Deterministic assisted clone of an unknown four-particle entangled cluster-type state*

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Two deterministic schemes are proposed to realize the assisted clone of an unknown four-particle entangled cluster-type state. The schemes include two stages. The first stage requires teleportation via maximal entanglement as the quantum channel. In the second stages of the protocols, two novel sets of mutually orthogonal basis vectors are constructed. With the assistance of the preparer through a four-particle or two-step two-particle projective measurement under these bases, the perfect copy of an original state can be produced. Comparing with the previous protocols which produce the unknown state and its orthogonal complement state at the site of the sender, the proposed schemes generate the unknown state deterministically.

Keywords: deterministic assisted clone, complete orthogonal basis, teleportation, remote preparation

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1. Introduction

One of the fundamental nonclassical properties of quantum systems is that an unknown quantum state cannot be cloned exactly. It is described by the celebrated no-cloning theorem.[1] However, approximate quantum cloning is necessary in quantum information and is applicable to other interesting quantum processes, e.g., quantum network coding,[2] etc. Although exact cloning is impossible, various cloning machines have been proposed in the literature, which operate in either a deterministic or probabilistic way.[3–8]

In 2000, Pati proposed a scheme for cloning an unknown single-qubit state with minimal assistance from a state preparer.[9] The perfect copies and their orthogonal-complement copies of the unknown state can be produced at the site of the sender. Inspired by Pati's original paper, a lot of schemes of assisted cloning have been proposed.[10–18] Those protocols have some features in common. One is that projective measurements are used by the sender to teleport the unknown state to the receiver. The other is that by making use of the leftover entangled resource after the teleportation,[19–22] the sender establishes a quantum channel between the preparer and her, and then the preparer carries out the remote state preparation (RSP).[23–28] However, these assisted-clone protocols succeed with a finite probability that is less than one, which means that they cannot always generate the perfect copy of the unknown state deterministically.

Recently, Xiao *et al.* investigated the assisted clone of an arbitrary three-qubit state.[18] To our knowledge, there is no scheme for assisted cloning an unknown four-particle state so far. It is known that a four-particle cluster state is a very important entangled resource in quantum information. For example, Zha *et al.* discussed the RSP of a two-particle state using a four-particle cluster state as the quantum channel.[29] Especially, a cluster state has the properties of the GHZ-class and the W-class entangled states, and is harder to be destroyed by local operations.[30] Some authors[26–28] put forward schemes to remotely prepare or jointly prepare a four-particle entangled cluster-type state

$$|\varphi\rangle = \alpha|0000\rangle + \beta|0011\rangle + \gamma|1100\rangle - \delta|1111\rangle, \quad (1)$$

where α is real, β, γ, δ are complex, and $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. Therefore, it is still meaningful to discuss the assisted clone of a cluster state due to its important roles in quantum information although the four-particle cluster state is not the most general four-qubit state. Moreover, our concern is to devise deterministic schemes as the success probability is of paramount importance for a quantum protocol.

In this paper, we investigate the assisted clone of a four-particle entangled cluster-type state given in Eq. (1). Two deterministic schemes via four Einstein–Podolsky–Rosen (EPR) pairs as the quantum channel are proposed. We construct a unitary transformation and two sets of appropriate basis vectors which play an important role in the proposed schemes. In detail, after the teleportation, the sender first performs this unitary transformation on her particles, and sends parts of the transformed particles to the preparer. Then the preparer performs projective measurements under this basis. By rigorous

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formulations, it is proved that the sender can reestablish the unknown state deterministically.

The rest of this paper is organized as follows. In Section 2, we propose the first deterministic scheme and construct a new set of measurement basis. If the preparer performs a four-particle projective measurement under this basis and the sender adopts some recovery unitary operations, the unknown state can be cloned with unit success probability. In Section 3, we propose the other deterministic scheme with the aid of another two-step two-particle projective measurement. Discussion and conclusion are given in Sections 4 and 5, respectively.

2. Deterministic assisted-clone scheme via four-particle projective measurement

Following the assisted-clone model proposed by Pati,[9] we make the following assumption. Suppose there are three participants, Victor, Alice, and Bob. The sender Alice has safely received a four-particle entangled cluster-type state in Eq. (1) from the preparer Victor before the assisted-clone process and will use it as her input state. This original state is unknown to both the sender Alice and the receiver Bob. The assisted-clone scheme concerns on how to teleport the input state from Alice to Bob and then create a perfect copy of the state at the site of the sender with minimal assistance from the preparer. For convenience, we use a shorthand $\oplus$ as plus module 2.

Assume that Alice and Bob share four entangled EPR pairs

$$|\psi\rangle_{\mathrm{A}_t\mathrm{B}_t}=\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)_{\mathrm{A}_t\mathrm{B}_t},\quad t=1,2,3,4, \tag{2}$$

where particles $\mathrm{A}_1,\mathrm{A}_2,\mathrm{A}_3,\mathrm{A}_4$ belong to Alice, while particles $\mathrm{B}_1,\mathrm{B}_2,\mathrm{B}_3,\mathrm{B}_4$ belong to Bob. Hence, the initial state of the whole system can be written as

$$\begin{aligned}|\Psi\rangle &= |\varphi\rangle_{1234}|\psi\rangle_{\mathrm{A}_1\mathrm{B}_1}|\psi\rangle_{\mathrm{A}_2\mathrm{B}_2}|\psi\rangle_{\mathrm{A}_3\mathrm{B}_3}|\psi\rangle_{\mathrm{A}_4\mathrm{B}_4}\\ &=\frac{1}{4}\sum_{p,q=0,1}\mu_{pq}|ppqq\rangle_{1234}\prod_{i=1}^{4}(\sum_{h_t=0}^{1}|h_th_t\rangle_{\mathrm{A}_t\mathrm{B}_t}),\end{aligned} \tag{3}$$

where $\mu_{00}=\alpha$, $\mu_{01}=\beta$, $\mu_{10}=\gamma$, $\mu_{11}=-\delta$. Alice first performs the joint measurement on her particles $(1,\mathrm{A}_1)$, $(2,\mathrm{A}_2)$, $(3,\mathrm{A}_3)$, $(4,\mathrm{A}_4)$ under the Bell basis

$$|\varphi_{nm}\rangle=\frac{1}{\sqrt{2}}\sum_{j=0}^{1}\mathrm{e}^{\pi\mathrm{i}jn}|j\rangle|j\oplus m\rangle,\quad n,m=0,1, \tag{4}$$

respectively. After these measurements, Alice sends the classical bit strings n_1m_1, n_2m_2, n_3m_3, n_4m_4 to Bob through a classical channel, where $nm=00$ $(10,01,11)$ corresponds to the Bell-state measurement result $|\varphi_{00}\rangle$ $(|\varphi_{10}\rangle,|\varphi_{01}\rangle,|\varphi_{11}\rangle)$, respectively. Since

$$|jh\rangle=\frac{1}{\sqrt{2}}\sum_{n=0}^{1}\mathrm{e}^{-\pi\mathrm{i}jn}|\varphi_{n,h\oplus j}\rangle,\quad j,h=0,1, \tag{5}$$

