Tunable zeroth-order resonator based on a ferrite metamaterial structure

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This paper proposes a tunable zeroth-order resonator on a composite right/left-handed transmission line consisting of a transversely magnetized ferrite substrate periodically loaded by microstrip inductors. Based on the propagation theory of edge guided modes, the analysis procedure of this structure is introduced. The numerical results demonstrate the tunability of the resonant frequency by changing the DC bias magnetic field applied to the ferrite. In contrast to previous work, the proposed structure is easy to design and fabricate and does not require a chip component.

Keywords: composite right/left-handed transmission line, magnetized ferrite substrate, tunable metamaterial, zeroth-order resonator

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1. Introduction

Over the last decade, the composite right/left-handed (CRLH) metamaterials with positive, negative, and zero propagation constants, have attracted considerable attention in optics, physics, and microwave engineering.[1–3] They often consist of periodic arrays of metallic particles effectively behaving as a homogeneous transmission line, for example, a transmission line periodically loaded with series capacitors and shunt inductors.[4]

Most CRLH transmission lines suffer from important drawbacks such as a narrow frequency band of operation and a lack of the ability to frequency-tune essential properties. Recently, the negative effective permeability values of ferrite materials at microwave frequencies and their dependences on applied bias field make these media a prime candidate for the implementation of tunable CRLH lines. For example, a tunable metamaterial consisting of ferrite slabs and copper wires was proposed in Ref. [5]. Some work has been reported by Kodera and Caloz[6,7] using a uniform ferrite-loaded open waveguide structure with a CRLH response. Left-handed transmission lines have also been realized by using a ferrite slabs in a coplanar waveguide (CPW) structure.[8]

As an application of the CRLH transmission line, the zeroth-order resonator (ZOR) whose resonant condition is based on achieving zero propagation constant, has been considered in some previous work. In this resonator, the resonant frequency is independent of the total physical length. Hence it can be very compact, especially at lower frequencies. Some works[9,10] have reported on ZORs formed by cascading left-handed unit cells in microstrips and CPW configurations.

In this paper, we present a novel tunable ZOR based on a ferrite CRLH microstrip line. The tuning nature of the proposed resonator is due to the dispersive permeability of the ferrite substrate.

The remainder of the present paper is organized as follows. In Section 2 the geometry of the proposed tunable CRLH transmission line and its behaviors are presented. In Section 3, the ZOR as an open-ended CRLH line is described. And finally, some conclusions are drawn from the present study in Section 4.

2. Geometry of the CRLH structure

As shown in Fig. 1, the proposed CRLH line in this paper consists of a transversely magnetized ferrite microstrip line periodically loaded by short-circuited inductive stubs on an adjoining dielectric substrate. The idea behind this structure is to realize a left-handed line by emulating a negative capacitance per unit length in the frequency range where inductance per unit length is already negative due to the properties of the ferrite itself.

As shown in the Appendix A, for a ferrite microstrip line on a normally magnetized ferrite substrate (along z), where the wave propagates along y, there are two kind of modes. The first mode, which is the desired mode in this paper, is an edge-guided mode with exponential behavior along x, and the second mode is a wave-guided mode with sinusoidal behavior along x, and with a cut-off frequency related to line width.

Within the commonly used approximation where fictitious magnetic walls are inserted along the lateral edges of the microstrip, the electric and magnetic fields of edge-guided mode inside the ferrite are given by (see Appendix A)[11]

$$\bar{E} = \mathrm{j}A\,\mathrm{e}^{\mu_a\beta x/\mu}\,\mathrm{e}^{(-\mathrm{j}\beta y)}\hat{a}_z, \tag{1a}$$

