Implementation of quantum partial search with superconducting quantum interference device qudits in cavity QED*

Li Hong-Yi(李虹轶)†, Wu Chun-Wang(吴春旺), Chen Yu-Bo(陈玉波), Lin Yuan-Gen(林源根), Chen Ping-Xing(陈平形), and Li Cheng-Zu(李承祖)

College of Science, National University of Defense Technology, Changsha 410073, *China*

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We present a method to implement the quantum partial search of the database separated into any number of blocks with qudits, D-level quantum systems. Compared with the partial search using qubits, our method needs fewer iteration steps and uses the carriers of the information more economically. To illustrate how to realize the idea with concrete physical systems, we propose a scheme to carry out a twelve-dimensional partial search of the database partitioned into three blocks with superconducting quantum interference devices (SQUIDs) in cavity QED. Through the appropriate modulation of the amplitudes of the microwave pulses, the scheme can overcome the non-identity of the cavity–SQUID coupling strengths due to the parameter variations resulting from the fabrication processes. Numerical simulation under the influence of the cavity and SQUID decays shows that the scheme could be achieved efficiently within current state-of-the-art technology.

Keywords: Grover search, quantum partial search, qudit, superconducting quantum interference device (SQUID)

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1. Introduction

Grover's algorithm[1] searching target elements in an unsorted database is one of the most celebrated applications of quantum information processing. It can provide a quadratic speedup over the best classical algorithm and has been shown to be optimal.[2] However, if we do not need the accurate location of the target item in practice, partial search algorithm[3,4] which finds a part of the database (a block) containing the target item, instead of the item itself, is applicable in this case and can work faster than a full search. For this reason, the partial search recently attracted much interest. It was suggested by Grover and Radhakrishnan[5] and optimized by Korepin.[4] Then, for a large number of blocks, a simple partial search algorithm was proposed.[6] The partial search algorithm was formulated in terms of group theory in Ref. [7], which provides a powerful mathematical tool to prove the optimality in a wide class of partial search algorithms. For being subroutines of larger algorithms, a sure success partial search algorithm was presented in Ref. [8].

In practice, the size of the database to be checked with the partial search and the number of blocks that the database is partitioned into can be arbitrary. However, if we use the two-dimensional (2D) information carriers, qubits, as usual to encode the database, the size of the database and the number of blocks must be the power of two, which could not match practical problems well. For example, a merit list consists of a ranking of students in a class sorted by the grade (say 39 students in the class). We may roughly want to know where a particular student stands, whether he/she ranks in the top $1/3$, the middle $1/3$, or the bottom $1/3$. For this issue, with qubits we cannot directly construct a partial search algorithm fitting in with the problem. Furthermore, if N items need to be searched with N being a positive integer, then we need to construct a database of size $M = 2^k$ with $k = \lceil \log_2 N \rceil$ qubits to index the searched items. Thus, in this case, $\pi\sqrt{M}/4 - c\sqrt{b}$ queries[4] are required to find the target block with b being the block size and c a positive coefficient. However, if we can establish a database of size M in the physical systems and efficiently perform the partial search in this database, only $\pi\sqrt{N}/4 - c\sqrt{b}$ queries need to be performed resulting in fewer operations (assuming the block size b is not changed). In this work, we present a method to carry out the partial search for the arbitrary size database separated into any number of blocks using multi-level systems, based on a previous idea for a fast quantum search algorithm with qudits.[9] Employing the higher-dimensional quantum states, with the same amount of physical resources, the available Hilbert space could increase exponentially and the carriers of the information are used more economically.

Recently, superconducting devices, such as Cooper pair boxes,[10,11] superconducting quantum interference devices (SQUIDs),[12,13] and Josephson junctions,[14–16] have attracted great interest due to their potential for scalability and compatibility to conventional electronics.[17] Among the superconducting devices, SQUIDs are a promising candidate to serve as the carriers of the information for quantum computing.[18,19] They have the favorable features of relatively long decoherence time,[20,21] design flexibility, and so

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†Corresponding author. E-mail: hongyili@nudt.edu.cn

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on. When they are embedded in a cavity, the strong-coupling limit of the cavity QED, which is difficult to achieve with atoms in a cavity, can be easily realized.[22]

Aiming to illustrate how to realize our idea in a concrete quantum system, we design a scheme with SQUIDs in cavity QED for the implementation of quantum partial search with qudits. For simplicity, but without loss of generality, a twelve-dimensional partial search of the database separated into three blocks is demonstrated using a qutrit, a three-level quantum system, and two qubits. When fabricating the superconducting devices, parameter nonuniformity unavoidably occurs. However, through the appropriate modulation of the amplitudes of the microwave pulses, our scheme could overcome the coupling strength difference of the SQUIDs to the cavity resulting from the parameter variations. Furthermore, since the cavity field is only required to be virtually excited, the system is insensitive to the cavity decay which releases the experimental requirement. To check the validity of our scheme more strictly, we numerically simulate the time evolution of the system under the influence of the cavity and SQUID decays and show that the scheme could be achieved efficiently with enough fidelity under the present experimental technology.

