TOPICAL REVIEW — Quantum information

Towards quantum-enhanced precision measurements: Promise and challenges*

Zhang Li-Jian(张利剑)[a)†] and Xiao Min(肖 敏)[a)b)]

[a)] College of Engineering and Applied Sciences, Nanjing University, Nanjing 210093, China
[b)] Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701, USA

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Quantum metrology holds the promise of improving the measurement precision beyond the limit of classical approaches. To achieve such enhancement in performance requires the development of quantum estimation theories as well as novel experimental techniques. In this article, we provide a brief review of some recent results in the field of quantum metrology. We emphasize that the unambiguous demonstration of the quantum-enhanced precision needs a careful analysis of the resources involved. In particular, the implementation of quantum metrology in practice requires us to take into account the experimental imperfections included, for example, particle loss and dephasing noise. For a detailed introduction to the experimental demonstrations of quantum metrology, we refer the reader to another article 'Quantum metrology' in the same issue.

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1. Introduction

Metrology, the precision measurement of unknown parameters, plays an important role in both fundamental science and practical technologies. A metrology process can be cased as three steps: the preparation of a probe, its interaction with the parameters to be estimated, and the measurements on the altered probe. The parameters to be estimated induce changes in the probe, therefore the precision of the estimation is determined by how well we can resolve these changes. The resolution is limited by both experimental imperfections and intrinsic quantum fluctuations of the probe state. The latter sets a fundamental limit of the achievable precision. A benchmark of a quantum metrology scheme is its precision when using a fixed amount of resources. A relevant measure of the resource to affect a measurement is the total energy or number of particles in the probe state, which is the product of the number of copies of identical probes ν and the average number of particles N in each probe state. When ν is large enough, due to the central-limit theorem, the error in the estimation is reduced by a factor of $1/\sqrt{\nu}$. The effect of N depends on the probe state utilized, the interaction process, and the measurement performed on the probe. When using coherent states or single-photon states as the probes, the best precision we can achieve is proportional to $1/\sqrt{N}$. In quantum metrology, this scaling is known as the 'standard quantum limit' (SQL), which is the best one can have when utilizing the classical resources. However, by employing genuine quantum effects, e.g., using entangled or squeezed states as the probe states,[1,2] it is possible to surpass this limit or even reach the Heisenberg limit (HL) which is proportional to $1/N$.[3]

Quantum metrology, as an emerging quantum technology, provides a more efficient method to utilize the resource in precision measurement compared to the classical means. This improved efficiency is particularly advantageous in searching for ultra weak signals such as gravitational wave detection as well as measuring fragile samples that can only tolerate a limited amount of probing disturbance. Yet this advantage comes with the technical challenges associated with the generation of large-scale quantum states and the engineering of complex quantum detection systems. Moreover, the delicate nature of quantum states makes them very sensitive to experimental imperfections such as loss and decoherence. Therefore to achieve the quantum-enhanced precision in practice requires, both the careful design of novel quantum metrology schemes and the development of quantum sources and detections are needed.

In this article, we review some recent developments in the field of quantum metrology. Section 2 introduces the basic theories of quantum metrology and several bounds for the precision. Section 3 discusses the performance of quantum metrology schemes in the presence of experimental imperfections. Demonstrations of quantum metrology, especially in interferometry, are reviewed in Section 4. We conclude and provide an outlook at the potential developments in Section 5.

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†Corresponding author. E-mail: lijian.zhang@nju.edu.cn
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2. Basics of quantum metrology

We consider ν copies of the probe states $\hat{\rho}$. Each replica interacts with the object such that the state of the probe is modified by the parameter of interest x. The goal is to estimate x from the ν copies of the modified probe states $\hat{\rho}^{\otimes\nu}(x)$. The precision we can achieve is determined by how well we can resolve $\hat{\rho}^{\otimes\nu}(x)$ from its neighboring state $\hat{\rho}^{\otimes\nu}(x')$. Here we briefly discuss several bounds on this problem.