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$$\bar{H} = \frac{\mathrm{j}A\beta}{\omega\mu_0\mu}\,\mathrm{e}^{\mu_a\beta x/\mu}\,\mathrm{e}^{(-\mathrm{j}\beta y)}\hat{a}_x, \tag{1b}$$

where

$$\frac{\mu_a}{\mu} = \frac{\omega_\mathrm{M}\omega}{\omega_\perp^2 - \omega^2}, \quad \mu = \frac{\omega_\perp^2 - \omega^2}{\omega_\mathrm{H}^2 - \omega^2}, \tag{1c}$$

with

$$\omega_\mathrm{H} = \gamma H_0 + \mathrm{j}\alpha\omega, \quad f_\mathrm{H} = \frac{\gamma H_0}{2\pi}, \quad \omega_\mathrm{M} = \gamma M_\mathrm{s},$$
$$\omega_\perp^2 = \omega_\mathrm{H}(\omega_\mathrm{M} + \omega_\mathrm{H}), \quad f_\perp = \frac{\omega_\perp}{2\pi}.$$

In these relations, H_0 is the bias magnetic field, M_s is the saturation magnetization, γ is the gyromagnetic ratio, α is the Gilbert damping constant that depends on magnetic loss, ΔH, and A is a constant. Moreover,

$$\beta = \pm k_0\sqrt{\mu\varepsilon_\mathrm{f}} \tag{2}$$

is the edge-guided propagation constant where k_0 is the free space wave number and ε_f is the dielectric constant of the ferrite material. The signs "+" and "−" in Eq. (2) denote the propagations in $+y$ direction and $-y$ direction, respectively. Take note that both fields show exponential behaviors along x axis (see Eqs. (1a) and (1b)). From Eq. (1c), in the frequency range between f_H and $f_\perp$, which is the range of interest in this paper, the real part of permeability (μ) is negative (i.e., the inductance per unit length of edge-guided mode is negative).

For implementing the negative capacitance per unit length, the easiest way is to load lines periodically by inductive stubs as shown in Fig. 1.

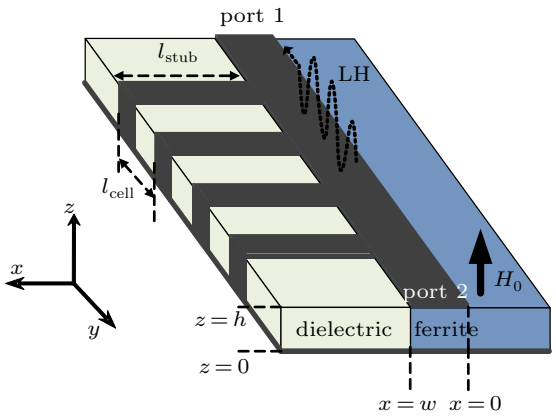


Fig. 1. (color online) Geometry of the proposed nonreciprocal CLRH line. The dashed line shows the left-handed response concentrated near the right edge ($x = 0$) when port 1 is excited.

In this figure, for the propagation in $+y$, where port 1 is excited, the left-handed response can be obtained by replacing β with $-|\beta_\mathrm{eff}|$ in Eqs. (1a) and (1b) as

$$\bar{E} = \mathrm{j}A\exp\left(-\frac{\mu_a}{\mu}|\beta_\mathrm{eff}|x\right)\exp\left(\mathrm{j}|\beta_\mathrm{eff}|y\right)\hat{a}_z, \tag{3a}$$

$$\bar{H} = \frac{\mathrm{j}A\beta}{\omega\mu_0\mu}\exp\left(-\frac{\mu_a}{\mu}|\beta_\mathrm{eff}|x\right)\exp\left(\mathrm{j}|\beta_\mathrm{eff}|y\right)\hat{a}_x, \tag{3b}$$

where β_eff is the effective propagation constant of the edge-guided mode in the presence of shorted stubs.

Based on Eq. (3a), the left-handed electric field is concentrated near the right edge ($x = 0$ in Fig. 1). Similarly, for the propagation in $-y$, where port 2 is excited, we can show that the left-handed electric field is concentrated near the left edge ($x = w$ in Fig. 1), where the shorted stubs are located. In this case, the electric field is shorted to ground and the transmission loss is increased. This means that the proposed structure in Fig. 1 is nonreciprocal and the metamaterial behavior is only obtained when port 1 is excited.