Eq. (3) can be rewritten as

$$\begin{aligned}|\Psi\rangle &= \frac{1}{16}\sum_{p,q,n_i,m_i=0}^{1}\Big[\mathrm{e}^{-\pi\mathrm{i}p(n_1\oplus n_2)-\pi\mathrm{i}q(n_3\oplus n_4)}\\ &\quad\times\mu_{pq}(\prod_{t=1}^{4}|\varphi_{n_tm_t}\rangle_{t\mathrm{A}_t})|\Lambda\rangle_{\mathrm{B}_1\mathrm{B}_2\mathrm{B}_3\mathrm{B}_4}\Big],\end{aligned} \tag{6}$$

with

$$|\Lambda\rangle_{\mathrm{B}_1\mathrm{B}_2\mathrm{B}_3\mathrm{B}_4}=|p\oplus m_1\rangle_{\mathrm{B}_1}|p\oplus m_2\rangle_{\mathrm{B}_2}|q\oplus m_3\rangle_{\mathrm{B}_3}|q\oplus m_4\rangle_{\mathrm{B}_4}.$$

According to the received classical information, Bob performs a unitary transformation

$$(U_{n_1m_1})_{\mathrm{B}_1}\otimes(U_{n_2m_2})_{\mathrm{B}_2}\otimes(U_{n_3m_3})_{\mathrm{B}_3}\otimes(U_{n_4m_4})_{\mathrm{B}_4} \tag{7}$$

on the remaining state

$$\sum_{p,q=0}^{1}\mathrm{e}^{-\pi\mathrm{i}p(n_1\oplus n_2)-\pi\mathrm{i}q(n_3\oplus n_4)}\mu_{pq}|\Lambda\rangle_{\mathrm{B}_1\mathrm{B}_2\mathrm{B}_3\mathrm{B}_4}, \tag{8}$$

where

$$U_{nm}=\sum_{j=0}^{1}\mathrm{e}^{\pi\mathrm{i}jn}|j\rangle\langle j\oplus m|. \tag{9}$$

Then Bob gets $\sum_{p,q=0}^{1}\mu_{pq}|ppqq\rangle_{\mathrm{B}_1\mathrm{B}_2\mathrm{B}_3\mathrm{B}_4}$, which is the state Alice wants to transmit. The teleportation is successfully realized.

To create a perfect copy of the unknown state, Alice needs the assistance of Victor. According to the projection postulate of quantum mechanics, if Alice applies projector $|\varphi_{n_1m_1}\rangle_{1\mathrm{A}_1}\langle\varphi_{n_1m_1}|\otimes|\varphi_{n_2m_2}\rangle_{2\mathrm{A}_2}\langle\varphi_{n_2m_2}|\otimes|\varphi_{n_3m_3}\rangle_{3\mathrm{A}_3}\langle\varphi_{n_3m_3}|\otimes|\varphi_{n_4m_4}\rangle_{4\mathrm{A}_4}\langle\varphi_{n_4m_4}|$ on the initial system $|\Psi\rangle$, the state of particles 1, A_1, 2, A_2, 3, A_3, 4, A_4 will collapse into

$$\prod_{t=1}^{4}|\varphi_{n_tm_t}\rangle_{t\mathrm{A}_t}. \tag{10}$$

Alice applies a transformation

$$C_{43}C_{\mathrm{A}_4\mathrm{3}}H_4C_{4\mathrm{A}_4}C_{21}C_{\mathrm{A}_2\mathrm{1}}H_2C_{2\mathrm{A}_2}\prod_{t=1}^{4}(U_{n_tm_t})_{\mathrm{A}_t} \tag{11}$$

on the state in Eq. (10), where U_{nm} is defined by Eq. (9), C_{pq} is a CNOT operation on particles p and q (with q the control qubit and p the target one), and H is the Hadamard transformation. As

$$(I\otimes U_{nm})|\varphi_{nm}\rangle=|\varphi_{00}\rangle=\frac{|00\rangle+|11\rangle}{\sqrt{2}}, \tag{12}$$

then Alice can obtain

$$\begin{aligned}|\Upsilon\rangle &= \frac{1}{2}(|0000\rangle+|1111\rangle)_{12\mathrm{A}_1\mathrm{A}_2}(|0000\rangle+|1111\rangle)_{34\mathrm{A}_3\mathrm{A}_4}\\ &=\frac{1}{2}(|00000000\rangle+|00110011\rangle\\ &\quad+|11001100\rangle+|11111111\rangle)_{1234\mathrm{A}_1\mathrm{A}_2\mathrm{A}_3\mathrm{A}_4}.\end{aligned} \tag{13}$$

Alice sends particles (1,2,3,4) to Victor and keeps particles $(\mathrm{A}_1,\mathrm{A}_2,\mathrm{A}_3,\mathrm{A}_4)$ in her possession.