2. Implementation of quantum partial search algorithm in arbitrary dimensional Hilbert spaces

2.1. Implementation of Grover algorithm in arbitrary dimensional Hilbert spaces

First, let us briefly review the standard Grover algorithm. The aim of the Grover algorithm is to identify a target state $|\tau\rangle$ among an unordered set of N states, which could be denoted by $|\Psi\rangle = (1/\sqrt{N})\sum_{i=0}^{N-1}|i\rangle$, where the computational states $|i\rangle$ $(i = 0,1,\ldots,N-1)$ represent the items to be checked. To find the target state, the Grover iteration, which consists of two operations,[23] is repeatedly performed on the entire database: (i) invert the amplitude of the target item $|\tau\rangle$ using a quantum phase gate $\boldsymbol{I}_\tau = \boldsymbol{I} - 2|\tau\rangle\langle\tau|$, where I is the $N\times N$ identity matrix; (ii) invert about the average of the amplitudes of all states using the diffusion transform $D_{ij} = 2/N$ for $i \neq j$ and $D_{ii} = -1 + 2/N$. After the Grover iteration $\boldsymbol{G} = \boldsymbol{D}\boldsymbol{I}_\tau$ is carried out $O(\sqrt{N})$ times, the probability of finding the marked state will be maximized, and then a measurement on the whole system is made to obtain the marked item with high probability.

2.1.1. Implementation of Grover algorithm with qubits

For a database of size $N = 2^n$, encoded by n two-level physical systems, qubits as usual, the Grover search could be completed via n-qubit controlled phase gate $\boldsymbol{U}_{\rm cp}^n$ and n-qubit Hadamard gate $\boldsymbol{H}^{\otimes n}$. $\boldsymbol{U}_{\rm cp}^n = \mathrm{diag}\{1,\ldots,1,-1\}$ is a $2^n\times 2^n$ diagonal matrix and

$$\boldsymbol{H}^{\otimes n} = \bigotimes_{i=1}^{n} H_i = \bigotimes_{i=1}^{n}\left(\frac{1}{\sqrt{2}}\begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix}\right), \tag{1}$$

where H_i is the Hadamard gate acting on the i-th qubit, transforming states $|0\rangle$ and $|1\rangle$ to $(1/\sqrt{2})(-|0\rangle + |1\rangle)$ and $(1/\sqrt{2})(|0\rangle + |1\rangle)$, respectively. The average state $|\Psi\rangle$ defined in the N-dimensional Hilbert space spanned by $\{|0_i\rangle, |1_i\rangle | i = 1,2,\ldots,n\}$ can be obtained via the n-qubit Hadamard gate: $|\Psi\rangle = \boldsymbol{H}^{\otimes n}|1_1 1_2\cdots 1_n\rangle$. The operations of inverting the amplitude of the marked item are carried out by $\boldsymbol{U}_{\rm cp}^n$ combined with single-qubit gates $\{\sigma_{x,i} | i = 1,2,\ldots,n\}$, where $\sigma_{x,i}$ is the NOT gate acting on the i-th qubit. The diffusion transform could be reached by combining the Hadamard gate with the controlled phase gate: $\boldsymbol{D} = -\boldsymbol{H}^{\otimes n}\boldsymbol{U}_{\rm cp}^n\boldsymbol{H}^{\otimes n}$.

2.1.2. Implementation of Grover algorithm with qudits

As a component of the partial search, the Grover search in databases of arbitrary size is required to realize the partial search for databases of arbitrary size. A method to implement the Grover algorithm in arbitrary-dimensional Hilbert spaces was given in Ref. [9]. For a database of arbitrary size N, if N can be decomposed into $N = N_1 N_2 \ldots N_m$, then the database could be encoded by N_1-, N_2-, ..., N_m-dimensional qudits. The Grover search in this database of size N can be carried out through m-qudit controlled phase gate $U_{\rm cp}^N$ and generalized Hadamard gates $\{U_{N_j} | j = 1,2,\ldots,m\}$. $\boldsymbol{U}_{\rm cp}^N$ is the same as $\boldsymbol{U}_{\rm cp}^n$ except that it is an $N\times N$ diagonal matrix, and $\boldsymbol{U}_{N_j}$ can be decomposed into a product of two-level unitary matrices in the individual qudit: $\boldsymbol{U}_{N_j} = \boldsymbol{U}_{N_j 1}\boldsymbol{U}_{N_j 2}\ldots\boldsymbol{U}_{N_j(N_j-1)}$, whose formalism, a little complicated, was given in Ref. [9]. $\boldsymbol{U}_{N_j i}$ for the j-th qudit, denoted by $\{|0\rangle_j, |1\rangle_j, \ldots, |N_j - 1\rangle_j\}$, acts on the subspace spanned by $\{|i-1\rangle_j, |i\rangle_j\}$, transforming states as

$$\begin{aligned} |i-1\rangle_j &\rightarrow -\sqrt{\frac{1}{i+1}}|i-1\rangle_j + \sqrt{\frac{i}{i+1}}|i\rangle_j,\\ |i\rangle_j &\rightarrow \sqrt{\frac{i}{i+1}}|i-1\rangle_j + \sqrt{\frac{1}{i+1}}|i\rangle_j. \end{aligned} \tag{2}$$

By the above transforms, the preparation of the average state $|\Psi\rangle$ can be achieved by

$$|\Psi\rangle = \bigotimes_{i=1}^{m}\left(\prod_{j=1}^{N_i-1}\boldsymbol{U}_{N_i,j}|N_i - 1\rangle\right). \tag{3}$$

The inversion of the amplitude of any target state I_τ can be accomplished by the conditional phase gate $\boldsymbol{U}_{\rm cp}^N$ combined with swapping operations between different levels in the individual qudits. The diffusion transform could be obtained by $\boldsymbol{D} = -\boldsymbol{U}_N\boldsymbol{U}_{\rm cp}^N\boldsymbol{U}_N^\dagger$, similar to the qubit situation, where $\boldsymbol{U}_N$ is given by $\boldsymbol{U}_N = \boldsymbol{U}_{N_1}\otimes\boldsymbol{U}_{N_2}\otimes\cdots\otimes\boldsymbol{U}_{N_m}$.