To verify this scenario, the nonreciprocal structure in Fig. 1 is simulated by commercial software HFSS based on the finite element method (FEM). In all simulations we use an RT/Duroid 5880 as the dielectric layer with a relative permittivity of 2.2 and dielectric loss tangent of 0.0009, and Yttrium Iron Garnet (YIG) with a saturation magnetization ($4\pi M_\mathrm{s}$) of 0.173 T, magnetic loss (ΔH) of 10 Oe (1 Oe = 79.5775 A·m^{-1}), relative permittivity of 15.3 is used as a ferrite layer.

Figure 2 shows the transmission characteristics of the nonreciprocal CRLH structure, when port 1 is excited. As shown in Fig. 2(a), the left-handed pass band is found to range from 1.58 GHz to 2.02 GHz, where the phase velocity ($v_\mathrm{p} = \omega/\beta$) is negative and the group velocity ($v_\mathrm{g} = \partial\omega/\partial\beta$) is positive. Above the frequency of 2.02 GHz, there is found an RH pass band, where both phase and group velocities are positive.

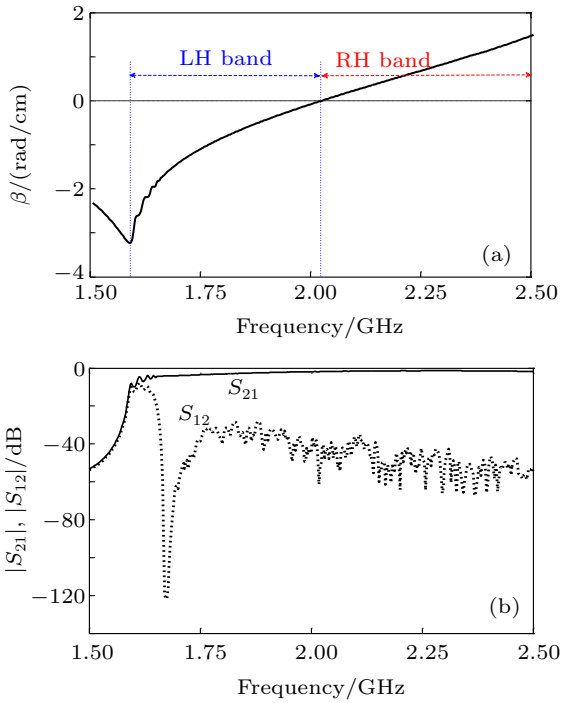


Fig. 2. (color online) Transmission characteristics of the CRLH structure in Fig. 1, showing (a) propagation constant β in the $+y$ direction, (b) magnitude of transmission coefficient, with the following parameters: $w = 3$ mm, $w_\mathrm{stub} = 1$ mm, $l_\mathrm{stub} = 4$ mm, $h = 0.5$ mm, the distance between stubs $l_\mathrm{cell} = 2.5$ mm, $H_0 = 600$ Oe, $4\pi M_\mathrm{s} = 0.173$ T, $\Delta H = 10$ Oe, and the number of stubs $N = 10$.

Remarkably, there is a gapless transition between the left- and right-handed range. This means that the proposed structure works as a balanced CRLH line. This unique trait can lead to several applications for the proposed structure, such as couplers, power dividers, and phase shifters.[12] Figure 2(b) shows the nonreciprocal response with 25-dB isolation, approximately. Probably, the main reason for the limited isolation is the excitation of the magnetostatic wave allowed by small z-variations of the fields.[13]

3. Tunable zeroth-order resonator

In a conventional right-handed transmission line (RH TL), the propagation constant β is always positive and the RH resonator can only resonate where the physical length of the structure (l) is a multiple of half a wavelength or, equivalently, the electrical length ($\theta = \beta \cdot l$) is a multiple of π. In contrast to the RH TL, the CRLH transmission line can have $\beta = 0$, $\beta < 0$, and $\beta > 0$. Therefore, the electrical length can be zero, negative, and positive correspondingly.

The ZOR is a resonator whose resonant condition is based on achieving zero propagation constant ($\beta = 0$). In this case, the electrical length θ is zero for each physical length. This is the unique property for zeroth order resonance.

As shown in Fig. 2(a), at transition frequency between left- and right-handed range ($f = 2.02$ GHz), the propagation constant β is zero. Therefore, the proposed CRLH structure in Fig. 1 can be a good candidate for the design of a ZOR.