In order to help the sender to get a perfect copy of the unknown state, we need to construct a useful measurement basis. Firstly, we construct a 4×16 matrix

$$(\boldsymbol{A}\ \boldsymbol{B}\ \boldsymbol{C}\ \boldsymbol{D}) \tag{14}$$

satisfying

$$(\boldsymbol{A}\ \boldsymbol{B}\ \boldsymbol{C}\ \boldsymbol{D})(\boldsymbol{A}\ \boldsymbol{B}\ \boldsymbol{C}\ \boldsymbol{D})^{\dagger}=\boldsymbol{E}_4, \tag{15}$$

where 4×4 matrices $\boldsymbol{A},\boldsymbol{B},\boldsymbol{C},\boldsymbol{D}$ are defined as

$$\boldsymbol{A}=\frac{1}{2}\begin{pmatrix}\alpha & \alpha & \alpha & \alpha\\ \beta & -\beta & \beta & -\beta\\ \gamma & \gamma & -\gamma & -\gamma\\ -\delta & \delta & \delta & -\delta\end{pmatrix},\ \boldsymbol{B}=\frac{1}{2}\begin{pmatrix}\beta & \beta & \beta & \beta\\ \alpha & -\alpha & \alpha & -\alpha\\ \delta & \delta & -\delta & -\delta\\ -\gamma & \gamma & \gamma & -\gamma\end{pmatrix},$$

$$\boldsymbol{C}=\frac{1}{2}\begin{pmatrix}\gamma & \gamma & \gamma & \gamma\\ \delta & -\delta & \delta & -\delta\\ \alpha & \alpha & -\alpha & -\alpha\\ -\beta & \beta & \beta & -\beta\end{pmatrix},\ \boldsymbol{D}=\frac{1}{2}\begin{pmatrix}\delta & \delta & \delta & \delta\\ \gamma & -\gamma & \gamma & -\gamma\\ \beta & \beta & -\beta & -\beta\\ -\alpha & \alpha & \alpha & -\alpha\end{pmatrix}. \tag{16}$$

Secondly, we construct an 8×16 complex matrix

$$\boldsymbol{K}=\begin{pmatrix}\boldsymbol{A} & \boldsymbol{B} & \boldsymbol{C} & \boldsymbol{D}\\ \boldsymbol{B}^* & -\boldsymbol{A}^* & \boldsymbol{D}^* & -\boldsymbol{C}^*\end{pmatrix}, \tag{17}$$

which satisfies

$$\boldsymbol{K}\boldsymbol{K}^{\dagger}=\boldsymbol{E}_8. \tag{18}$$

Accordingly, an 8×16 matrix $\boldsymbol{L}$ can always be found such that $\begin{pmatrix}\boldsymbol{K}\\ \boldsymbol{L}\end{pmatrix}$ is a 16×16 unitary matrix. Here we give an explicit expression of such a matrix $\boldsymbol{L}=(\boldsymbol{V}\ \boldsymbol{W})$, where 8×8 complex matrices $\boldsymbol{V},\boldsymbol{W}$ are taken as

$$\boldsymbol{V}=\begin{pmatrix}\boldsymbol{J} & \boldsymbol{J}^*\\ \boldsymbol{J} & -\boldsymbol{J}^*\end{pmatrix},\quad \boldsymbol{W}=-\boldsymbol{V}(\boldsymbol{Q}^{-1}\boldsymbol{P})^{\dagger}, \tag{19}$$

with

$$\boldsymbol{J}=\frac{1}{2}\begin{pmatrix}\gamma & \delta & \gamma & \delta\\ \delta^* & -\gamma^* & \delta^* & -\gamma^*\\ \alpha & \beta & -\alpha & -\beta\\ -\beta^* & \alpha & \beta^* & -\alpha\end{pmatrix},$$

$$\boldsymbol{P}=\begin{pmatrix}\boldsymbol{A} & \boldsymbol{B}\\ \boldsymbol{B}^* & -\boldsymbol{A}^*\end{pmatrix},\quad \boldsymbol{Q}=\begin{pmatrix}\boldsymbol{C} & \boldsymbol{D}\\ \boldsymbol{D}^* & -\boldsymbol{C}^*\end{pmatrix}. \tag{20}$$

From the fact

$$[\boldsymbol{E}+(\boldsymbol{Q}^{-1}\boldsymbol{P})^{\dagger}(\boldsymbol{Q}^{-1}\boldsymbol{P})](\boldsymbol{E}-\boldsymbol{P}^{\dagger}\boldsymbol{P})=\boldsymbol{E}, \tag{21}$$

$$\boldsymbol{E}-\boldsymbol{P}^{\dagger}\boldsymbol{P}=\boldsymbol{V}^{\dagger}\boldsymbol{V}, \tag{22}$$

it is not hard to verify

$$\begin{pmatrix}\boldsymbol{K}\\ \boldsymbol{L}\end{pmatrix}=\begin{pmatrix}\boldsymbol{P} & \boldsymbol{Q}\\ \boldsymbol{V} & \boldsymbol{W}\end{pmatrix} \tag{23}$$

is a 16×16 unitary matrix.

Since Victor knows the original state $|\varphi\rangle$, he knows α, β, γ, δ completely. Construct a set of mutually orthogonal basis vectors $|\xi_1\rangle,\ldots,|\xi_{16}\rangle$ which are related to the computation basis $|0000\rangle$, $|0011\rangle$, $|1100\rangle$, $|1111\rangle$, $|0001\rangle$, $|0010\rangle$, $|0100\rangle$, $|0101\rangle$, $|0110\rangle$, $|0111\rangle$, $|1000\rangle$, $|1001\rangle$, $|1010\rangle$, $|1011\rangle$, $|1101\rangle$, $|1110\rangle$ (with this order) as follows:

$$\begin{pmatrix}|\xi_1\rangle\\ |\xi_2\rangle\\ \vdots\\ |\xi_{16}\rangle\end{pmatrix}=\begin{pmatrix}\boldsymbol{K}\\ \boldsymbol{L}\end{pmatrix}^{\dagger}\begin{pmatrix}|0000\rangle\\ |0011\rangle\\ \vdots\\ |1110\rangle\end{pmatrix}. \tag{24}$$

Then Victor performs a four-qubit projective measurement on his particles $(1,2,3,4)$ under the basis vectors $|\xi_1\rangle,\ldots,|\xi_{16}\rangle$. After the measurement, Victor sends the measurement outcome to Alice through a classical channel. With the basis $|\xi_1\rangle,\ldots,|\xi_{16}\rangle$ of qubits $(1,2,3,4)$, the joint system in Eq. (13) can be rewritten as