2.2. Implementation of quantum partial search algorithm with qudits

The partial search algorithm considers the following problem: a database containing N items is partitioned into K blocks each with the same size $b = N/K$, and the aim is to find the block with the target item, called target block, instead of the target item itself. The partial search, which is essentially based on the Grover algorithm, consists of the global Grover iteration $\boldsymbol{G}_{\rm g}$ performed on the entire database and the local Grover iteration $G_{\rm l}$ performed on each block of the database in parallel.

The operation steps of the partial search algorithm are given below.

Step 1 Prepare a superposition state $|\Psi\rangle$ as an unsorted database and apply $l_{\rm g}$ global iterations on it. Then, the state of the database is $|\Psi_1\rangle = (\boldsymbol{G}_{\rm g})^{l_{\rm g}}|\Psi\rangle$.

Step 2 Applying $l_{\rm l}$ local iterations on $|\Psi_1\rangle$, we can obtain $|\Psi_2\rangle = (\boldsymbol{G}_{\rm l})^{l_{\rm l}}(\boldsymbol{G}_{\rm g})^{l_{\rm g}}|\Psi\rangle$.

Step 3 A final global iteration on $|\Psi_2\rangle$. The resulting state is $|\Psi_3\rangle = \boldsymbol{G}_{\rm g}(\boldsymbol{G}_{\rm l})^{l_{\rm l}}(\boldsymbol{G}_{\rm g})^{l_{\rm g}}|\Psi\rangle$.

After the partial search, the states in non-target blocks have amplitude of zero and the state measured is the item in the target block. The algorithm uses $l_{\rm g}+1$ global iterations and $l_{\rm l}$ local iterations. Korepin optimized this procedure[4] and gave the numbers of the global and local iterations that minimize the total number of iterations $l_{\rm g}+l_{\rm l}+1$. The numbers of iterations $l_{\rm g}$ and $l_{\rm l}$ for large b ($b\to\infty$) are given as follows:

$$l_{\rm g} = \frac{\pi}{4}\sqrt{N} - \eta\sqrt{b}, \qquad l_{\rm l} = \alpha\sqrt{b}. \tag{4}$$

Here η and α are parameters of order of 1 and can be expressed as functions of number of blocks K

$$\tan\frac{2\eta}{\sqrt{K}} = \frac{\sqrt{3K-4}}{K-2}, \qquad \cos 2\alpha = \frac{K-2}{2(K-1)}. \tag{5}$$

The minimized total number of iterations is $\pi\sqrt{N}/4 - c\sqrt{b}$ with $c = \eta - \alpha$. Compared with the standard Grover search, the partial search reduces the queries to the oracle by $c\sqrt{b}$. For instance, for $K = 3$, $\alpha \approx 0.66$, $\eta \approx 1$, and $c \approx 0.34$.

The implementation of standard Grover algorithm in arbitrary size databases is shown above. The local Grover search could be expressed by $\boldsymbol{G}_{\rm l} = \boldsymbol{D}_{\rm l}\boldsymbol{I}_\tau$, where $\boldsymbol{I}_\tau$ is the same as the inversion of the amplitude of the target state in standard Grover search, while the local diffusion transform is defined by

$$\boldsymbol{D}_{\rm l} = \bigoplus_{i=1}^{K} \boldsymbol{D}_i, \tag{6}$$

with $\boldsymbol{D}_i$ representing the diffusion transform in the i-th block. It is a direct sum of diffusion transforms defined in each block. In the following, we realize the local diffusion transform with qudits by a Kronecker product. For the database of size $N = Kb$, assuming that K, b can be decomposed into $K = K_1K_2\cdots K_m$ and $b = b_1b_2\cdots b_n$, then the database could be encoded by K_1, K_2, …, K_m, b_1, b_2, …, b_n-dimensional qudits and the local average states in the blocks can be described by

$$|k_1\rangle_1|k_2\rangle_2\cdots|k_m\rangle_m \bigotimes_{i=1}^{n}\left(\frac{1}{\sqrt{b_i}}\sum_{j=0}^{b_i-1}|j\rangle_{m+i}\right),$$
$$k_l = 0,1,\ldots,K_l-1; \quad l = 1,2,\ldots,m. \tag{7}$$

We define a block diffusion transform $\boldsymbol{D}_{\rm b} = -\boldsymbol{U}_{\rm b}\boldsymbol{U}_{\rm cp}^{\rm b}\boldsymbol{U}_{\rm b}^{\dagger}$ in the block subspace spanned by $\{|0\rangle_{m+i}, |1\rangle_{m+i}, \ldots, |b_i-1\rangle_{m+i}\ |i = 1,2,\ldots,n\}$, where $\boldsymbol{U}_{\rm b} = \boldsymbol{U}_{b_1}\otimes\boldsymbol{U}_{b_2}\otimes\ldots\otimes\boldsymbol{U}_{b_n}$ and $\boldsymbol{U}_{\rm cp}^{\rm b}$, the n-qudit controlled phase gate, is a $b\times b$ diagonal matrix defined in the block space. The local operations in the subspace can be realized via selectively manipulating the qudits. For the SQUIDs embedded in a cavity, we could adjust the level spacings of SQUIDs such that there are no interactions between the cavity mode and the SQUIDs not in the block space. Now, the local Grover iteration can be expressed as

$$\boldsymbol{G}_{\rm l} = (\boldsymbol{I}_K\otimes\boldsymbol{D}_{\rm b})\boldsymbol{I}_\tau, \tag{8}$$

where $\boldsymbol{I}_K$ is the $K\times K$ identity matrix defined in the subspace spanned by $\{|0\rangle_i, |1\rangle_i, \ldots, |K_i-1\rangle_i\ |i = 1,2,\ldots,m\}$. So far, we obtain all the operations to accomplish the partial search for the arbitrary size database separated into any number of blocks. In what follows, we show how to realize our idea in a concrete physical system by designing a scheme with SQUIDs in cavity QED to implement a twelve-dimensional partial search of the database partitioned into three blocks using a qutrit and two qubits.