2.1. Generalized uncertainty relation and the quantum Cramér–Rao bound

To estimate x, a measurement denoted by a positive-operator-valued measure (POVM) $\{\hat{\Pi}_j\}$, $j=1,\ldots,J$ is performed on each copy of $\hat{\rho}(x)$. (A joint measurement on $\hat{\rho}^{\otimes\nu}$ is also possible. Yet in the framework of the Cramér–Rao bound, the joint measurement will not increase the precision.[4]) The probability of registering result j is given by the Born rule

$$P(j|x)=\mathrm{Tr}(\hat{\Pi}_j\hat{\rho}(x)). \tag{1}$$

The estimation of x is a function of the measurement results on ν replica of $\hat{\rho}(x)$ given by $x_{\mathrm{est}}(j_1,j_2,j_\nu)$. The proper measure to quantify the deviation of x_{est} from x is given by[5,6]

$$\delta x=\frac{x_{\mathrm{est}}}{|\mathrm{d}x_{\mathrm{est}}/\mathrm{d}x|}-x, \tag{2}$$

which takes into account both the systematic bias and the difference in units of x_{est} and x. A statistical measure of the precision is given by the second moment of δx, which is lower bounded by the Cramér–Rao bound[7]

$$\Delta^2 x=\langle(\delta x)^2\rangle_x\geq\frac{1}{\nu F(x)}, \tag{3}$$

where Δx is the root mean square error (RMSE), and $F(x)$ is the Fisher information

$$F(x)=\sum_j\frac{1}{P(j|x)}\left(\frac{\mathrm{d}P(j|x)}{\mathrm{d}x}\right)^2. \tag{4}$$

Fisher's theorem[8,9] indicates that the Cramér–Rao bound (CRB) can be achieved asymptotically for a large ν with the maximum-likelihood estimation. It is shown that for any POVM $\{\hat{\Pi}_j\}$, $j=1,\ldots,J$, there is an upper bound of F

$$F\leq\mathrm{Tr}\left(\hat{L}_x^2\hat{\rho}_x\right)=Q_x, \tag{5}$$

where $\hat{L}_x$ is known as the symmetric logarithmic derivative (SLD) satisfying the linear algebraic equation

$$\frac{\mathrm{d}\hat{\rho}_x}{\mathrm{d}x}=\frac{\hat{L}_x\hat{\rho}_x+\hat{\rho}_x\hat{L}_x}{2}. \tag{6}$$

The maximum of Fisher information Q_x is called the quantum Fisher information (QFI) and is achieved with the optimal POVM. The QFI is only determined by the probe state and the parameter to be estimated. If the path from $\hat{\rho}$ to $\hat{\rho}_x$ is generated by a unitary transformation

$$\hat{\rho}_x=\mathrm{e}^{\mathrm{i}x\hat{h}}\hat{\rho}\,\mathrm{e}^{-\mathrm{i}x\hat{h}}, \tag{7}$$

we have $Q_x\leq 4\Delta^2\hat{h}$, with the equality achieved when the probe state is pure.[5,6] Substituting this condition into Eqs. (3) and (5), we have the generalized uncertainty relation

$$\Delta x\Delta\hat{h}\geq\frac{1}{2\sqrt{\nu}}. \tag{8}$$

Equation (8) is instructive to the precision analysis of a quantum metrology scheme. As an example, we consider the phase estimation in interferometry, in which $x=\phi$ is the phase to be estimated and $\hat{h}=\hat{n}=\hat{a}^\dagger\hat{a}$ is the photon number operator, therefore the precision is directly related to the photon number variance of the probe state $\Delta\phi\geq 1/2\sqrt{\nu\langle\Delta^2\hat{n}\rangle}$. For a coherent state $|\alpha\rangle$, the variance of the photon number is $\Delta^2\hat{n}=|\alpha|^2=N$, where N is the averaged photon number. Therefore, the RMSE is bounded from below by $\Delta\phi\geq 1/2\sqrt{\nu N}$, which is the SQL or shot-noise limit. Yet for the squeezed vacuum state, we have $\Delta^2\hat{n}=2N(N+1)$, and the lower bound of the RMSE is $\Delta\phi\geq 1/2\sqrt{\nu}N$, therefore the sub-shot-noise limit or even the HL is achieved.[3,10] Similarly, the HL can also be achieved by interfering a coherent state with a squeezed vacuum state.[11] For a probe with a fixed number of photons, the maximum precision is achieved with the celebrated $N00N$ state[12–14]

$$|\psi_{N00N}\rangle=\frac{1}{\sqrt{2}}\left(|N0\rangle+|0N\rangle\right), \tag{9}$$

which has a photon-number variance of $N^2/4$, allowing us to reach the HL.

2.2. Bounds beyond the generalized uncertainty relation

The generalized uncertainty relation and the QCRB are the most widely used tools in the field of quantum metrology. Yet they have some limitations. One is that the QCRB can only be achieved asymptotically when the repetition number of measurement ν is sufficiently large,[9,15–17] which is not always possible in practice. It is also favorable to have a bound directly related to the averaged photon number $\langle\hat{n}\rangle$[18–20] instead of through the photon number variance $\langle\Delta^2\hat{n}\rangle$ since the relation between $\langle\hat{n}\rangle$ and $\langle\Delta^2\hat{n}\rangle$ can be arbitrary. There have been a lot of efforts on searching for alternative bounds recently. Here we discuss several progresses in this field. Similar to the QCRB, these bounds are also related to the distinguishability of the probe states in the Hilbert space.