$$\begin{aligned}|\Upsilon\rangle\ =\ &\frac{1}{4}\big[|\xi_1\rangle_{1234}f(\alpha,\beta,\gamma,-\delta)_{\mathrm{A_1A_2A_3A_4}}\\ &+|\xi_2\rangle_{1234}f(\alpha,-\beta,\gamma,\delta)_{\mathrm{A_1A_2A_3A_4}}\\ &+|\xi_3\rangle_{1234}f(\alpha,\beta,-\gamma,\delta)_{\mathrm{A_1A_2A_3A_4}}\\ &+|\xi_4\rangle_{1234}f(\alpha,-\beta,-\gamma,-\delta)_{\mathrm{A_1A_2A_3A_4}}\\ &+|\xi_5\rangle_{1234}f(\beta,\alpha,\delta,-\gamma)_{\mathrm{A_1A_2A_3A_4}}\\ &+|\xi_6\rangle_{1234}f(\beta,-\alpha,\delta,\gamma)_{\mathrm{A_1A_2A_3A_4}}\\ &+|\xi_7\rangle_{1234}f(\beta,\alpha,-\delta,\gamma)_{\mathrm{A_1A_2A_3A_4}}\\ &+|\xi_8\rangle_{1234}f(\beta,-\alpha,-\delta,-\gamma)_{\mathrm{A_1A_2A_3A_4}}\\ &+|\xi_9\rangle_{1234}f(\gamma,\delta,\alpha,-\beta)_{\mathrm{A_1A_2A_3A_4}}\\ &+|\xi_{10}\rangle_{1234}f(\gamma,-\delta,\alpha,\beta)_{\mathrm{A_1A_2A_3A_4}}\\ &+|\xi_{11}\rangle_{1234}f(\gamma,\delta,-\alpha,\beta)_{\mathrm{A_1A_2A_3A_4}}\\ &+|\xi_{12}\rangle_{1234}f(\gamma,-\delta,-\alpha,-\beta)_{\mathrm{A_1A_2A_3A_4}}\\ &+|\xi_{13}\rangle_{1234}f(\delta,\gamma,\beta,-\alpha)_{\mathrm{A_1A_2A_3A_4}}\\ &+|\xi_{14}\rangle_{1234}f(\delta,-\gamma,\beta,\alpha)_{\mathrm{A_1A_2A_3A_4}}\\ &+|\xi_{15}\rangle_{1234}f(\delta,\gamma,-\beta,\alpha)_{\mathrm{A_1A_2A_3A_4}}\\ &+|\xi_{16}\rangle_{1234}f(\delta,-\gamma,-\beta,-\alpha)_{\mathrm{A_1A_2A_3A_4}}\big], \end{aligned}\tag{25}$$

with $f(x,y,z,u):=x|0000\rangle+y|0011\rangle+z|1100\rangle+u|1111\rangle$. From the above equation, one can see that whatever Victor's measurement outcome is, Alice's particles collapse into the corresponding state which can be transformed into the original state $|\varphi\rangle$. As a summary, Alice's recovery unitary operations conditioned on Victor's measurement results are shown in Table 1.

Therefore, the clone of an unknown four-particle entangled cluster-type state is deterministically realized. The quantum circuit of the scheme is shown in Fig. 1.

Table 1. The relation among Victor's measurement outcome (VMO), the collapsed state (CS), and Alice's appropriate unitary operation (AAUO).

VMO	CS	AAUO
$\|\xi_1\rangle_{1234}$	$f(\alpha,\beta,\gamma,-\delta)_{\mathrm{A_1A_2A_3A_4}}$	$I_{\mathrm{A_1}}I_{\mathrm{A_2}}I_{\mathrm{A_3}}I_{\mathrm{A_4}}$
$\|\xi_2\rangle_{1234}$	$f(\alpha,-\beta,\gamma,\delta)_{\mathrm{A_1A_2A_3A_4}}$	$I_{\mathrm{A_1}}I_{\mathrm{A_2}}I_{\mathrm{A_3}}Z_{\mathrm{A_4}}$
$\|\xi_3\rangle_{1234}$	$f(\alpha,\beta,-\gamma,\delta)_{\mathrm{A_1A_2A_3A_4}}$	$Z_{\mathrm{A_1}}I_{\mathrm{A_2}}I_{\mathrm{A_3}}I_{\mathrm{A_4}}$
$\|\xi_4\rangle_{1234}$	$f(\alpha,-\beta,-\gamma,-\delta)_{\mathrm{A_1A_2A_3A_4}}$	$I_{\mathrm{A_1}}Z_{\mathrm{A_2}}Z_{\mathrm{A_3}}I_{\mathrm{A_4}}$
$\|\xi_5\rangle_{1234}$	$f(\beta,\alpha,\delta,-\gamma)_{\mathrm{A_1A_2A_3A_4}}$	$Z_{\mathrm{A_1}}I_{\mathrm{A_2}}X_{\mathrm{A_3}}X_{\mathrm{A_4}}$
$\|\xi_6\rangle_{1234}$	$f(\beta,-\alpha,\delta,\gamma)_{\mathrm{A_1A_2A_3A_4}}$	$I_{\mathrm{A_1}}Z_{\mathrm{A_2}}Y_{\mathrm{A_3}}X_{\mathrm{A_4}}$
$\|\xi_7\rangle_{1234}$	$f(\beta,\alpha,-\delta,\gamma)_{\mathrm{A_1A_2A_3A_4}}$	$I_{\mathrm{A_1}}I_{\mathrm{A_2}}X_{\mathrm{A_3}}X_{\mathrm{A_4}}$
$\|\xi_8\rangle_{1234}$	$f(\beta,-\alpha,-\delta,-\gamma)_{\mathrm{A_1A_2A_3A_4}}$	$I_{\mathrm{A_1}}I_{\mathrm{A_2}}Y_{\mathrm{A_3}}X_{\mathrm{A_4}}$
$\|\xi_9\rangle_{1234}$	$f(\gamma,\delta,\alpha,-\beta)_{\mathrm{A_1A_2A_3A_4}}$	$X_{\mathrm{A_1}}X_{\mathrm{A_2}}Z_{\mathrm{A_3}}I_{\mathrm{A_4}}$
$\|\xi_{10}\rangle_{1234}$	$f(\gamma,-\delta,\alpha,\beta)_{\mathrm{A_1A_2A_3A_4}}$	$X_{\mathrm{A_1}}X_{\mathrm{A_2}}I_{\mathrm{A_3}}I_{\mathrm{A_4}}$
$\|\xi_{11}\rangle_{1234}$	$f(\gamma,\delta,-\alpha,\beta)_{\mathrm{A_1A_2A_3A_4}}$	$X_{\mathrm{A_1}}Y_{\mathrm{A_2}}Z_{\mathrm{A_3}}I_{\mathrm{A_4}}$
$\|\xi_{12}\rangle_{1234}$	$f(\gamma,-\delta,-\alpha,-\beta)_{\mathrm{A_1A_2A_3A_4}}$	$Y_{\mathrm{A_1}}X_{\mathrm{A_2}}I_{\mathrm{A_3}}I_{\mathrm{A_4}}$
$\|\xi_{13}\rangle_{1234}$	$f(\delta,\gamma,\beta,-\alpha)_{\mathrm{A_1A_2A_3A_4}}$	$X_{\mathrm{A_1}}Y_{\mathrm{A_2}}Y_{\mathrm{A_3}}X_{\mathrm{A_4}}$
$\|\xi_{14}\rangle_{1234}$	$f(\delta,-\gamma,\beta,\alpha)_{\mathrm{A_1A_2A_3A_4}}$	$Y_{\mathrm{A_1}}X_{\mathrm{A_2}}X_{\mathrm{A_3}}X_{\mathrm{A_4}}$
$\|\xi_{15}\rangle_{1234}$	$f(\delta,\gamma,-\beta,\alpha)_{\mathrm{A_1A_2A_3A_4}}$	$X_{\mathrm{A_1}}X_{\mathrm{A_2}}Y_{\mathrm{A_3}}X_{\mathrm{A_4}}$
$\|\xi_{16}\rangle_{1234}$	$f(\delta,-\gamma,-\beta,-\alpha)_{\mathrm{A_1A_2A_3A_4}}$	$X_{\mathrm{A_1}}X_{\mathrm{A_2}}X_{\mathrm{A_3}}X_{\mathrm{A_4}}$

