0,2\pi)$ being unknown, the unitary transformation of $1\to2$ asymmetric EPCCM (AEPCCM) in d dimensions is given by[56]

$$\begin{aligned}&|i\rangle_1|i\rangle_2\to|ii\rangle_{12},\\&|j\rangle_1|0\rangle_2\to\cos\phi|ji\rangle_{12}+\sin\phi|ij\rangle_{12},\end{aligned} \tag{44}$$

where $i,j=0,1,\ldots,d-1$ $(j\neq i)$, and $\phi\in[0,\pi/2]$. The fidelities of two clones can be expressed as

$$\begin{aligned}F_{1\to2,1}^{\mathrm{AEPCC},d}&=\frac{1}{d^2}[d+2p(d-1)+p^2(d^2-3d+2)],\\F_{1\to2,2}^{\mathrm{AEPCC},d}&=\frac{1}{d^2}[d+2q(d-1)+q^2(d^2-3d+2)].\end{aligned} \tag{45}$$

Obviously, if $\phi=\pi/4$, the state of Eq. (38) is

$$\begin{aligned}&|i\rangle_1|i\rangle_2\to|ii\rangle_{12},\\&|j\rangle_1|0\rangle_2\to(|ji\rangle_{12}+|ij\rangle_{12})/\sqrt{2}.\end{aligned} \tag{46}$$

This is the unitary transformation of the $1\to2$ symmetric EPCCM (SEPCCM) in d dimensions, and the fidelity of the two quantum clones is

$$F_{1\to2}^{\mathrm{SEPCC},d}=\frac{1}{2d^2}[d-1+(d+\sqrt{2}-1)^2]. \tag{47}$$

2.4. Real state quantum cloning machine

If the input state is $|\psi\rangle_1=\cos\theta/2|0\rangle+\mathrm{e}^{\mathrm{i}\varphi}\sin\theta/2|1\rangle$, here $\theta\in[0,\pi)$ is unknown, but $\varphi\in[0,2\pi)$ is known. In other words, the amplitude information of the input state is completely unknown, but the phase information is known. This type of restrictive quantum cloning is referred to as the real state quantum cloning machine (RSCM).[15,16,22]

2.4.1. The optimal $1\to2$ symmetric (asymmetric) RSCM in 2-dimensional Hilbert space

First, we consider the symmetric case. The unitary transformation of the $1\to2$ symmetric RSCM (SRSCM) in two dimensions is given by

$$\begin{aligned}&|0\rangle_1|0\rangle_2|0\rangle_{\mathrm{a}}\\&\to\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{8}}\right)|0\rangle_1|0\rangle_2|0\rangle_{\mathrm{a}}+\frac{1}{\sqrt{8}}(|01\rangle+|10\rangle)_{12}|1\rangle_{\mathrm{a}}\\&\quad+\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{8}}\right)|1\rangle_1|1\rangle_2|0\rangle_{\mathrm{a}},\\&|1\rangle_1|0\rangle_2|0\rangle_{\mathrm{a}}\\&\to\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{8}}\right)|1\rangle_1|1\rangle_2|1\rangle_{\mathrm{a}}+\frac{1}{\sqrt{8}}(|10\rangle+|01\rangle)_{12}|0\rangle_{\mathrm{a}}\\&\quad+\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{8}}\right)|0\rangle_1|0\rangle_2|1\rangle_{\mathrm{a}}.\end{aligned} \tag{48}$$

The optimal fidelity of each clone is

$$F_{1\to2}^{\mathrm{SRSC},2}=\left(1+1/\sqrt{2}\right)/2\approx0.853. \tag{49}$$

Obviously, the fidelity of the cloning described by Eq. (43) is equal to $F_{1\to2}^{\mathrm{SPCC},2}$.