3. The scheme with SQUIDs in cavity QED

The SQUIDs to be discussed below are rf SQUIDs, each consisting of an enclosed superconducting loop interrupted by a Josephson junction. Consider three SQUIDs simultaneously interacting with a single-mode cavity field and driven by classical microwave pulses. The SQUIDs are biased properly to have identical four-level configurations of three lowest states $|0\rangle$, $|1\rangle$, and $|2\rangle$, and an excited state $|e\rangle$ (see Fig. 1).[24–26] The qutrit is denoted by $|0\rangle$, $|1\rangle$, and $|2\rangle$ of SQUID I, and the two qubits are denoted by $|0\rangle$ and $|1\rangle$ of SQUID II and SQUID III. As shown in Fig. 1, the SQUID transition $|1\rangle \leftrightarrow |e\rangle$ is non-resonantly coupled by the cavity mode with coupling strength g_i and detuning δ, and driven by the classical field with the Rabi frequency Ω_i and detuning Δ, respectively. The states $|0\rangle$ and $|2\rangle$ are not affected by the cavity and classical fields. The Hamiltonian for the SQUID with junction capacitance C, loop inductance L, and externally applied biased flux Φ_x can be written in the usual form[27–29]

$$H_{\rm s} = \frac{Q^2}{2C} + \frac{(\Phi-\Phi_x)^2}{2L} - E_{\rm J}\cos\left(2\pi\frac{\Phi}{\Phi_0}\right), \tag{9}$$

where Φ is the magnetic flux threading the loop, Q is the total charge on the capacitor, and $E_{\rm J} = I_{\rm c}\Phi_0/2\pi$ is the Josephson

coupling energy with I_c being the critical current of the junction and $\Phi_0 = h/2e$ the flux quantum. The Hamiltonian of the three SQUIDs interacting simultaneously with a single cavity mode and microwave pulses in the interaction picture is described by ($\hbar = 1$)

$$H_{\text{total}} = H_c + H_{\mu w}, \tag{10}$$

with

$$H_c = \sum_{j=1}^{3} g_j \left(a^{\dagger} \sigma_j^{-} \, e^{i\delta t} + a \sigma_j^{+} \, e^{-i\delta t} \right), \tag{11}$$

$$H_{\mu w} = \sum_{j=1}^{3} \Omega_j \left(\sigma_j^{-} \, e^{i\Delta t} + \sigma_j^{+} \, e^{-i\Delta t} \right), \tag{12}$$

where the subscript j represents the j-th SQUID, and a and $a^{\dagger}$ are, respectively, the annihilation and creation operators for the cavity mode. Here, $\sigma_j^{+} = |e_j\rangle\langle 1_j|$, $\sigma_j^{-} = |1_j\rangle\langle e_j|$, and the expressions of g_j and Ω_j are given, respectively, by[28,30]

$$g_j = \frac{1}{L_j} \sqrt{\frac{\omega_c}{2\mu_0 \hbar}} \langle 1_j | \Phi_j | e_j \rangle \int_{S_j} \boldsymbol{B}_c^j(\boldsymbol{r}) \cdot d\boldsymbol{S},$$
$$\Omega_j = \frac{1}{\hbar L_j} \langle 1_j | \Phi_j | e_j \rangle \int_{S_j} \boldsymbol{B}_{\mu w}^j(\boldsymbol{r}) \cdot d\boldsymbol{S}, \tag{13}$$

with ω_c being the cavity frequency, L_j the inductance of the j-th SQUID loop, μ_0 the vacuum permeability, $\boldsymbol{B}_c^j(\boldsymbol{r})$ and $\boldsymbol{B}_{\mu w}^j(\boldsymbol{r})$ the magnetic components of the cavity mode and classical microwave in the superconducting loop of the j-th SQUID, respectively.

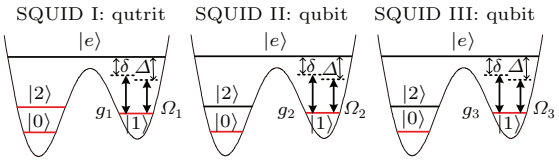


Fig. 1. (color online) Schematic illustration of relevant energy-level structure of the three SQUIDs embedded in a cavity. The SQUID transition $|1\rangle \leftrightarrow |e\rangle$ is nonresonantly coupled to the quantized cavity mode and the classical field with the coupling strengths g and Ω, respectively. The states $|0\rangle$ and $|2\rangle$ are not affected by the cavity and classical fields. The qutrit is denoted by SQUID I, and the two qubits are denoted by SQUID II and SQUID III.