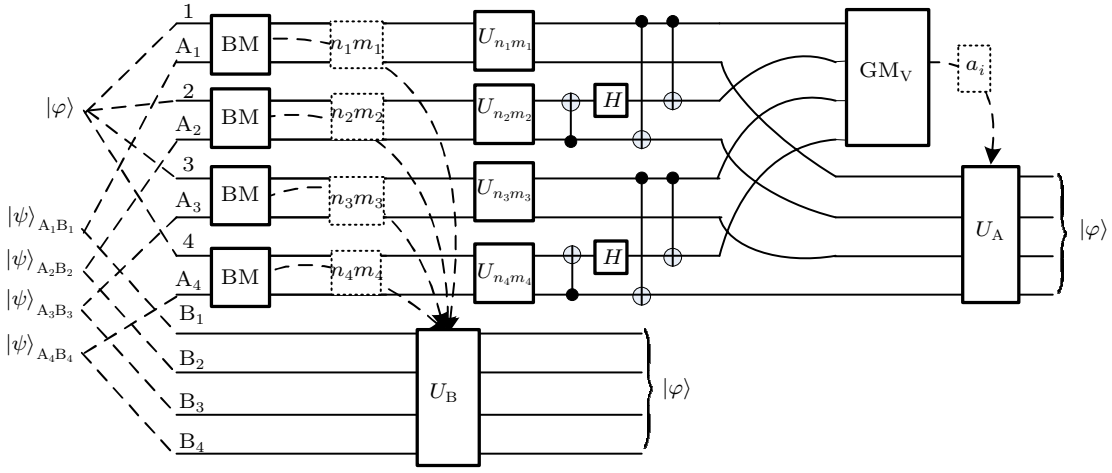


Fig. 1. Quantum circuit of the first assisted-clone scheme. Here BMs denote the projective measurements defined in Eq. (4) and $n_i m_i$ $(i=1,2,3,4)$ denote the coding forms of the corresponding measurement outcomes. The unitary transformation $U_{\rm B}$ performed by Bob is defined in Eq. (7); the local transformations $U_{n_i m_i}$ are defined in Eq. (9); and H denotes the Hadamard operation. After Alice's transformations on $(1,{\rm A}_1,\ldots,4,{\rm A}_4)$, she sends her particles $(1,2,3,4)$ to Victor. ${\rm GM_V}$ denotes the general projective measurement under the basis in Eq. (24) performed by Victor and a_i denotes the corresponding classical coding of outcome $\{|\xi_1\rangle_{1234},\ldots,|\xi_{16}\rangle_{1234}\}$; $U_{\rm A}$ is defined in Table 1.

3. Deterministic assisted clone via two-step two-particle projective measurement

In this section, we demonstrate the other deterministic scheme by performing two-step two-particle projective measurements and adding local operations.

As mentioned in Section 2, the assisted-clone process is combined with quantum teleportation and remote state preparation. In the first stage, Alice teleports the unknown state to Bob. Then Alice applies a unitary operation in Eq. (11) on the leftover entangled resource after the teleportation and sends parts of the particles (1,2,3,4) to Victor. As a result, Victor and Alice share the entangled channel in Eq. (13). Now, we describe the following RSP stage in detail.

Notice that the four-particle entangled cluster-type state in Eq. (1) can be rewritten as

$$a_0|0000\rangle + a_1 {\rm e}^{{\rm i}\theta_1}|0011\rangle + a_2 {\rm e}^{{\rm i}\theta_2}|1100\rangle - a_3 {\rm e}^{{\rm i}\theta_3}|1111\rangle, \quad (26)$$

where $a_0=\alpha$, $a_1{\rm e}^{{\rm i}\theta_1}=\beta$, $a_2{\rm e}^{{\rm i}\theta_2}=\gamma$, $a_3{\rm e}^{{\rm i}\theta_3}=\delta$, $a_0,a_1,a_2,a_3\geq 0$ are real coefficients with the normalization condition $a_0^2+a_1^2+a_2^2+a_3^2=1$, and $\theta_1,\theta_2,\theta_3\in[0,2\pi)$.