In the asymmetric case, the unitary transformation of $1\to2$ asymmetric RSCM (ARSCM) in two dimensions can be expressed as[57]

$$\begin{aligned}|0\rangle_1|0\rangle_2|0\rangle_a &\to A|0\rangle_1|0\rangle_2|0\rangle_a+(B|01\rangle+C|10\rangle)_{12}|1\rangle_a\\ &\quad+D|1\rangle_1|1\rangle_2|0\rangle_a,\\ |1\rangle_1|0\rangle_2|0\rangle_a &\to A|1\rangle_1|1\rangle_2|1\rangle_a+(B|10\rangle+C|01\rangle)_{12}|0\rangle_a\\ &\quad+D|0\rangle_1|0\rangle_2|1\rangle_a,\end{aligned}\tag{50}$$

with A, B, C, $D\geq0$ satisfying the normalization condition $A^2+B^2+C^2+D^2=1$. Thus, the two fidelities read

$$F_{1\to2,1}^{\mathrm{ASRSC},2}=A^2+B^2,\quad F_{1\to2,2}^{\mathrm{ASRSC},2}=A^2+C^2.\tag{51}$$

In fact, $F_{1\to2,1}^{\mathrm{ASRSC},2}$, $F_{1\to2,2}^{\mathrm{ASRSC},2}\in[1/2,1]$. If we set the value of $F_{1\to2,1}^{\mathrm{ASRSC},2}=(1+\cos\phi)/2$ ($\phi\in[0,\pi/2]$), the fidelity of clone 2 is maximal. Through calculation, we can obtain the optimal fidelity $F_{1\to2,2}^{\mathrm{ASRSC},2}=(1+\sin\phi)/2$. Obviously, the fidelity $F_{1\to2}^{\mathrm{ASRSC},2}$ is equal to the fidelity $F_{1\to2}^{\mathrm{APCC},2}$.

2.4.2. Extend RSC to multi-dimensional case

The symmetric RSC can be generalized into the d-dimensional case. Assume the input state is in the form of $\psi_2=\sum_{i=0}^{d-1}c_i|i\rangle$ with c_i being real and unknown and $\sum_{i=0}^{d-1}c_i^2=1$, the unitary transformation of $1\to2$ SRSCM in d dimensions can be written as[57]

$$\begin{aligned}|i\rangle_1|0\rangle_2|0\rangle_a\to&\left(\alpha|i\rangle_1|i\rangle_2+\beta\sum_{\substack{j=0\\j\neq i}}^{d-1}|j\rangle_1|j\rangle_2\right)|i\rangle_a\\ &+\gamma\sum_{\substack{j=0\\j\neq i}}^{d-1}(|j\rangle_1|i\rangle_2+|i\rangle_1|j\rangle_2)|j\rangle_a,\end{aligned}\tag{52}$$

where α, β, $\lambda\geq0$, and $\alpha^2+(d-1)\beta^2+2(d-1)\gamma^2=1$.

The cloning coefficients are given by

$$\begin{aligned}\alpha&=\left(4+d+\sqrt{d^2+4d+20}\right)\\ &\quad\times[20+24d+5d^2+d^3\\ &\quad+\left(6+3d+d^2\right)\sqrt{d^2+4d+20}]^{-1/2},\\ \beta&=2[20+24d+5d^2+d^3\\ &\quad+\left(6+3d+d^2\right)\sqrt{d^2+4d+20}]^{-1/2},\\ \gamma&=\frac{1}{2}\left(2+d+\sqrt{d^2+4d+20}\right)\\ &\quad\times[20+24d+5d^2+d^3\\ &\quad+\left(6+3d+d^2\right)\sqrt{d^2+4d+20}]^{-1/2}.\end{aligned}\tag{53}$$

The corresponding fidelity of the two clones is written as

$$F_{1\to2}^{\mathrm{SRSC},d}=1/2+\left(\sqrt{d^2+4d+20}-d+2\right)/4(d+2).\tag{54}$$

In d dimensions, the explicit unitary cloning transformation of the optimal $1\to2$ ARSCM can be written as[56]

$$\begin{aligned}|i\rangle_1|0\rangle_2|0\rangle_a\to&\left(\alpha|i\rangle_1|i\rangle_2+\beta\sum_{\substack{j=0\\j\neq i}}^{d-1}|j\rangle_1|j\rangle_2\right)|i\rangle_a\\ &+\sum_{\substack{j=0\\j\neq i}}^{d-1}(\gamma|j\rangle_1|i\rangle_2+\chi|i\rangle_1|j\rangle_2)|j\rangle_a.\end{aligned}\tag{55}$$