To show the zero order resonant behavior of the proposed CRLH structure, the CRLH line in Fig. 1 is open-ended in port 2, and the input impedance in port 1 is given in Fig. 3(a). As shown in this figure, the open-ended structure of Fig. 1 has a resonant behavior at transition frequency, where the propagation constant β and the electrical length θ are zero. Based on the field distribution given in Fig. 3(b), at this resonant frequency ($f = 2.02$ GHz), the guided wavelength is infinite, as may be seen from the fact that the maximum of the field does not move along the structure. Note that this wave is a propagating wave since its group velocity, corresponding to the dispersion relation in Fig. 2(a), is different from zero.

In fact, by varying the magnetic bias field H_0, the proposed CRLH line would experience a rapid change in permeability which can generate a change in the propagation constant. Therefore, a magnetically tunable ZOR based on the proposed CRLH line can be realized. The tuning capability of the ZOR is studied for different values of DC magnetic bias (See Fig. 4). It can be concluded from results given in Fig. 4 that the resonant frequency can be shifted toward higher frequencies when the bias magnetic field (H_0) is increased.

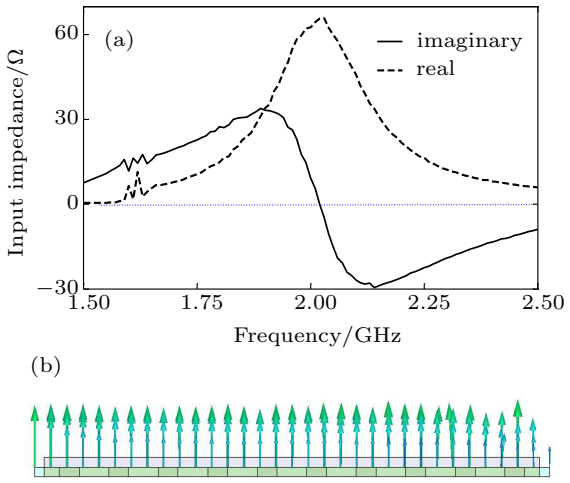


Fig. 3. (color online) Simulated results of the proposed ZOR in Fig. 1 when port 2 is open, showing (a) input impedance at port 1, and (b) the field distribution at resonant frequency, with the following parameters: $w = 3$ mm, $w_{\text{stub}} = 1$ mm, $l_{\text{stub}} = 4$ mm, $h = 0.5$ mm, the distance between stubs $l_{\text{cell}} = 2.5$ mm, $H_0 = 600$ Oe, $4\pi M_{\text{s}} = 0.173$ T, $\Delta H = 10$ Oe, and the number of stubs $N = 10$.

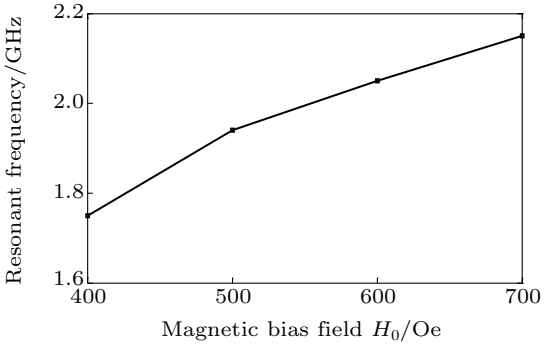


Fig. 4. Resonant frequency of the ZOR versus magnetic bias field.

4. Conclusions

In this paper, a ferrite-based composite right/left handed line is proposed. The main idea behind this structure is to achieve left-handed behavior by the combination of the negative permeability of ferrite layer and negative permittivity of shorted stubs. The numerical results show a balanced metamaterial behavior for this structure. As an application of the CRLH line proposed in this paper, a tunable ZOR is presented.

Compared with previous structure, the proposed tunable ZOR is easy to design and fabricate and does not require a chip component. Therefore, it can be a good candidate for design of microwave devices.