Since Victor knows α, β, γ, δ, he knows a_0, a_1, a_2, a_3 and θ_1, θ_2, θ_3 completely. Victor first performs a two-particle projective measurement on particles $(1,3)$ under the complete orthogonal basis $\{|\zeta_{00}\rangle,|\zeta_{01}\rangle,|\zeta_{10}\rangle,|\zeta_{11}\rangle\}$, which is related to the computation basis $\{|00\rangle,|01\rangle,|10\rangle,|11\rangle\}$ by the following relation:

$$\begin{pmatrix}|\zeta_{00}\rangle\\|\zeta_{01}\rangle\\|\zeta_{10}\rangle\\|\zeta_{11}\rangle\end{pmatrix}=\begin{pmatrix}a_0&a_1&a_2&a_3\\a_1&-a_0&-a_3&a_2\\a_2&a_3&-a_0&-a_1\\-a_3&a_2&-a_1&a_0\end{pmatrix}\begin{pmatrix}|00\rangle\\|01\rangle\\|10\rangle\\|11\rangle\end{pmatrix}. \quad (27)$$

After the measurement, Victor does not perform the second-step measurement immediately but performs a unitary operation $U^{r_1 s_1}$ on his particles (2,4) conditioned on his first measurement result $|\zeta_{r_1 s_1}\rangle_{13}$ $(r_1,s_1\in\{0,1\})$. Here

$$U^{00}=I\otimes I,\ U^{01}=I\otimes\sigma_x,\ U^{10}=\sigma_x\otimes I,\ U^{11}=\sigma_x\otimes\sigma_x, \quad (28)$$

with I being the identity operation, and σ_x being the Pauli operation. Then Victor performs a two-particle projective measurement on particles $(2,4)$ under the complete orthogonal basis $\{|\eta_{00}\rangle,|\eta_{01}\rangle,|\eta_{10}\rangle,|\eta_{11}\rangle\}$ which are defined by

$$\begin{pmatrix}|\eta_{00}\rangle\\|\eta_{01}\rangle\\|\eta_{10}\rangle\\|\eta_{11}\rangle\end{pmatrix}=\frac{1}{2}\begin{pmatrix}1&{\rm e}^{-{\rm i}\theta_1}&{\rm e}^{-{\rm i}\theta_2}&-{\rm e}^{-{\rm i}\theta_3}\\1&-{\rm e}^{-{\rm i}\theta_1}&{\rm e}^{-{\rm i}\theta_2}&{\rm e}^{-{\rm i}\theta_3}\\1&{\rm e}^{-{\rm i}\theta_1}&-{\rm e}^{-{\rm i}\theta_2}&{\rm e}^{-{\rm i}\theta_3}\\1&-{\rm e}^{-{\rm i}\theta_1}&-{\rm e}^{-{\rm i}\theta_2}&-{\rm e}^{-{\rm i}\theta_3}\end{pmatrix}\begin{pmatrix}|00\rangle\\|01\rangle\\|10\rangle\\|11\rangle\end{pmatrix}. \quad (29)$$

To see how our protocol works, let us express the shared quantum channel in Eq. (13) in terms of the measurement basis $\{|\zeta_{00}\rangle,|\zeta_{01}\rangle,|\zeta_{10}\rangle,|\zeta_{11}\rangle\}$

$$\begin{aligned}|\Upsilon\rangle=\frac{1}{2}\big[&|\zeta_{00}\rangle_{13}g(a_0,a_1,a_2,a_3)_{24{\rm A}_1{\rm A}_2{\rm A}_3{\rm A}_4}\\&+|\zeta_{01}\rangle_{13}g(a_1,-a_0,-a_3,a_2)_{24{\rm A}_1{\rm A}_2{\rm A}_3{\rm A}_4}\\&+|\zeta_{10}\rangle_{13}g(a_2,a_3,-a_0,-a_1)_{24{\rm A}_1{\rm A}_2{\rm A}_3{\rm A}_4}\\&+|\zeta_{11}\rangle_{13}g(-a_3,a_2,-a_1,a_0)_{24{\rm A}_1{\rm A}_2{\rm A}_3{\rm A}_4}\big], \quad (30)\end{aligned}$$

where $g(x,y,z,u):=x|000000\rangle+y|010011\rangle+z|101100\rangle+u|111111\rangle)$. Without loss of generality, we assume that Victor's first measurement result is $|\zeta_{01}\rangle_{13}$. In this situation, Victor performs unitary operation $U^{01}=I\otimes\sigma_x$ on his particles (2,4), and gets

$$\begin{aligned}&\big(a_1|010000\rangle-a_0|000011\rangle-a_3|111100\rangle\\&+a_2|101111\rangle\big)_{24{\rm A}_1{\rm A}_2{\rm A}_3{\rm A}_4}. \quad (31)\end{aligned}$$

Then Victor performs a two-particle projective measurement on his particles (2,4) under the basis $\{|\eta_{00}\rangle,|\eta_{01}\rangle,|\eta_{10}\rangle,|\eta_{11}\rangle\}$ defined by Eq. (29). From the fact that the state in Eq. (31) can

be rewritten as

$$
\begin{aligned}
&|\eta_{00}\rangle_{24} f(a_1 \mathrm{e}^{\mathrm{i}\theta_1}, -a_0, a_3 \mathrm{e}^{\mathrm{i}\theta_3}, a_2 \mathrm{e}^{\mathrm{i}\theta_2})_{\mathrm{A_1A_2A_3A_4}} \\
&+|\eta_{01}\rangle_{24} f(-a_1 \mathrm{e}^{\mathrm{i}\theta_1}, -a_0, -a_3 \mathrm{e}^{\mathrm{i}\theta_3}, a_2 \mathrm{e}^{\mathrm{i}\theta_2})_{\mathrm{A_1A_2A_3A_4}} \\
&+|\eta_{10}\rangle_{24} f(a_1 \mathrm{e}^{\mathrm{i}\theta_1}, -a_0, -a_3 \mathrm{e}^{\mathrm{i}\theta_3}, -a_2 \mathrm{e}^{\mathrm{i}\theta_2})_{\mathrm{A_1A_2A_3A_4}} \\
&+|\eta_{11}\rangle_{24} f(-a_1 \mathrm{e}^{\mathrm{i}\theta_1}, -a_0, a_3 \mathrm{e}^{\mathrm{i}\theta_3}, -a_2 \mathrm{e}^{\mathrm{i}\theta_2})_{\mathrm{A_1A_2A_3A_4}}, \quad (32)
\end{aligned}
$$

one can see whatever Victor's second measurement outcome $|\eta_{r_2s_2}\rangle_{24}$ ($r_2, s_2 \in \{0,1\}$) is, Alice can obtain the perfect copy of the unknown state by performing an appropriate unitary operation on her collapsed particles.

As for the other three cases corresponding to Victor's first measurement results $|\zeta_{00}\rangle_{13}$, $|\zeta_{10}\rangle_{13}$, $|\zeta_{11}\rangle_{13}$, similar analysis processes can be made. As a summary, Alice's recovery operations conditioned on Victor's two-step measurement results $|\zeta_{r_1s_1}\rangle_{13}$ and $|\eta_{r_2s_2}\rangle_{24}$ are shown in Table 2 (X, Y, and Z are Pauli operations).