The cloning coefficients are given by

$$\begin{aligned}\alpha&=\left[(d-1)(\gamma+\chi)+\sqrt{(\gamma+\chi)}-d\left(d\gamma^2+2\gamma\chi+d\chi^2-1\right)\right]/d,\\ \beta&=\left[\sqrt{(\gamma+\chi)^2-d\left(d\gamma^2+2\gamma\chi+d\chi^2-1\right)}-\gamma-\chi\right]/d,\\ \gamma&=\frac{\left(a^2+1\right)\chi-\sqrt{2}b+\sqrt{2a^2-2a^4+2a^2b^2-4\sqrt{2}a^2b\chi+4a^2\chi^2}}{a^2-1}.\end{aligned}\tag{56}$$

Here,

$$\begin{aligned}a&=\left[\sqrt{2}d+\sqrt{d(2d-1)}\right]\left(\Delta-\frac{1}{\sqrt{d}}\right),\quad b=a+\frac{\Delta}{\sqrt{2}},\\ \Delta&=\left(d+2+\sqrt{d^2+4d+20}\right)\\ &\quad\times[d^3+5d^2+24d+20\\ &\quad+\left(d^2+3d+6\right)\sqrt{d^2+4d+20}]^{-1/2}.\end{aligned}\tag{57}$$

The cloning coefficients contain only one parameter, $\chi\in\left[0,1/\sqrt{d}\right]$. The two fidelities are given by

$$\begin{aligned}F_{1\to2,1}^{\mathrm{ARSC},d}&=\beta^2+\gamma^2+(d-2)\chi^2+2(\alpha\chi+\beta\gamma),\\ F_{1\to2,2}^{\mathrm{ARSC},d}&=\beta^2+\chi^2+(d-2)\gamma^2+2(\alpha\gamma+\beta\chi).\end{aligned}\tag{58}$$

2.5. State-dependent quantum cloning machine (SDC)

A state-dependent quantum cloning machine was proposed in Ref. [6]. Two pure nonorthogonal states in a two-

dimensional Hilbert space can be defined as

$$|a\rangle = \cos\theta\,|0\rangle + \sin\theta\,|1\rangle\,, \quad |b\rangle = \sin\theta\,|0\rangle + \cos\theta\,|1\rangle\,. \tag{59}$$

Here, $\{|0\rangle, |1\rangle\}$ is an orthonormal basis, and $\theta \in [0, \pi/4]$.

Consider a unitary operator U acting on the two nonorthogonal states. We have

$$|\alpha\rangle = U\,|a\rangle\,|0\rangle\,, \quad |\beta\rangle = U\,|b\rangle\,|0\rangle\,. \tag{60}$$

Unitarity gives the following constraint on the scalar product of the final states:

$$\langle\alpha/\beta\rangle = \langle a/b\rangle = \sin 2\theta \equiv S. \tag{61}$$

As a criterion for optimality of the state-dependent cloner, we take the transformation that maximizes the global fidelity $F_{\rm g}$ of both final states $|\alpha\rangle$ and $|\beta\rangle$ with respect to the perfect cloned states $|aa\rangle$ and $|bb\rangle$. The global fidelity is defined as

$$F_{\rm g} = \frac{1}{2}\left(|\langle\alpha/aa\rangle|^2 + |\langle\beta/bb\rangle|^2\right). \tag{62}$$

The value of the optimal global fidelity is

$$\begin{aligned} F \;=\; & \frac{1}{4}(\sqrt{1+\sin^2 2\theta}\sqrt{1+\sin 2\theta} \\ & + \cos 2\theta\sqrt{1-\sin 2\theta})^2. \end{aligned} \tag{63}$$

The corresponding unitary transformation U is given by

$$\begin{aligned} U\,|00\rangle &= a\,|00\rangle + b\,(|01\rangle + |10\rangle) + c\,|11\rangle\,, \\ U\,|10\rangle &= c\,|00\rangle + b\,(|01\rangle + |10\rangle) + a\,|11\rangle\,. \end{aligned} \tag{64}$$