A bound in terms of the expectation value of the generator of transformation $\hat{h}$ is given by[21]

$$\Delta x\geq\frac{\kappa}{\nu(\langle\hat{h}\rangle-E_0)}, \tag{10}$$

where E_0 is the energy of a 'ground state' and $\kappa\approx 0.091$. This bound is derived through the quantum speed limit,[22]

which sets a limit on how fast a quantum state can evolve for a given and fixed amount of energy $\langle\hat{h}\rangle - E_0$. A bound similar to Eq. (11) is derived for a more specific situation in which the coherence between different photon-number components of the probe state is ignored.[23]

To resolve the problem that the QCRB is overly optimistic when the number of trials ν is small, the quantum Ziv–Zakai bound (QZZB)[24] is developed by combining the classical ZZB[25] with the Helstrom's theorem.[26] A simplified QZZB is given by

$$\langle\Delta x\rangle \geq \left\{\frac{1}{2}\int_0^{\infty} \mathrm{d}\tau\tau \int_{-\infty}^{+\infty} \min\left[P_X(x)+P_X(x+\tau)\right] \times \left[1-\sqrt{1-F(\hat{\rho}_x^{\otimes\nu},\hat{\rho}_{x+\tau}^{\otimes\nu})}\right]\right\}^{1/2}, \tag{11}$$

where P_X is the prior probability density of x, $\langle\Delta x\rangle$ is the RMSE averaged over P_X, and $F(\hat{\rho}_x^{\otimes\nu},\hat{\rho}_{x+\tau}^{\otimes\nu})$ is the quantum fidelity between $\hat{\rho}_x^{\otimes\nu}$ and $\hat{\rho}_{x+\tau}^{\otimes\nu}$. The QZZB is usually a tighter bound than the QCRB when ν is small. It is also shown that a bound similar to that in Eq. (11) can be derived from the QZZB.[24] Moreover, the QZZB is used to prove that if without the prior information of x, i.e., P_X is a uniform distribution, it is impossible to achieve a super Heisenberg scaling[27] in linear phase estimation.[28]

2.3. Beyond the Heisenberg limit: nonlinear metrology

Equations (8) and (11) suggest that if the Hamiltonian generator $\hat{h}$ is a higher-order term of the photon-number operator $\hat{n}$, i.e., $\hat{h} = \hat{n}^k$ with $k > 1$, the best scaling of precision can be N^{-k} and the super Heisenberg scaling is possible.[29–33] This type of generator can be achieved through the multi-body interactions, which couples the probes non-trivially. To illustrate the physics behind this super Heisenberg scaling, we consider an example of Kerr-like nonlinearity.[34,35] The probe state experiences a self-phase modulation $\exp(\mathrm{i}\chi\hat{n}^2)$, where χ is the nonlinear coefficient. When χ is small, the probe state picks up a phase $\phi = \chi N$, while the best precision to estimate this phase is $\Delta\phi \sim 1/N$. Therefore the best precision to estimate χ is $\Delta\chi = \Delta\phi/N \sim 1/N^2$. In particular, if the probe state is a coherent state, we still have a super Heisenberg scaling $\Delta\chi \sim N^{-1.5}$. Apart from Kerr-like nonlinearities, nonlinear quantum metrology has also been proposed for the Bose–Einstein condensates,[36,37] opto-mechanical systems,[38] and atomic ensembles.[39,40] An experimental demonstration has been achieved with atomic ensembles.[41]

3. Quantum metrology in the real world

To achieve the genuine quantum-enhanced precision in weak signal detection requires sophisticated non-classical probe states, the efficient and accurate generation of which is still challenging with current technologies. Moreover, noises in the interaction processes and the limited efficiency of the detectors further deteriorate the performance of the non-perfect probe states. To implement quantum metrology in the real world, we need to both quantify the effects of these experimental imperfections and find solutions to minimize these effects. The analysis of quantum metrology with the presence of noise requires us to consider the mixed probe states as well as the general dynamical evolution which is not a unitary process. A general framework is given in Ref. [42] with further tools developed in Refs. [43] and [44].