Therefore, the total success probability for cloning the unknown four-particle entangled cluster-type state is unit. The quantum circuit of this scheme is shown in Fig. 2.

Table 2. The relation between Victor's two-step measurement outcomes $|\zeta_{r_1s_1}\rangle_{13}$, $|\eta_{r_2s_2}\rangle_{24}$, and Alice's appropriate unitary operation (AAUO) on the particles $(\mathrm{A_1}, \mathrm{A_2}, \mathrm{A_3}, \mathrm{A_4})$.

AAUO	$r_2s_2 = 00$	$r_2s_2 = 01$
$r_1s_1 = 00$	$I_{\mathrm{A_1}} I_{\mathrm{A_2}} I_{\mathrm{A_3}} I_{\mathrm{A_4}}$	$I_{\mathrm{A_1}} I_{\mathrm{A_2}} I_{\mathrm{A_3}} Z_{\mathrm{A_4}}$
$r_1s_1 = 01$	$I_{\mathrm{A_1}} Z_{\mathrm{A_2}} Y_{\mathrm{A_3}} X_{\mathrm{A_4}}$	$Z_{\mathrm{A_1}} I_{\mathrm{A_2}} X_{\mathrm{A_3}} X_{\mathrm{A_4}}$
$r_1s_1 = 10$	$Y_{\mathrm{A_1}} X_{\mathrm{A_2}} I_{\mathrm{A_3}} I_{\mathrm{A_4}}$	$X_{\mathrm{A_1}} Y_{\mathrm{A_2}} Z_{\mathrm{A_3}} I_{\mathrm{A_4}}$
$r_1s_1 = 11$	$X_{\mathrm{A_1}} X_{\mathrm{A_2}} Y_{\mathrm{A_3}} X_{\mathrm{A_4}}$	$X_{\mathrm{A_1}} X_{\mathrm{A_2}} X_{\mathrm{A_3}} X_{\mathrm{A_4}}$
AAUO	$r_2s_2 = 10$	$r_2s_2 = 11$
$r_1s_1 = 00$	$Z_{\mathrm{A_1}} I_{\mathrm{A_2}} I_{\mathrm{A_3}} I_{\mathrm{A_4}}$	$I_{\mathrm{A_1}} Z_{\mathrm{A_2}} Z_{\mathrm{A_3}} I_{\mathrm{A_4}}$
$r_1s_1 = 01$	$I_{\mathrm{A_1}} I_{\mathrm{A_2}} Y_{\mathrm{A_3}} X_{\mathrm{A_4}}$	$I_{\mathrm{A_1}} I_{\mathrm{A_2}} X_{\mathrm{A_3}} X_{\mathrm{A_4}}$
$r_1s_1 = 10$	$X_{\mathrm{A_1}} X_{\mathrm{A_2}} I_{\mathrm{A_3}} I_{\mathrm{A_4}}$	$X_{\mathrm{A_1}} X_{\mathrm{A_2}} Z_{\mathrm{A_3}} I_{\mathrm{A_4}}$
$r_1s_1 = 11$	$X_{\mathrm{A_1}} Y_{\mathrm{A_2}} Y_{\mathrm{A_3}} X_{\mathrm{A_4}}$	$Y_{\mathrm{A_1}} X_{\mathrm{A_2}} X_{\mathrm{A_3}} X_{\mathrm{A_4}}$

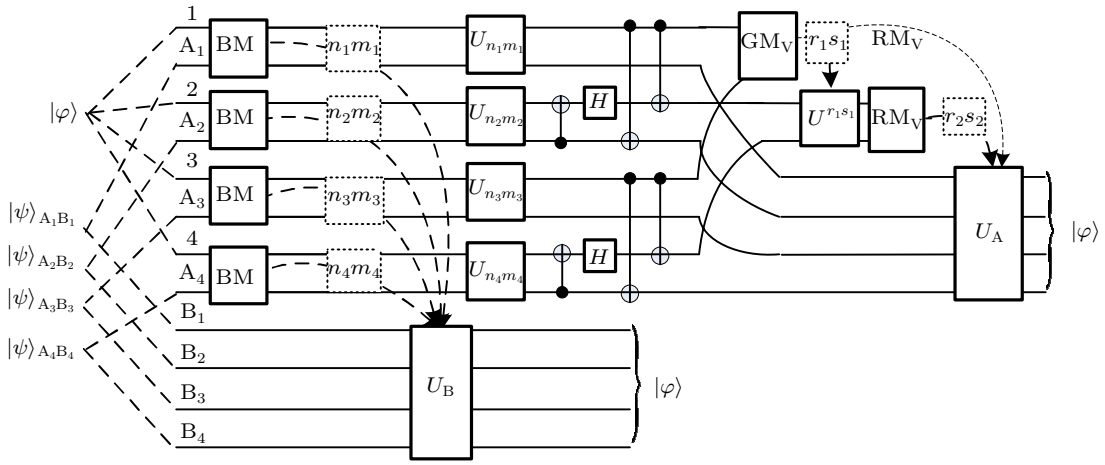


Fig. 2. Quantum circuit of the second assisted-clone scheme. Here $\mathrm{GM_V}$ denotes the general measurement defined in Eq. (27) performed by Victor, $U^{r_1s_1}$ denotes the unitary transformations in Eq. (28) performed by Victor, $\mathrm{RM_V}$ denotes the projective measurement defined in Eq. (29) performed by Victor. The U_{A} dependence on r_1s_1 and r_2s_2 is shown in Table 2. All the other symbols are the same as those in Fig. 1.