Appendix A

In this section, a microstrip line on a grounded ferrite substrate, magnetized in the direction perpendicular to the substrate, is considered.[11]

Generally, the sourceless Maxwell's equations for a ferrite-based microstrip line in Fig. A1 can be introduced as

follows:

$$\nabla \times \bar{E} = -\mathrm{j}\omega\mu_0\bar{\bar{\mu}}\bar{H}, \tag{A1}$$

$$\nabla \times \bar{H} = \mathrm{j}\omega\varepsilon\bar{E}, \tag{A2}$$

$$\nabla \cdot (\bar{\bar{\mu}}\bar{H}) = 0, \tag{A3}$$

$$\nabla \cdot \bar{E} = 0, \tag{A4}$$

where the permeability tensor of ferrite is given by

$$\bar{\bar{\mu}} = \begin{bmatrix} \mu & \mathrm{j}\mu_a & 0 \\ -\mathrm{j}\mu_a & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{A5}$$

with μ and μ_a given in Eq. (1c). Inserting Eq. (A5) in Eqs. (A1) and (A2) for Cartesian coordinate system yields:

$$-\frac{\partial E_z}{\partial y} + \frac{\partial E_y}{\partial z} = \mathrm{j}\omega\mu_0\mu H_x - \omega\mu_0\mu_a H_y, \tag{A6a}$$

$$-\frac{\partial E_x}{\partial z} + \frac{\partial E_z}{\partial x} = \omega\mu_0\mu_a H_x + \mathrm{j}\omega\mu_0\mu H_y, \tag{A6b}$$

$$-\frac{\partial E_y}{\partial x} + \frac{\partial E_x}{\partial y} = \mathrm{j}\omega\mu_0 H_z, \tag{A6c}$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \mathrm{j}\omega\varepsilon E_x, \tag{A6d}$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = \mathrm{j}\omega\varepsilon E_y, \tag{A6e}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = \mathrm{j}\omega\varepsilon E_z. \tag{A6f}$$

If we assume that the width of the microstrip (w) is much larger than the thickness of the ferrite substrate (h), we can neglect the variation of field along the z direction ($\partial/\partial z = 0$). In this case, the above equations can be split into two groups, one with E_z component:

$$-\frac{\partial E_z}{\partial y} = \mathrm{j}\omega\mu_0\mu H_x - \omega\mu_0\mu_a H_y, \tag{A7a}$$

$$\frac{\partial E_z}{\partial x} = \omega\mu_0\mu_a H_x + \mathrm{j}\omega\mu_0\mu H_y, \tag{A7b}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = \mathrm{j}\omega\varepsilon E_z, \tag{A7c}$$

and the other one with H_z component:

$$\frac{\partial H_z}{\partial y} = \mathrm{j}\omega\varepsilon E_x, \tag{A8a}$$

$$-\frac{\partial H_z}{\partial x} = \mathrm{j}\omega\varepsilon E_y, \tag{A8b}$$

$$-\frac{\partial E_y}{\partial x} + \frac{\partial E_x}{\partial y} = \mathrm{j}\omega\mu_0 H_z. \tag{A8c}$$

Solving Eqs. (A7a) and (A7b) for H_x and H_y and then inserting these two expressions into Eq. (A7c) yield

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \omega^2\varepsilon\mu_0\left(\mu - \frac{\mu_a^2}{\mu}\right)E_z = 0. \tag{A9}$$

Inserting Eqs. (A8a) and (A8b) into Eq. (A8c) yields

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \omega^2\varepsilon\mu_0 H_z = 0. \tag{A10}$$

Equations (A9) and (A10) represent TM_z and TE_z modes for structure in Fig. A1, where the z direction is the direction of magnetic bias.

Since the width of the microstrip (w) is much larger than the thickness of the ferrite substrate (h), most of the fields are confined into the substrate region between the two metals. So, the fringing field becomes less important and the walls of $x = 0$ and $x = w$ can be replaced by boundary perfect magnetic conductor (PMC). Based on the above assumptions, the problem of microstrip line in Fig. A1 is similar to that of a rectangular waveguide completely filled with ferrite, when the boundary conditions are

$$H_z\big|_{\substack{x=0\\x=w}} = 0, \quad H_y\big|_{\substack{x=0\\x=w}} = 0, \tag{A11a}$$

$$E_x\big|_{\substack{z=0\\z=h}} = 0, \quad E_y\big|_{\substack{z=0\\z=h}} = 0. \tag{A11b}$$

The TE_z mode does not satisfy the above boundary conditions, so the only TM_z mode propagates in the microstrip line.