Here,

$$\begin{aligned} a &= \frac{1}{\cos 2\theta}\left[\cos\theta\,(P + Q\cos 2\theta) - \sin\theta\,(P - Q\cos 2\theta)\right], \\ b &= \frac{1}{\cos 2\theta} P \sin 2\theta\,(\cos\theta - \sin\theta)\,, \\ c &= \frac{1}{\cos 2\theta}\left[\cos\theta\,(P - Q\cos 2\theta) - \sin\theta\,(P + Q\cos 2\theta)\right], \end{aligned} \tag{65}$$

with

$$Q = \frac{1}{2}\frac{\sqrt{1-\sin 2\theta}}{\cos 2\theta}, \quad Q = \frac{1}{2}\frac{\sqrt{1-\sin 2\theta}}{\cos 2\theta}.$$

2.6. Probabilistic cloning machine

The probabilistic cloning machine, which is constructed as a unitary-reduction process, where a unitary transformation U is concatenated to a projective measurement, is first proposed by Duan *et al.*[7] Considering $|\psi^0\rangle_1$ and $|\psi^1\rangle_1$ as two input non-orthogonal states, we define the unitary transformation U as

$$\begin{aligned} U\,|\psi^i\rangle_1\,|0\rangle_2\,|0\rangle_{\rm a} \;=\; & \sqrt{p}\,|\psi^i\rangle_1\,|\psi^i\rangle_2\,|0\rangle_{\rm a} \\ & + \sqrt{1-p}\,|\phi\rangle_{12}\,|1\rangle_{\rm a}\,, \quad i = 0, 1, \end{aligned} \tag{66}$$

where $|0\rangle_2\,|0\rangle_{\rm a}$ is a blank initial bipartite state, p is the corresponding probability of successfully obtaining two perfect clones, and $|\varphi\rangle_{12}$ is the normalized states of the composite system. After the application of the unitary transformation, we perform a measurement $O = \lambda_0\,|0\rangle_{\rm a}\langle 0| + \lambda_1\,|1\rangle_{\rm a}\langle 1|$ on the ancillary system. If the state of the ancillary system is $|0\rangle_{\rm a}$ with probability p, the original and copy systems are described by exactly the same states, and the cloning succeeds. Otherwise, if the state of the ancillary system is $|1\rangle_{\rm a}$ with probability $1-p$, the remain system is left in the state $|\varphi\rangle_{12}$ independent of the initial state of the original system. In this case, the cloning fails. Assume the states that are to be cloned are generated with the same *a priori* probability. Then, the optimal cloning probability p is given by[7]

$$p = \frac{1}{1 + |\langle\psi^i/\psi^j\rangle|}. \tag{67}$$

For the optimal asymmetric $1 \to M$ PPQC,[58] the optimal success probabilities $p^i_{1\to M}$ and $p^j_{1\to M}$ of the input states $|\psi^i\rangle_1$ satisfy the relation

$$\frac{p^i_{1\to M} + p^j_{1\to M}}{2} \le \frac{1 - |\langle\psi^i/\psi^j\rangle|}{1 - |\langle\psi^i/\psi^j\rangle|^M}.$$

Here, the equality holds if $p^i_{1\to M} = p^j_{1\to M}$.