4. Discussion

In a strict sense, a quantum cloning machine is not supposed to communicate with the state preparer Victor because he can produce an arbitrary number of copies and send them to the interested parties with the assumption that he knows the original state. For example, Victor may design a unitary transformation which is able to transform a given product state into the desired cluster state. In detail, Victor constructs a 4×4 unitary matrix U_1 by performing the Gram–Schmidt orthogonal procedure on a linear independent vector set $\{(\alpha, \beta, \gamma, -\delta), (\beta^*, -\alpha, \delta^*, \gamma^*)\}$ of the complex Hilbert space $\mathcal{H} = \mathbf{C}^4$. Without loss of generality, we denote

$$
\boldsymbol{U}_1 = \begin{pmatrix} \alpha & \beta & \gamma & -\delta \\ \beta^* & -\alpha & \delta^* & \gamma^* \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \end{pmatrix}.
$$

To obtain the four-particle cluster state, Victor introduces four auxiliary qubits with the original state $|0000\rangle_{\mathrm{V_1V_2V_3V_4}}$ and makes a unitary transformation $\boldsymbol{U} = \mathrm{diag}\{\boldsymbol{U}_1, \boldsymbol{U}_1, \boldsymbol{U}_1, \boldsymbol{U}_1\}$ on his particles $(\mathrm{V_1}, \mathrm{V_2}, \mathrm{V_3}, \mathrm{V_4})$ under the computation basis $\{|0000\rangle, |0011\rangle, |1100\rangle, |1111\rangle, |0001\rangle, |0010\rangle, |0100\rangle, |0101\rangle, |0110\rangle, |0111\rangle, |1000\rangle, |1001\rangle, |1010\rangle, |1011\rangle, |1101\rangle, |1110\rangle\}$. Victor can get $(\alpha|0000\rangle + \beta|0011\rangle + \gamma|1100\rangle - \delta|1111\rangle)_{\mathrm{V_1V_2V_3V_4}}$. Similarly, Victor can obtain another copy of the original state. Then, Victor directly teleports one copy to Alice and another to Bob. At the end, Alice and Bob both get the perfect copy of the original state which has the same effect as the proposed schemes.

The above communication protocol seems simpler than the proposed ones in the sense that the preparer Victor broadcasts the original state to the two receivers Alice and Bob. However, the motivation and quantum task of the trivial proposal are different from ours, see Fig. 3. In fact, this protocol transmits the known state to each receiver with the shared quantum channel between the preparer and the receiver. In this paper, the question we raise is: can we produce a per-

fect copy of an unknown four-particle cluster state with minimal assistance from the preparer Victor? If Victor has only one input state and does not want to send infinite bits of information, then without violating any principles of quantum theory and using quantum mechanical operations can we go around the no-cloning theorem? If Victor agrees to assist Alice with a minimum amount of help without revealing what the input state is, then what we can do is described in the proposed schemes. Comparing with the trivial scheme, the importance of our approach can be in those situations where Victor is one-way (cut-off) from the rest of the world and he cannot send out any quantum state but has a classical channel to send only cbits, as shown in Fig. 4. Moreover, the present schemes may be more applicable and safer in the situations where there is no shared quantum channel between the assistant Victor and the receiver Bob, and the role of Alice is a transmission relay or a controller.

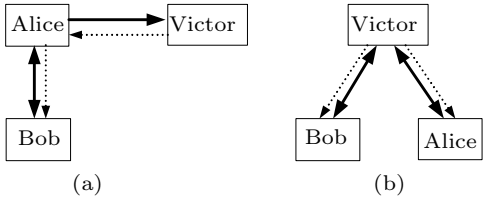


Fig. 3. The schematic representations of the proposed schemes and the trivial scheme: (a) our assisted-clone model; (b) the trivial quantum communication model. The solid line with one arrow (two arrows) denotes a one-way (double-way) quantum transmission channel, while the dotted line with one arrow denotes a one-way classical transmission channel. The one-way transmission means to transmit a quantum state according to the physical channel while the double-way transmission means to teleport a quantum state via quantum channel constructed by quantum entanglement resources.

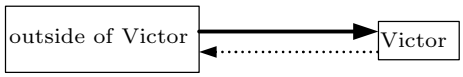


Fig. 4. The schematic representation of the assisted-clone environment. Here the solid line denotes a one-way quantum transmission channel, while the dotted line denotes a one-way classical transmission channel.

5. Conclusion

In conclusion, we have proposed two deterministic assisted-clone schemes which can produce perfect copies of an unknown four-particle entangled cluster-type state when maximal entanglement is used as the quantum channel. Comparing with the original protocols which produce the unknown state and its orthogonal complement state, the proposed schemes generate a perfect copy of the unknown state with unit success probability.

The proposed schemes include two steps. In the first step, Alice performs Bell measurements and sends the classical bits to Bob via a classical channel. Based on Alice's classical information, Bob can reconstruct the original state on his particles. In the second step, a new transformation performed by Alice is constructed. Then we construct a set of four-particle orthogonal basis in the first scheme, and two sets of two-particle orthogonal bases in the second scheme. Under these bases, Victor performs a projective measurement on his particles which are sent by Alice. After receiving Victor's measurement result, Alice can reestablish the original state. The probabilities of successful teleportation and clone of the original state are both unit. The classical communication costs of the two schemes are both 8 cbits, which are divided into two parts. One is 4 cbits in the teleportation, i.e., the sender Alice broadcasts the measurement outcome on her particles $(1, \mathrm{A}_1)$, $(2, \mathrm{A}_2)$, $(3, \mathrm{A}_3)$, $(4, \mathrm{A}_4)$ to the receiver Bob. The other 4 cbits are consumed in the process of RSP, i.e., the preparer Victor sends his measurement outcome on particles $(1, 2, 3, 4)$ to the sender Alice.

It is noticed that the two schemes can both realize the assisted clone with unit success probability by using the same entangled resources. Compared to the first scheme in Section 2, the second scheme has favorable and unfavorable aspects. On one hand, the two-particle projective measurement is easier to perform than the four-particle projective measurement. On the other hand, Victor needs to use the information splitting and divide the complex coefficients $\{a+b\mathrm{i}\ (=r\mathrm{e}^{\mathrm{i}\theta})\}$ of the unknown state into $\{r\}$ and $\{\theta\}$ before constructing the measurement basis. Also, before performing the second-step two-particle projective measurement, Victor needs to perform an additional unitary operation conditioned on his first-step projective measurement outcome.

Moreover, the proposed schemes can be extended to clone an unknown state via non-maximally entangled EPR pairs as the quantum channel. Under this assumption, the receiver can re-establish the unknown state with a certain probability. After the probabilistic teleportation, Alice still ends up with four Bell pairs which are the results of her measurements. Similar to the procedure, as the quantum channel is maximally entangled, the perfect copy of the input state can be produced at the sender's location deterministically. It is worth mentioning that the proposed schemes can be reduced to other deterministic schemes such as cloning other classes of four-qubit states with no more than four coefficients, for example, four-particle W-state, GHZ-state, etc.

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