3.1. Effect of inefficiency and optimal state design

Loss is a major source of experimental imperfections in photonic systems. As shown in Section 2, $N00N$ states achieve the optimal precision, the Heisenberg limit, in linear quantum metrology. Yet at the presence of photon loss, this advantage disappears rapidly. Even with one photon lost, the state becomes a completely mixed state, which possesses no phase sensitivity. Thus to maintain its usefulness for phase estimation, all the photons in the $N00N$ state must survive the loss, the probability of which is η^N with $1-\eta$ the total loss in the system. This probability decreases exponentially with the increase of the photon number. Such scaling makes the overall precision worse than the SQL when a moderate loss is present in the system.

The reason that the loss of one photon completely removes the coherence of the $N00N$ state is due to the 'macroscopic' superposition of two well distinguishable photon-number components. Intuitively, one would expect that a state with a superposition of more photon-number components

$$|\psi\rangle = \sum_{M=0}^{N} C_{M,N-M}|M,N-M\rangle \tag{12}$$

will possibly be more tolerable to loss.[45,46] Recent works have performed a full numerical optimization over the probe states,[47–49] and a proof-of-principle experimental demonstration was reported in Ref. [50] for $N = 2$. Yet there are two major drawbacks which make the optimal states unfavorable for practical quantum metrology: their structures strongly depend on the value of the loss, and the optimal POVM to achieve the QFI is hard for implementation. Therefore, to implement quantum metrology in practice we need to find a more universal solution.

One possible solution is the Holland–Burnett (HB) state,[51] which is generated by interfering two Fock states $|N\rangle|N\rangle$ at a 50:50 beam splitter. Fock states can be generated in a heralded manner through parametric nonlinear processes. The QCRB of the HB(N) state can be achieved by a beam splitter followed by photon-number-resolving detectors, which are available with current technologies.[52] The HB(N)

state and its associated optimal measurement are experimentally more feasible than the $N00N$ state and the optimal loss-tolerant states. Yet its performance is not drastically reduced in the presence of loss. Figure 1 shows the QFI of $N00N$ states, HB(N) states, and optimal states. It is clear that both the optimal states and the HB(N) states are more resilient to losses than the $N00N$ states. In a practical setup, the interaction process is not the only one that contributes to the total inefficiency. There are also losses at the state generation and detection stages. The effect of these losses on the precision measurements with the HB(N) states has been studied in Ref. [53]. A feasibility region analysis shows that the detector imperfection is the bottleneck in beating the SQL. It has also been shown that to beat the SQL unambiguously requires the combination of photon sources and detectors with highest efficiencies to date, which is yet to be attained.

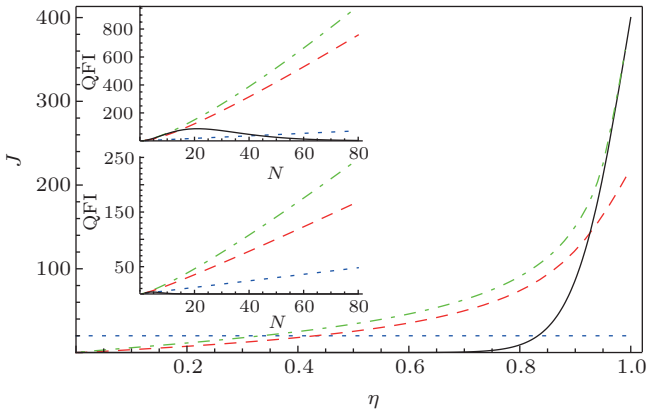


Fig. 1. Quantum Fisher information for different states as a function of the transmissivity η for 20 input photons:[53] blue (dotted): standard quantum limit, red (dashed): HB(10) states, black (solid): $N00N$ states, green (dot-dashed): optimal loss-tolerant states. Inset shows the QFI as a function of the photon number N for $\eta = 0.9$ (top) and $\eta = 0.6$ (bottom).

Probe states with an uncertain photon number, e.g. coherent states and squeezed states, may have different performance compared to the states with a fixed photon number. Analyses on this problem were given in Refs. [54] and [55]. Some results have been used to derive the fundamental bound for the precision of a squeezed-light-enhanced gravitational-wave detector.[55]

3.2. The filtering protocol and resource counting

As shown in the previous section, to beat the SQL unambiguously can be highly demanding, especially the sophisticated probe states, for example, the $N00N$ states, can only be generated in a probabilistic manner with current technology. Therefore, many experimental demonstrations of quantum metrology utilize the filtering or post-selection protocol. Instead of the ideal probe state, those experiments generate states that are more experimentally feasible or even the classical states in nature, and then filter the high-resolution states at the detection stage by selecting particular detector outcomes.[56] This is useful for a proof-of-principle demonstration of the performance of the high-resolution state, since the wavefunction collapse can happen at either the state preparation or detection stage. Yet there is a fundamental difference in the resource counting. Using the filtering protocol, a part of the resources are wasted. The object system is probed with much more resources which do not contribute directly to the parameter estimation, yet they may still, for a delicate sample, cause certain damage to the system.