In the absence of the classical field, the system interaction Hamiltonian is given by H_c. Considering the large SQUID–cavity detuning condition $\delta \gg g_i$ ($i = 1,2,3$), we can obtain an effective Hamiltonian[31]

$$H_c^1 = \frac{1}{\delta} a^{\dagger} a \sum_{i=1}^{3} g_i^2 \sigma_i^{-} \sigma_i^{+} - \frac{1}{\delta} a a^{\dagger} \sum_{i=1}^{3} g_i^2 \sigma_i^{+} \sigma_i^{-} - \frac{1}{\delta} \sum_{\substack{i,j=1 \\ i \neq j}}^{3} g_i g_j \sigma_i^{+} \sigma_j^{-}. \tag{14}$$

Due to the strong detuning between the cavity and SQUIDs, if the cavity field is initially in the vacuum state, then photons in the cavity mode play a negligible role in the temporal evolution, and the effective Hamiltonian in Eq. (14) can be reduced to[32]

$$H_c^2 = -\frac{1}{\delta} \sum_{i=1}^{3} g_i^2 \sigma_i^{+} \sigma_i^{-} - \frac{1}{\delta} \sum_{\substack{i,j=1 \\ i \neq j}}^{3} g_i g_j \sigma_i^{+} \sigma_j^{-}. \tag{15}$$

Then, take the weak external driving field into account and adjust the Rabi frequencies so that $\Omega_i = \eta g_i$ ($i = 1,2,3$) with η being a positive real number. To this end, if the non-identity of coupling strengths g_i merely results from position inaccuracy of the SQUIDs, we could let the microwave pulse input collinearly with the cavity axis. In this case the driving pulse has the same spatial mode structure as the cavity mode.[33,34] Then, according to Eqs. (13), Ω_i/g_i will be a constant. However, if the non-identity of SQUID–cavity coupling strengths g_i results from the the non-uniformity in device parameters or combined with position inaccuracy mentioned just now, the SQUIDs should be addressed individually by the classical pulses with appropriate intensity. In this condition the full Hamiltonian for the dynamics of the whole system can be redefined as

$$H = -\frac{1}{\delta} \sum_{i=1}^{3} g_i^2 \sigma_i^{+} \sigma_i^{-} - \frac{1}{\delta} \sum_{\substack{i,j=1 \\ i \neq j}}^{3} g_i g_j \sigma_i^{+} \sigma_j^{-} + \eta \sum_{j=1}^{3} g_j \left(\sigma_j^{-} \, e^{i\Delta t} + \sigma_j^{+} \, e^{-i\Delta t} \right). \tag{16}$$

The SQUIDs are initially prepared in the computational basis states $\{|h_1 h_2 h_3\rangle\}$ where $h_1 = 0,1,2$ and $h_2, h_3 = 0,1$. When the SQUIDs are in the states $|000\rangle$ and $|200\rangle$, they do not evolve with the time. Thus, we only need to consider the temporal evolution of the states in the form

$$|\Phi\rangle_{i_1,i_2,\ldots,i_p} = |\cdots 1_{i_1} \cdots 1_{i_2} \cdots 1_{i_p} \cdots\rangle, \tag{17}$$

where the SQUIDs $i_1, i_2, \ldots, i_p$ ($p = 1,2,3$) are initially in the state $|1\rangle$ and other SQUIDs are in the state $|0\rangle$ or $|2\rangle$.

Since the states $|0\rangle$ and $|2\rangle$ do not participate in the interaction, the Hamiltonian in Eq. (16) can be spanned by the finite basic states[35]

$$|\Psi_{p,q}\rangle = \frac{1}{q! C_{p,q}} J^q_{i_1,\cdots,i_p,+} |\Phi\rangle_{i_1,i_2,\cdots,i_p}, \quad (0 \leq q \leq p), \tag{18}$$

where

$$J_{i_1,\ldots,i_p,+} = \sum_{j=i_1,\ldots,i_p} g_j |e_j\rangle\langle 1_j|,$$
$$C_{p,q} = \sqrt{\sum_{\substack{j_1,j_2,\ldots,j_q \in \{i_1,\ldots,i_p\} \\ j_1 \neq j_2 \neq \cdots \neq j_q}} \left(g_{j_1} g_{j_2} \cdots g_{j_q}\right)^2}. \tag{19}$$

The state $|\Psi_{p,q}\rangle$ describes that q SQUIDs transfer into the state $|e\rangle$ from $|1\rangle$. As two special cases, $|\Psi_{p,0}\rangle$ is given by Eq. (17) and $|\Psi_{p,p}\rangle = |\cdots e_{i_1} \cdots e_{i_2} \cdots e_{i_p} \cdots\rangle$ describes that the SQUIDs $i_1, i_2, \ldots, i_p$ ($p = 1,2,3$) occupy the state $|e\rangle$ and other

SQUIDs are in the state $|0\rangle$ or $|2\rangle$. In these basic states, the Hamiltonian H can be rewritten as

$$H_{\rm p} = -\frac{1}{\delta}\sum_{q=0}^{p}\omega_{p,q}\left|\Psi_{p,q}\right\rangle\left\langle\Psi_{p,q}\right| + \left(\sum_{q=0}^{p-1}\lambda_{p,q}\left|\Psi_{p,q+1}\right\rangle\left\langle\Psi_{p,q}\right|\mathrm{e}^{-\mathrm{i}\Delta t}+\mathrm{H.c.}\right), \quad (20)$$

where

$$\omega_{p,0}=0, \quad \omega_{p,q}=C_{p,1}^2+\left(q^2-1\right)\frac{C_{p,q+1}^2}{C_{p,q}^2},$$
$$\lambda_{p,q}=(q+1)\,\eta\,\frac{C_{p,q+1}}{C_{p,q}}. \quad (21)$$