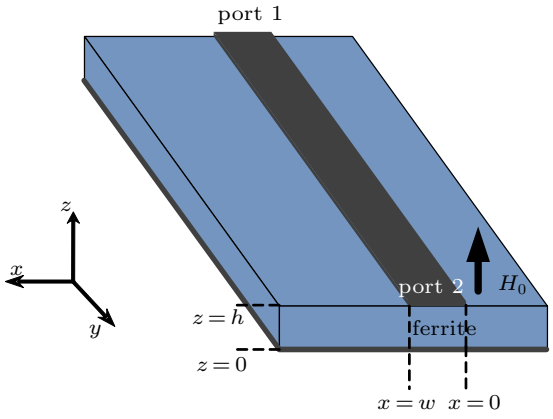


Fig. A1. A microstrip line on a grounded ferrite substrate, magnetized in the direction perpendicular to the substrate.

For the propagation in the y direction, the electric field of TM_z mode in Eq. (A9) has the general solution as follows:

$$E_z = [A\sin(k_x x) + B\cos(k_x x)]\,\mathrm{e}^{-\mathrm{j}\beta y}, \tag{A12a}$$

with

$$k_x^2 = \omega^2\mu_0\varepsilon\left(\mu - \frac{\mu_a^2}{\mu}\right) - \beta^2,$$

where A and B are the constants, and β is the propagation constant in the y direction. The H_x and H_y are obtained by inserting Eq. (A12a) into Eqs. (A7a) and (A7b) as

$$H_x = \frac{1}{\mathrm{j}\omega\mu_0(\mu^2 - \mu_a^2)}\left[\mathrm{j}\mu\beta E_z - \mathrm{j}\mu_a\frac{\partial E_z}{\partial x}\right], \tag{A12b}$$

$$H_y = \frac{1}{\mathrm{j}\omega\mu_0(\mu^2 - \mu_a^2)}\left[-\beta\mu_a E_z + \mu\frac{\partial E_z}{\partial x}\right]. \tag{A12c}$$

Appling the boundary conditions of Eq. (A11a) to the field expressions of Eq. (A12c) leads to two groups of modes. The modes of the first group are named edge-guided modes with the following propagation constant and electric and magnetic fields:

$$\beta = \pm\sqrt{\omega^2\varepsilon\mu_0\mu}, \tag{A13a}$$

$$\bar{E} = \mathrm{j}A\exp\left(\frac{\mu_a}{\mu}\beta x\right)\exp(-\mathrm{j}\beta y)\hat{a}_z, \tag{A13b}$$

$$\bar{H} = \frac{\mathrm{j}A\beta}{\omega\mu_0\mu}\exp\left(\frac{\mu_a}{\mu}\beta x\right)\exp(-\mathrm{j}\beta y)\hat{a}_x. \tag{A13c}$$

Based on Eqs. (A13b) and (A13c), the electric and magnetic fields of edge guided modes show exponential behaviors along the x direction.

The modes of the second group are named wave-guided modes with the following propagation constants and electric fields:

$$\beta = \pm\sqrt{\omega^2\varepsilon\mu_0\mu_\perp - \left(\frac{n\pi}{w}\right)^2}, \qquad n = 1,2,\ldots, \tag{A14a}$$

$$\bar{E} = A\left[\sin\left(\frac{n\pi}{w}x\right) + \frac{n\pi\mu}{w\mu_a\beta}\cos\left(\frac{n\pi}{w}x\right)\right]\exp(-\mathrm{j}\beta y)\hat{a}_z, \tag{A14b}$$

where $\mu_\perp$ is the effective permeability of these modes, given by:

$$\mu_\perp = \left(\mu - \frac{\mu_a^2}{\mu}\right). \tag{A15}$$

The magnetic fields of these modes are obtained by inserting Eq. (A14b) into Eqs. (A12b) and (A12c). Based on Eqs. (A14a) and (A14b), the wave-guided modes each have a cut-off frequency related to line width and show a sinusoidal behavior along the x direction.

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