2.7. Brief summary and new findings

From the above discussion, we find that scalability has been a principal focus of quantum cloning research. In fact, scalability is a key concept of the quantum information processing. In quantum cloning, scalability is realized in two ways: by generalizing to the multiple copies case or by extending to the multi-dimensional case. Comparing the two scalabilities, fidelity will be reduced with an increased number of quantum clones or an increased number of dimensions in the quantum system, but fidelity is more stable in the multiple copies case. On the other hand, comparing different types of quantum cloning processes with regard to fidelity, one can obtain $F^{\rm SUQC,2}_{1\to M} < F^{\rm SPCC,2}_{1\to M} = F^{\rm SEPCC,2}_{1\to M}$ and $F^{\rm ASUQC,}_{1\to 2}{}^{d} < F^{\rm AEPCC,}_{1\to 2}{}^{d} \le F^{\rm APCC,}_{1\to 2}{}^{d} \le F^{\rm ARSC,}_{1\to 2}{}^{d}$. Comparing approximate quantum cloning and probabilistic quantum cloning, approximate method ensures that one can obtain the quantum copy at the expense of fidelity, while the probabilistic method ensures that one will obtain a perfect quantum copy at the expense of probability. Furthermore, very recent progress has been reported by Zhang *et al.*[59] In 2012, they designed a uniform transformation by combining the concepts of state-dependent quantum cloning machines (SDC) and probabilistic cloning machines (PQC). In addition, they derived an explicit expression of copy fidelity as a function of the probability of success, together with the explicit unitary transformation. In other words, the fidelity of their quantum clones is tunable by choosing the probability of success.

3. Implementations of quantum cloning machines in physical systems

In this section, we review the schemes for implementing various quantum cloning machines. The implementation of the quantum cloning machine not only verifies the theory of the quantum cloning, but also shines light on the security issues particular to quantum communications and quantum computation. Thus, it has attracted much attention. Reviewing the proposed schemes in the recent ten years, we find that almost every type of quantum cloning machine has been repeatedly proposed. For example, plausible implementations of quantum cloning machines have been proposed in several physical systems for engineering quantum information processing, including cavity quantum electrodynamics systems,[60–78] superconducting device systems,[79–84] ion-trap systems,[84–89] quantum optics systems,[90–108] spin systems,[109–111] etc. When researchers try to design a quantum cloning machine, what should they pay attention to? First, what type of quantum cloning machine should be implemented? Second, why should this type of quantum cloning be realized? Third, how should this type of quantum cloning be performed?

3.1. Selecting the quantum system and operators for implementing QCM

In theory, an arbitrary QCM can be implemented by using several single-qubit rotation operators and two-qubit quantum controlled-not (CNOT) operators, as Bužek and Hillery proposed in Ref. [2]. However, the realization of the quantum CNOT operator is an important and difficult problem in quantum information processing. Most of the proposed schemes were to adopt other two-qubit quantum operators or multi-qubit quantum operators.[87–89,104] Hence, the realization of the quantum cloning machine depends upon the physical system chosen. At present, the several possible candidates for engineering quantum information processing attract much attention, including cavity quantum electrodynamics systems,[112,113] superconducting device systems, ion-trap systems,[114] quantum optics systems,[115] and the nuclear magnetic resonance technique. One emerging strategy is the use of the characteristics of the physical system to simplify the quantum cloning operator. For example, in Ref. [78], Yu *et al.* proposed a scheme to realize an ancilla-free EPCCM with superconducting quantum interference device (SQUID) qubits. In their scheme, the EPQCM can be realized with multiple SQUIDs by adiabatic passage. Analogously, in Refs. [110] and [111], the quantum cloning process is greatly simplified, benefitting from characteristics of the physical system. Of course, each scheme has the advantages of the adopted platform. For instance, trapped ions can store information for a long time,[85] and the photons in an optical system can transfer the quantum information over a long distance.[105] Furthermore, the cavity QED system can provide a platform to achieve strong coherent interactions between atoms and photons. Using this strategy, some schemes[77–79,99] have been proposed to design a QCM in a quantum network.

3.2. Selecting the type of QCM and the corresponding scheme

In the above subsection, we point out that the proposed schemes should use advantages of the chosen physical system. The realization efficiency of a proposed scheme also should be given more consideration. On one hand, designers have tried to build scalable QCMs.[68,69,110,111] On the other hand, the designers have tried to build general QCMs that can implement several types of quantum cloning processes.[74–76,88,89,105] As is well known, the scalability of QCM is tied to the conservation of quantum information and quantum state estimation, so a general QCM has some advantages for being used in quantum communication and computation processes. In Ref. [81], as shown in Fig. 1, the authors proposed a unitary transformation U, which is

$$U = R^{1,2}G^{1,3}G^{1,2}R_4^{1,2,3}R_2^3G^{1,3}(\phi_0)G^{1,2}R_4^1R_2^2R_1^1, \tag{68}$$