To realize the three-qudits phase gate, we let the parameters η and Δ satisfy the following conditions:

$$\Delta=-\frac{\omega_{3,1}}{\delta}=-\frac{g_1^2+g_2^2+g_3^2}{\delta}, \quad (22a)$$
$$\left|-\frac{\omega_{p,q+1}}{\delta}+\frac{\omega_{p,q}}{\delta}-\Delta\right|\gg\lambda_{p,q}, \quad (p,q\neq 3,0). \quad (22b)$$

The physical idea behind these equations is that Eq. (22a) leads to the resonant transition between $|\Psi_{3,0}\rangle$ and $|\Psi_{3,1}\rangle$, while Eq. (22b) makes other transitions largely detuned. Through this selective resonant interaction,[36] we have the effective interaction

$$H_{\rm eff}=\lambda_{3,0}\left|\Psi_{3,1}\right\rangle\left\langle\Psi_{3,0}\right|+\mathrm{H.c.},$$
$$\lambda_{3,0}=\sqrt{\Omega_1^2+\Omega_2^2+\Omega_3^2}, \quad (23)$$

for the initial state where all three SQUIDs are in the state $|1\rangle$. For other initial states, similar to the above analysis, the effective interaction is dispersive from Eq. (22b). Choosing the interaction time $t=\pi/\lambda_{3,0}$, the state $|111\rangle$ will acquire a $\mathrm{e}^{\mathrm{i}\pi}$ phase factor, while other states in computational bases remain unchanged.

To implement the three-qudit phase gate, we define the swap operations between two levels in the single qutrit σ_{ij}: $\sigma_{ij}|i\rangle=|j\rangle$, $\sigma_{ij}|j\rangle=|i\rangle$ $(i,j=0,1,2)$, like the Pauli X operator with the qubits. These operations can be obtained via the Raman processes using two external classical microwave pulses with appropriate frequency,[37] which provide an effective interaction $\Omega_{\rm eff}|i\rangle\langle j|+\mathrm{H.c.}$, where $\Omega_{\rm eff}=\Omega_i\Omega_j/\Delta$, with $\Omega_{i,j}$ being the Rabi frequencies and Δ the detuning from the excited state $|e\rangle$ $(\Delta\gg\Omega_{i,j})$. By the operation from the Hamiltonian (10) under the conditions (22) combined with the single-qutrit gate σ_{12}, we can obtain the controlled phase gate $U_{\rm cp}^{12}$ in the twelve-dimensional Hilbert space

$$\begin{aligned}
&|000\rangle\to|000\rangle, \quad |100\rangle\to|100\rangle, \quad |200\rangle\to|200\rangle,\\
&|001\rangle\to|001\rangle, \quad |101\rangle\to|101\rangle, \quad |201\rangle\to|201\rangle,\\
&|010\rangle\to|010\rangle, \quad |110\rangle\to|110\rangle, \quad |210\rangle\to|210\rangle,\\
&|011\rangle\to|011\rangle, \quad |111\rangle\to|111\rangle, \quad |211\rangle\to-|211\rangle. \quad (24)
\end{aligned}$$

For the controlled phase gate $U_{\rm cp}^4$ acting on the block space spanned by the two qubits, we could adjust, by the externally applied biased flux Φ_x, the level spacing of the SQUID I to be decoupled from the cavity mode and the microwave,[28,38] then repeat the above analysis, yet, for the coupling system involving only the two qubits interacting with the cavity field and microwave pulses. In this case, the interaction time $t'=\pi/\lambda_{2,0}=\pi/(\Omega_2^2+\Omega_3^2)^{1/2}$, and the condition for the resonant transition between $|\Psi'_{2,0}\rangle$ and $|\Psi'_{2,1}\rangle$ is $\Delta'=-\omega'_{2,1}/\delta=-(g_2^2+g_3^2)/\delta$, with $|\Psi'_{2,0}\rangle=|1_2 1_3\rangle$ and $|\Psi'_{2,1}\rangle=1/(g_2^2+g_3^2)^{1/2}(g_2|e_2 1_3\rangle+g_3|1_2 e_3\rangle)$.

The partial search for the database of size 12, partitioned into three blocks, is performed by a local iteration followed by a global iteration[6]

$$|\Psi_{\rm f}\rangle=G_{\rm g}G_{\rm l}|\Psi\rangle, \quad (25)$$

where $|\Psi\rangle$ is the initial average state in the computational bases, $|\Psi_{\rm f}\rangle$ is the final state wherein the components in the blocks not containing the target item have amplitude of zero, and

$$G_{\rm l}=[\boldsymbol{I}_3\otimes(\boldsymbol{U}_4\boldsymbol{U}_{\rm cp}^4\boldsymbol{U}_4^{\dagger})]\boldsymbol{I}_\tau,$$
$$G_{\rm g}=\boldsymbol{U}_{12}\boldsymbol{U}_{\rm cp}^{12}\boldsymbol{U}_{12}^{\dagger}\boldsymbol{I}_\tau, \quad (26)$$

with

$$\boldsymbol{U}_{12}=\boldsymbol{U}_3\otimes\boldsymbol{U}_2\otimes\boldsymbol{U}_2,$$
$$\boldsymbol{U}_4=\boldsymbol{U}_2\otimes\boldsymbol{U}_2. \quad (27)$$