An important effect in the quantum metrology using the filtering protocol is the phase super resolution, which gives an N-fold increase in the oscillation rate of the sinusoidal detection signal as a function of the phase ϕ to be estimated. It is shown that the visibility of the oscillating fringes distinguishes between the quantum and classical probe states.[57] Phase super sensitivity, which takes into account the preparation efficiency of the probe state as well as the oscillation rate and the visibility of the detection signal, is proposed as a measure of the performance of a quantum metrology setup.[56] Yet this measure neglects the losses in the interaction and detection processes, therefore still overestimates the measurement precision.[58] The proper estimation of the measurement precision involves the calculation of the QFI for the prepared probe state, not the post-selected one.

3.3. The effect of dephasing noise

Quantum metrology is also proposed in the high precision spectroscopy to improve the frequency standard.[12,59] In this situation, a highly entangled state such as the $N00N$ state can also provide an HL scaling in a perfect experimental environment. Yet in such a system, a common source of decoherence is the Markovian dephasing. The $N00N$ state still offers a factor of $\sqrt{N}$ enhancement compared to the uncorrelated probes in the measurement of phase $\phi = \omega t$, where t is the measurement time. Thus for given an uncertainty in ϕ, the longer the measurement time t is, the more precision can be obtained in the estimation of the frequency ω. Nonetheless, the entangled state is exponentially more sensitive to the dephasing noise than the uncorrelated probe states, therefore the optimal measurement time for the entangled state is much shorter than that with the uncorrelated probe states, which cancels the enhancement in the measurement precision in ϕ. Again, a partially entangled state can be more tolerable to the dephasing noise, yet the best achievable scaling is $1/\sqrt{N}$, the SQL.[59] Recently, it has been shown that in the presence of the non-Markovian dephasing, the entangled probe states can still beat the SQL.[60]

4. Applications of quantum metrology

The early stage of the development of quantum metrology was focused on achieving the precision measurements be-

yond the shot-noise limit with the squeezed states of light. Such precision enhancements were demonstrated more than twenty-five years ago for both the detections of small phase modulation[61] and amplitude modulation,[62] and have been employed later for laser Doppler anemometry[63] as well as optical heterodyne detection.[64] It has also been proposed to utilize the squeezed light to improve the performance of a gravitational-wave detector,[65,66] which has been implemented recently[67] and sub-shot-noise sensitivities have been demonstrated in the real GEO 600 gravitational wave detection system.[68,69] More than phase sensing, a quantum-enhanced phase tracking system has also been demonstrated with the squeezed light.[70]

A lot of efforts have also been made on the generation of photonic $N00N$ states and a super resolution with high visibility has been demonstrated.[71–81] Yet those experiments all employed the filtering protocol, i.e., the $N00N$ states were generated in a post-selected manner. The generation of a photonic $N00N$ state in a deterministic or heralded manner requires either a complex linear-optical network,[87–91] or large nonlinearity[92] or actively controlled cavities.[93] In contrast, a scalable route to prepare a highly pure HB(N) state has been proposed and demonstrated.[58] An alternative quantum metrology scheme with HL scaling has been demonstrated by passing a single-photon superposition state through a lossless phase shifter N times,[4,94–96] which has also been utilized in a quantum-enhanced magnetometer.[97]

A state similar to the $N00N$ state in a Ramsey interferometer is referred to as the spin-squeezed state[98,99] or the Schrödinger-cat state,[100–102] which have been demonstrated in ion-trap systems,[100–102] Bose–Einstein condensates,[103] nuclear spins,[104] and atomic ensembles.[105–107] A measurement precision beyond SQL has been achieved with these states.[101,104,105]

5. Concluding remarks

Over last decades we have greatly improved our knowledge about the advantages and limitations of quantum metrology. Many proof-of-principle experimental demonstrations have confirmed the great potential of quantum metrology. Yet to fully explore its potential in the real world requires the development of the generation, manipulation, and detection of sophisticated quantum states as well as numerical estimation algorithms. The design of a practical quantum metrology system should try to balance the enhancement in performance and its experimental feasibility. In particular, experimental imperfections must be taken into account in the design. It has been shown that with current technologies it is possible to beat the standard quantum limit unambiguously. With further improvement of the quantum technologies, we can expect that quantum metrology will play a more and more important role in precision measurements.

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