where

$$R_1^1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$
$$R_2^2 = \begin{pmatrix} \cos\phi_0 & -\mathrm{i}\,\mathrm{e}^{\mathrm{i}\phi_1}\sin\phi_0 \\ -\mathrm{i}\,\mathrm{e}^{-\mathrm{i}\phi_1}\sin\phi_0 & \cos\phi_0 \end{pmatrix},$$
$$R_3^3 = \begin{pmatrix} 1 & 0 \\ 0 & \mathrm{i} \end{pmatrix},$$

and

$$R_4^1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

are the single-qubit operators, and the superscript implies the operated qubit.

$$G^{1,3}(\varphi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}\phi}\cos\phi & -\mathrm{i}\,\mathrm{e}^{-\mathrm{i}\phi}\sin\phi & 0 \\ 0 & -\mathrm{i}\,\mathrm{e}^{-\mathrm{i}\phi}\sin\phi & \mathrm{e}^{-\mathrm{i}\phi}\cos\phi & 0 \\ 0 & 0 & 0 & \mathrm{e}^{-2\mathrm{i}\phi} \end{pmatrix}$$

and

$$G^{1,2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

are the two qubit operators, and the superscript denotes the two operated qubits.

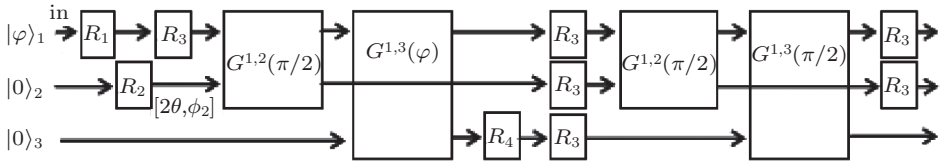


Fig. 1. This is the schematic diagram of the PGQCM. R_i $(i = 1,2,3,4)$ is the single-qubit rotation. $G^{1,j}(\pi/2)$ $(j = 2,3)$ is the two-qubit operation which pictures the interaction between two SQUIDs and the nonresonant cavity, where 1 and j represent No. 1 and No. j qubits.

If the input qubit is in the state described by Eq. (1), and the initial state of the cloning system is $|0\rangle_2|0\rangle_a$, then the output state is

$$\begin{aligned}&\cos\phi[\cos\theta|0\rangle_1|0\rangle_2|0\rangle_a+\sin\theta\,e^{i\varphi}|1\rangle_1|1\rangle_2|1\rangle_a]\\&+i\sin\phi\,e^{-i(\phi+\phi_1)}[\cos\phi_0(\cos\theta|1\rangle_1|0\rangle_2|1\rangle_a\\&+\sin\theta\,e^{i\varphi}|0\rangle_1|1\rangle_2|0\rangle_a)\\&+\sin\phi_0(\cos\theta|0\rangle_1|1\rangle_2|1\rangle_a+\sin\theta\,e^{i\eta}|1\rangle_1|0\rangle_2|0\rangle_a)].\end{aligned}\quad(69)$$

Choosing suitable values of ϕ, ϕ_0, and φ_1, the state given by Eq. (69) should neatly transform to Eqs. (5), (20), (22), (23), and (42) with $M = 3$.

4. Experimental evidence

Each proposed scheme is experimentally feasible in theory. As is well known, the realization of most quantum cloning machines requires multiple steps. Although today's experimental techniques are, in principle, sufficient to verify most of the schemes of the QCM, due to extreme requirements on the experimental environment and the practical cost factors, the experimental evidence is something of luxury. Moreover, even if some authors declared that their schemes need only one step, the preparation processes still make the experiments very challenging. Thus, we must be grateful that some experiments have been reported. Most of the experiments are implemented by using the linear optical systems,[99–101,116–118] or the nuclear magnetic resonance technique.[109,119,120] Achievements of optical systems in this area are amazing. In 2008, Xu *et al.*[100] reported an experimental realization of the optimal universal and phase-covariant quantum cloning machine that is based on parametric down-conversion, a design first proposed by Zou *et al.*[105] The prepared input state is illustrated in Fig. 3 of Ref. [105]. The UV pulses are focused into a beta-barium-borate (BBO) crystal twice to generate four photons with special polarizations, which can be confirmed by the detector D. The half-wave plate (HWP) and the quarter wave plate (QWP) are used to prepare any required pure input state. Then, the experimental setup is connected to the setup in Fig. 2 of Ref. [105]. In Ref. [100], those cloning machines described by Eqs. (2) and (42) with $M = 3$ are observed according to the proposed reflection coefficient, $R = 2/3$ or $R = 3/4$. The corresponding observed fidelities of these cloning machines are equal to 71% and 73%, respectively, and the theoretical values for both are 83.3%.