From Eq. (2), we obtain the $\boldsymbol{U}_2$ and $\boldsymbol{U}_3$ transformations for the qubit and qutrit, respectively,

$$\boldsymbol{U}_2=\begin{bmatrix}-\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}}\\ \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}}\end{bmatrix},$$
$$\boldsymbol{U}_3=\boldsymbol{U}_{31}\boldsymbol{U}_{32}=\begin{bmatrix}-\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0\\ \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & -\sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}}\\ 0 & \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}}\end{bmatrix}, \quad (28)$$

where $\boldsymbol{U}_3$ for the qutrit acts on the three-dimensional Hilbert space spanned by $\{|0\rangle,|1\rangle,|2\rangle\}$, $\boldsymbol{U}_{31}$ acts on the subspace spanned by $\{|0\rangle,|1\rangle\}$, $\boldsymbol{U}_{32}$ acts on the subspace spanned by $\{|1\rangle,|2\rangle\}$, and $\boldsymbol{U}_2$ for the qubits acts on the two-dimensional Hilbert space spanned by $\{|0\rangle,|1\rangle\}$. These gates can also be achieved by the above Raman processes for the swap gates σ_{ij}. Based on the conditional phase gate $\boldsymbol{U}_{\rm cp}^{12}=\boldsymbol{I}_{211}$ marking the state $|211\rangle$, we could construct eleven other gates for the marking job, taking the marked state $|001\rangle$ as an example

$$\boldsymbol{I}_{001}=\sigma_{02}^{(1)}\sigma_x^{(2)}\boldsymbol{I}_{211}\sigma_{02}^{(1)}\sigma_x^{(2)}. \quad (29)$$

So far, we have given all the gate operations needed to complete the twelve-dimensional partial search of the database partitioned into three blocks. To carry out our scheme, we consider three SQUIDs embedded in a cavity, simultaneously

interacting with the cavity mode and driven by the classical fields. Taking the marked state $|001\rangle$ as an example, the three SQUIDs are initially prepared in the superposition state through Raman processes using external microwave pulses, then they undergo the operations in Fig. 2 from the left to the right. L denotes the SQUID I beyond the interaction with the cavity mode and classical field, through the adjustment of its level spacing. Finally the SQUIDs are separately read out in the computational bases.

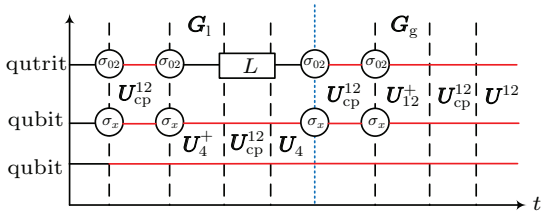


Fig. 2. (color online) Schematic plot for the marked state $|001\rangle$ in a twelve-item partial search of database separated into three blocks, where the three SQUIDs are initially prepared in the average state through Raman processes using external classical fields. The single qubit and qutrit gate σ_x and σ_{02} added on the two ends of the conditional phase gate U_{cp}^{12} are for generating I_{001}. L represents the qutrit beyond the interaction with the cavity mode and classical field. The operations at the left and right side of the dotted line constitute the local and global Grover iterations, respectively.

4. Numerical simulation under dissipations

In the ideal case, the twelve-item partial search of database separated into three blocks is carried out with certainty. However, in the presence of the cavity and SQUID decays, the search is performed only probabilistically. We numerically simulate the partial search by the quantum jump approach[39] under the condition that no photon leakage actually happens, due to either the spontaneous emission from the SQUIDs' excited states or the decay from the cavity mode during the operation. As an example, we demonstrate the search for the marked state $|001\rangle$ (the first block being the target block) with the non-identical SQUID–cavity coupling strengths, in the cases of $\kappa = \gamma = 0$, $\kappa = \gamma = g/300$, and $\kappa = \gamma = g/200$, with κ and γ being the cavity and SQUID decay rates, respectively. Figure 3 shows the probabilistic distribution of the computational bases after the partial search, and under the cavity and SQUID dissipations the states in the non-target blocks appear to be populated.

We also simulate the impact of decays on the success rate and fidelity. Figure 4 plots how the success rate and fidelity simultaneously depend on the cavity decay rate κ and the SQUID spontaneous emission rate γ. Under the limits of large SQUID–cavity detuning and extremely weak external excitation, the cavity decay plays a neglected role in our scheme. In fact, it is not difficult to note that the dominant source of decoherence is the fact that the state $|\Psi_{3,1}\rangle$ ($|\Psi'_{2,1}\rangle$) can decay spontaneously to $|\Psi_{3,0}\rangle$ ($|\Psi'_{2,0}\rangle$) due to the excited state remarkably populated during the temporal evolution. The two decay effects are reflected clearly in Fig. 4. For a set of actual parameters,[28] $(g_1, g_2, g_3, \kappa, \gamma) = (180, 160, 140, 1.32, 0.4)$ MHz, $\delta = 16g_3$, $\Delta = -(g_1^2 + g_2^2 + g_3^2)/\delta$, $\eta = 1/(0.0039\sqrt{3}\delta)$, $\Delta' = -(g_2^2 + g_3^2)/\delta$, and $\eta' = 1/(0.0029\sqrt{2}\delta)$, as the time for single qutrit and qubit operations is negligible, the partial search takes 8.36 μs wherein the local iteration needs $(\pi/\lambda_{3,0} + \pi/\lambda_{2,0}) \sim 3.97$ μs and the global iteration needs $2\pi/(\lambda_{3,0}) \sim 4.39$ μs, and the fidelity and success rate are 0.97 and 0.98, respectively.

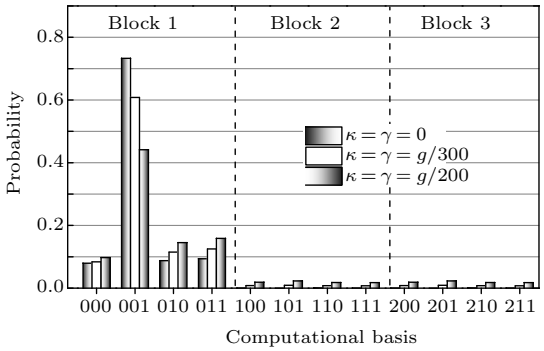


Fig. 3. The probability of the computational basis for the twelve-item partial search of database separated into three blocks. Parameters: $g_1 = 1.4g$, $g_2 = 1.2g$, $g_3 = g$, $\delta = 30g$, $\Delta = -(g_1^2 + g_2^2 + g_3^2)/\delta$, $\eta = 1/(11\sqrt{3}\delta)$, $\Delta' = -(g_2^2 + g_3^2)/\delta$, $\eta' = 1/(10\sqrt{2}\delta)$.

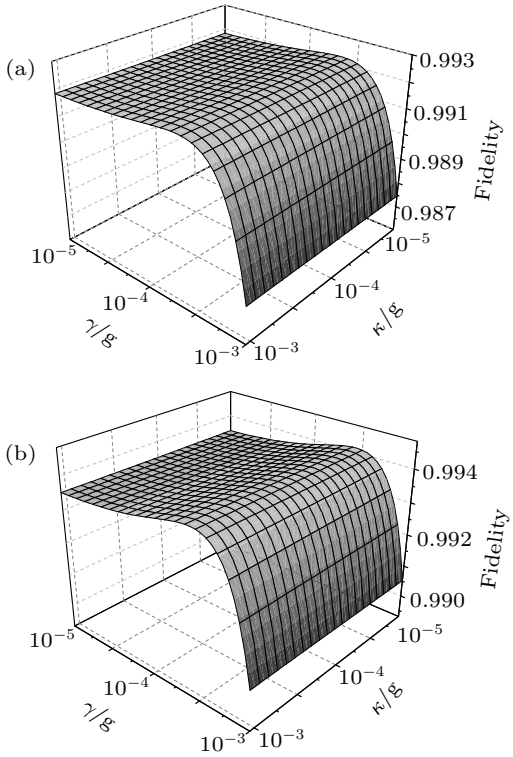


Fig. 4. The fidelity of the final state after the partial search and the probability for finding the states in the target block versus the decay rates. Parameters: $g_1 = 1.4g$, $g_2 = 1.2g$, $g_3 = g$, $\delta = 30g$, $\Delta = -(g_1^2 + g_2^2 + g_3^2)/\delta$, $\eta = 1/(11\sqrt{3}\delta)$, $\Delta' = -(g_2^2 + g_3^2)/\delta$, $\eta' = 1/(10\sqrt{2}\delta)$.

To show that our scheme is tolerable to the inevitable non-uniformity in device parameters, figure 5 compares the fidelity of the final state after the partial search using our method with that regardless of the non-identity of the cavity–SQUID coupling strengths (meaning that the microwave pulses have

the same intensity). We can see that the fidelity of the final state regardless of the non-identity of the cavity–SQUID coupling strengths is highly sensitive to the non-identity of cavity–SQUID coupling constants, however the fidelity using our scheme is hardly affected.

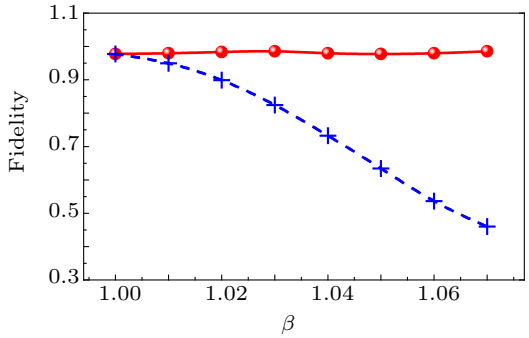


Fig. 5. (color online) Fidelity of the final state after the partial search. The solid line denotes the fidelity of the final state using our scheme, while the dashed line denotes the fidelity regardless of the non-identity of the cavity-SQUID coupling strengths (meaning that the microwave pulses have the same intensity). Parameters: $g_1 = \beta^2 g$, $g_2 = \beta g$, $g_3 = g$, $\delta = 20g_1$, $\Delta = -(g_1^2 + g_2^2 + g_3^2)/\delta$, $\eta = 1/(11\sqrt{3}\delta)$, $\Delta' = -(g_2^2 + g_3^2)/\delta$, $\eta' = 1/(10\sqrt{2}\delta)$.

5. Conclusion

In conclusion, we have suggested a method to implement the partial search for the arbitrary size database partitioned into any number of blocks using multi-level systems. With this method fewer iteration steps are needed and carriers of the information are used more economically compared with the partial search with qubits. To crystalize the idea, we have proposed a scheme for the implementation of a twelve-item partial search of the database separated into three blocks using a qutrit and two qubits with SQUIDs in cavity QED. Through the appropriate modulation of the amplitudes of the microwave pulses, our scheme could overcome the non-identity of the cavity–SQUID coupling strengths. In addition, since the cavity field is only required to be virtually excited, the system is insensitive to the cavity decay which releases the experimental requirement. Numerical simulation under the influence of the cavity and SQUID decays shows that the scheme could be achieved with high fidelity under current state-of-the-art technology.

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