In 2010, Nagali *et al.*[101] reported an experimental realization of the optimal quantum cloning of four-dimension quantum states, encoded in the polarization and orbital angular momentum degrees of freedom of photons. Engagingly, very recent progress has been reported by Chang *et al.*[121] in 2013: the first room-temperature realization of quantum phase cloning. The cloning was implemented in the nitrogen-vacancy (NV) defect center of diamond at room temperature. The logic states are encoded in the spin energy levels of the electron spin of the NV. The quantum states corresponding to $|m_s = 1\rangle$, $|m_s = -1\rangle$, and $|m_s = 0\rangle$ are denoted by $|00\rangle$, $|01\rangle$, and $|10\rangle$. The transition between $|m_s = 0\rangle$ and $|m_s = 1\rangle$ is coupled to the first microwave pulse, and the transition between $|m_s = 0\rangle$ and $|m_s = -1\rangle$ is resonance coupled to the second microwave pulse. The input state $\left(|0\rangle + e^{i\phi}|1\rangle\right)/\sqrt{2}$ is prepared by the first microwave pulse, which is encoded in $\left(|m_s = 1\rangle + e^{i\phi}|m_s = 0\rangle\right)/\sqrt{2}$. Then, the phase cloning machine described by Eq. (38) can be completed by the second microwave pulse. The systemic state is $[|00\rangle + e^{i\phi}(|10\rangle + |01\rangle)/\sqrt{2}]/\sqrt{2}$. The average experimental fidelity is 80.3%, compared with the theoretical value of 85.4%. The quantum phase cloner works well for all qubits located on the equator of the Bloch sphere. The phase is controlled and can be measured with high accuracy, and the experimental results are consistent with theoretical expectations.

5. Conclusion

This paper reviews the basic theory of QCMs for discrete variables. We discuss the difference between the approximate quantum cloning and the probabilistic quantum cloning. About the approximate quantum cloning, we review and compare several different types of quantum cloning machines, including UQCM, PCCM, EPCC, RSCM, and SDC. In addition, we give a brief discussion on the probabilistic quantum cloning. On the other hand, we review the implementation for these various quantum cloning machines (QCMs) in different quantum systems. About that, we mainly discuss the utilization efficiency of the designed scheme of the QCM and the use of the characteristics of the physical system. In addition, we also discuss the two directions about the design of QCM:

versatility and scalability. Lastly, we give a brief review on the experimental evidence.

Quantum cloning is one of the most intriguing topics in quantum physics. It is important not only because of its fundamental characteristics but also because of its direct applications to quantum communications and quantum computation. Quantum cloning has developed considerably over the past two decades, but some open questions remain. The study of optimal cloning has always supposed pure input states, temporarily leaving aside the optimal cloning of mixed states. We compute the single-copy fidelity for each cloning scheme to evaluate the quality of the cloning, and we explore the relation between optimal fidelity and state estimation. Quantum information processing is not secure when the fidelity is optimal. In the future, we think that designing experimentally feasible QCMs will continue to be important in order to move toward practical applications of quantum information. In addition, other study directions may aim to optimize the cloning of mixed states, optimize cloning in d dimensions, transform a one-input qubit to M identical copies, and explore the connection between optimal cloning and state estimation.

Lastly, due to the limited knowledge and ability of the authors, omissions and deficiencies in this paper are inevitable. We welcome the readers to give us any comments.

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